NONLINEAR DYNAMIC BEHAVIOR OF A PORTAL FRAME UNDER SUPPORT EXCITATION

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Abstract. *We present a study of the nonlinear dynamic behavior of a simple portal frame excited by ground motion. Two vertical columns clamped in their bases supporting a horizontal beam pinned in both ends compose the structure. Nonlinearity is introduced by considering the elements shortening due to bending. A two-degrees-of-freedom mathematical model is derived via Lagrange's formulation leading to a set of two second order differential equations with quadratic nonlinearities. The geometry and mass distribution is chosen in such a way as to render a 1:2 relationship between the frequency of the first mode (sway mode), related with the lateral motion of the vertical columns, and the frequency of the second mode (symmetrical mode), related to the vertical motions of the horizontal beam axis. This ratio may cause internal resonance between those modes with possible energy transfer. Further, we impose harmonic support motions in the vertical and horizontal directions in near resonance with the modes. Several interesting nonlinear phenomena are observed. In one case, when the support excitation frequency is near the second frequency of the structure, as its amplitude is increased the vertical motion of the horizontal beam axis will increase accordingly up to a point when it stops growing, that is, saturates. At this point, the energy pumped into the system via the second mode is transferred to the first mode, not directly excited, causing large amplitude sway motions, potentially dangerous. These are the saturation and energy transference phenomena. In another case, when the support excitation frequency is near the first frequency of the structure, as its amplitude is increased again energy transfer between modes occurs in an intermittent fashion.*

Keywords*: Nonlinear dynamics, Structural Dynamics, Earthquake Engineering*

1. INTRODUCTION

It is well known that earthquakes are rare and of little importance in Brazil. Nevertheless, Brazilian engineers are more and more called to design structures in other Latin American countries along the Pacific Coast. Such countries are notoriously prone to strong seismic motions.

Earthquake engineering has always been a major motivation for research in structural dynamics, including nonlinear dynamics, due to elastoplastic behavior of structural materials and the geometric nonlinearities resultant of slender dimensions of members.

In this paper we study the nonlinear dynamic behavior of a portal frame represented by a two degree of freedom mathematical model related to the first (sway) mode and second (symmetrical) mode. Their frequencies are set to a 1:2 relationship that will cause internal resonance. Further, harmonic ground motions resonant with the second mode of the frame excite the structure.

The nonlinearities are due to the consideration of the shortening of the columns due to sway bending. The equations of motion are derived via a Lagrangian formulation and display quadratic nonlinearities, (Nayfeh and Mook, 1979) mathematically studied similar equations via the Multiple Scales Perturbations Method. The phenomena of mode saturation and energy transference between modes are observed. The research group the first author belongs has studied several framed structures under excitation of supported machines (Brasil and Mazzilli, 1990; Mazzilli and Brasil, 1995; Balthazar et all, 2003, 2004a, 2004b, 2005). Here we present a similar frame excited by ground motions.

2. THE MATHEMATICAL MODEL

The frame of Fig. 1 is composed by two vertical columns, of h height end *Ic* sectional moment of inertia, clamped at their bases, and a horizontal beam, of *L* length and *IB* sectional moment of inertia, pinned to the upper end of the columns. Lumped masses *m* are considered at the top of the columns and a lumped mass *M* at the center section of the beam. We adopt as generalized coordinates *q1* and *q2*, the horizontal and vertical motions of this center section of the beam, which are also related to the fist (sway) mode and the second (symmetrical) mode. In the calculation of the Total Potential Energy of the system, we consider the shortening of the columns due to the sway bending.

The displacements of the lumped masses are considered to be given by:

$$u_1 = u_2 = u_3 = q_1 \tag{1}$$

$$v_2 = q_2 \tag{2}$$

$$v_1 = v_3 = \frac{1}{2} C q_1^2 \tag{3}$$

The coefficient *C*, related to the column shortening can be computed by a Raylegh-Ritz procedure adopting a cubic shape function for the column bending in the sway mode. The resulting value is:

$$C = \frac{6}{5h} \tag{4}$$

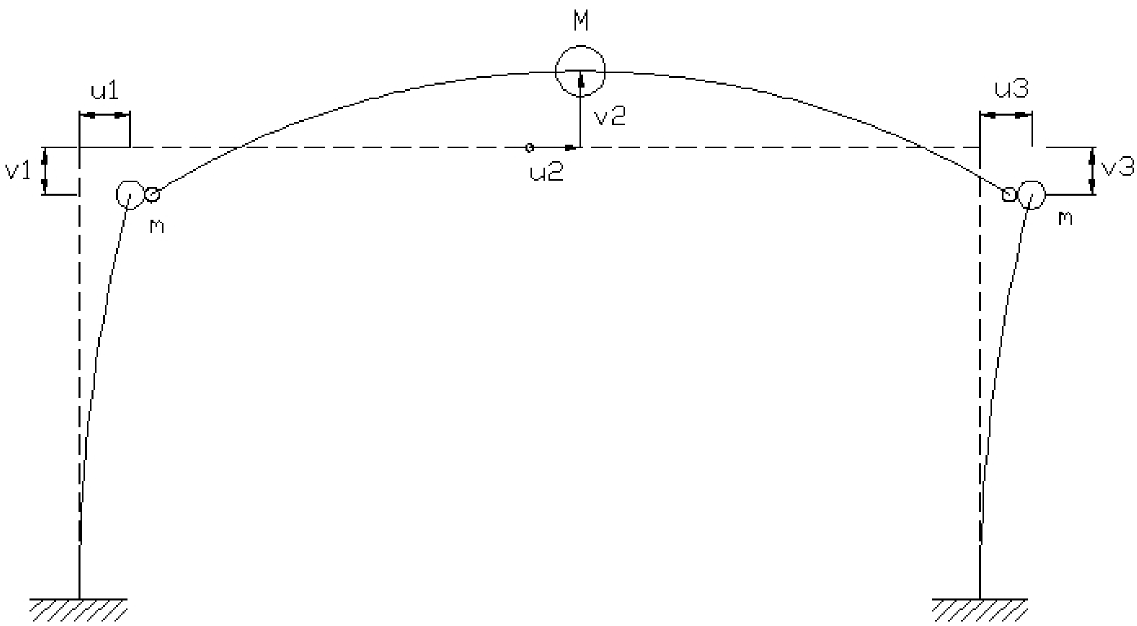

Figure 1: the model

Horizontal and vertical ground motions S_1 and S_2 are considered in the form:

$$S_1(t) = S_{10} \cos \Omega t \qquad S_2(t) = S_{20} \sin \Omega t \tag{5}$$

Following the Lagrangian procedure, we firs compute the Kinetic Energy

$$T = \frac{1}{2}\left\{2m\left[\left(\dot{q}_1 + \dot{S}_1\right)^2 + \dot{S}_2^2\right] + M\left[\left(\dot{q}_1 + \dot{S}_1\right)^2 + \left(\dot{q}_2 + \dot{S}_2\right)^2\right]\right\} \tag{6}$$

Next, the Strain Energy, considering shortening of columns due to sway bending

$$U = \frac{1}{2}\left[k_B\left(q_2 + \frac{1}{2}Cq_1^2\right)^2 + 2k_C\, q_1^2\right] \tag{7}$$

Now, we compute the Work of Conservative Forces also considering shortening of the columns

$$W = -Mg(S_2 + q_2) - 2mg(S_2 - \frac{1}{2}Cq_1^2) \tag{8}$$

The Total Potential Energy is

$$V = U - W \tag{9}$$

In the sequence, Lagrange's equations are applied

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) + \frac{\partial V}{\partial q_i} = N_i \tag{10}$$

The only non-conservative forces *Ni* considered are modal linear viscous damping, represented by damping coefficients *c1* and *c2* in the resulting the Equations of Motion, which, to quadratic terms, are:

$$\ddot{q}_1 + \frac{2(k_C - Cmg)}{2m+M} q_1 = -\frac{c_1}{2m+M}\dot{q}_1 - \frac{Ck_B}{2m+M} q_1 q_2 - \ddot{S}_1 \tag{11}$$

$$\ddot{q}_2 + \frac{k_B}{M} q_2 = -\frac{c_2}{M}\dot{q}_2 - \frac{Ck_B}{2M} q_1^{\ 2} - \ddot{S}_2 - g \tag{12}$$

where

$\omega_1^2 = \frac{2(k_C - Cmg)}{2m+M}$, frequency of the first mode (sway mode)

$\omega_2^2 = \frac{k_B}{M}$, frequency of the second mode (symmetric mode)

$$k_B = \frac{48EI_B}{L^3}$$

$$k_C = \frac{3EI_C}{h^3}$$

3. NUMERICAL SIMULATIONS

In this section, we present numerical simulations of the equations of motion, Eqs (11) and (12), that is, we numerically integrate those equations up to the steady state condition for variable excitation conditions and determine the response amplitude of the two coordinates.

The adopted numerical parameters are: E=2.05E+11, I_B=Ic= 1.08394E-09 m^4, M=1.849 kg, m=2.50 kg, h=1 m, L=1.98 m, k_B=1374.05 N/m, kc= 666.62 N/m, C=1.2, $2\omega_1$=27.26 rad/s, ω_2=27.26 rad/s.

As commented we first set the frequencies of the system as follows:

$$\Omega \approx \omega_2 \approx 2\omega_1 \tag{13}$$

In the first series of simulations, we plot in Fig. 2 the steady-state amplitude of vibrations of the two coordinates as function of the amplitude of the ground motion. In the beginning, we note that the second mode amplitude (shown as a dashed red line), related to the vertical motion of the center section of the beam, grows as the excitation amplitude also grows. The amplitude of the fist (sway) mode (shown as a solid blue line) is very small, almost zero. When a certain value of the excitation is reached, the amplitude of the second mode stops growing and the energy pumped into the system makes the sway mode start vibrating at large amplitude, potentially dangerous. This is the mode saturation and energy transfer phenomena.

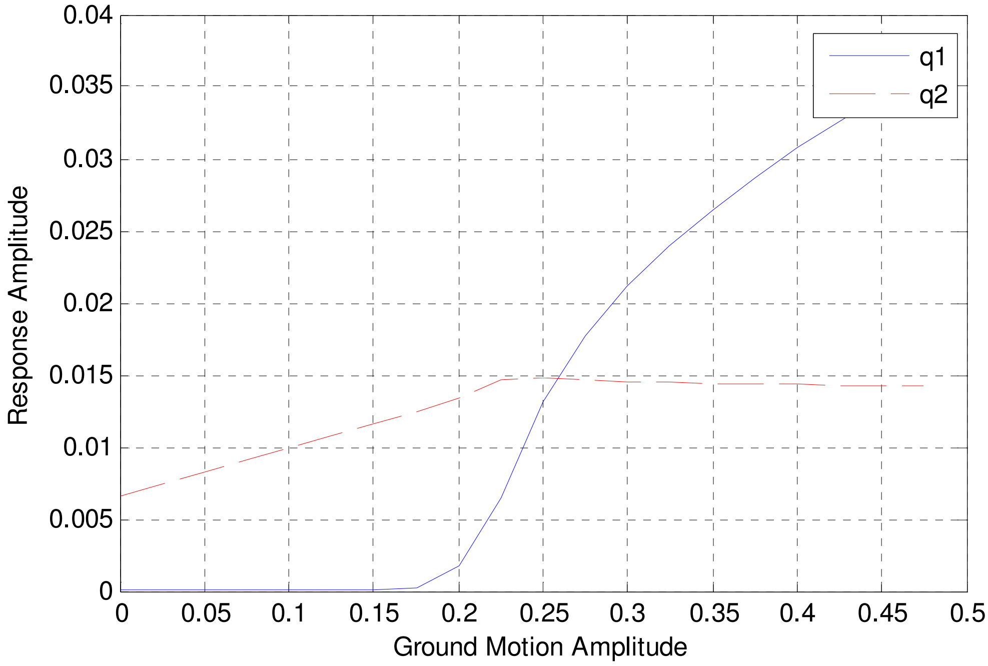

Figure 2:

In the second series of simulations, we plot in Fig. 3 the steady-state amplitude of vibrations of the two coordinates as function of the ground motion frequency near the resonance region of the second mode that is directly excited. We note that the response amplitude of this mode (shown as a dashed red line) will not grow as expected in the linear theory (shown as a dotted line) and the response amplitude of the first mode (shown as a solid blue line) that should be very small will grow considerably due to energy transfer from one mode to the other.

Next, we set the frequencies of the system as follows:

$$\Omega \approx \omega_1 \qquad \text{and} \qquad \omega_2 \approx 2\omega_1 \tag{14}$$

In the third series of simulations, we plot in Fig. 4 frequency response curves of the steady-state amplitude of vibrations of the two coordinates as function of the ground motion frequency near the resonance region of the first mode that is directly excited. We note that the response amplitude of this mode (shown as a solid blue line) will not grow as expected and the response amplitude of the second mode (shown as a dashed red line) that should be very small will grow considerably due to energy transfer from one mode to the other.

4. CONCLUSIONS

- we studied the nonlinear dynamic behavior of a framed structure under ground motion.
- the shortening of the columns due to bending is considered
- internal and external resonance conditions are enforced
- the mode saturation and energy transfer between modes was observed
- these phenomena are quite different from the expected behavior of linear models

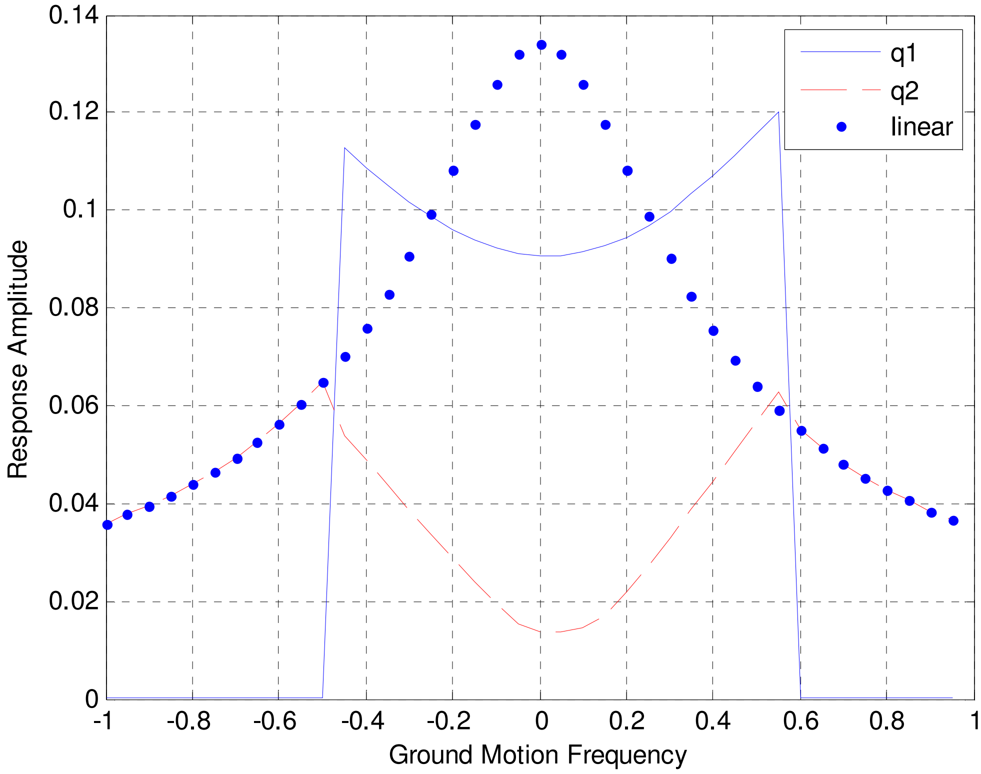

Figure 3: Frequency-response curves

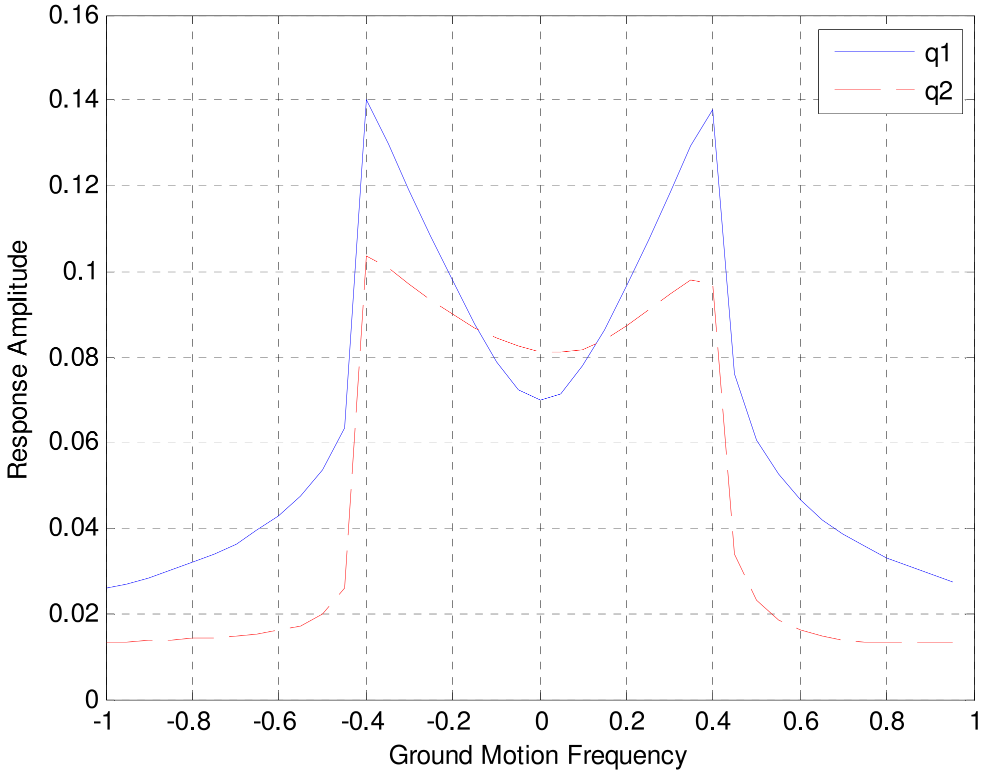

Figure 4: frequency-response curves

5. ACKNOWLEDGEMENTS

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