HYBRID SOLUTION OF THE MHD LAMINAR FLOW AND HEAT TRANSFER IN THE ENTRANCE REGION OF A PARALLEL-PLATE CHANNEL

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Abstract. *The study of the magnetohydrodynamic (MHD) flow and heat transfer has important applications in such devices as MHD generators, nuclear reactors, and in metallurgic and aluminium industries as well. Therefore, a hybrid solution is obtained through the so-called Generalized Integral Transform Technique (GITT) for the MHD flow with heat transfer of a Newtonian electrically conducting fluid in the entrance region of a parallel-plate channel. The flow, modeled through the boundary layer formulation, is sustained by a constant gradient pressure and the magnetic field is applied in a direction normal to flow. The magnetic Reynolds number is assumed to be small, thus permitting the normal magnetic field to be kept uniform and to remain much larger than any fields in the others coordinate directions. Hall and ion-slip effects are neglected. To evaluate the effects of the applied magnetic field on both entrance regions (flow and heat transfer), two types of inlet conditions are used: uniform and non-MHD fully-developed parabolic velocity profiles. Results for the velocity, temperature and related fields are computed within the main governing parameters, namely, Reynolds number, Hartmann number and electric field parameter, for typical situations. A convergence analysis is also performed showing the consistency of the results. In addition, the present results are confronted with those previously reported in the literature showing excellent agreements. Finally, due to its hybrid numerical-analytical nature, it is expected that the present methodology could be employed as an appropriate benchmarking tool in this kind of physics.*

Keywords*: Integral Transforms, Magnetohydrodynamic (MHD), Laminar Flow, Heat Transfer*

1. INTRODUCTION

Starting on the early twenty century, studies on magnetohydrodynamics (MHD), either experimentally or numerically, reappeared on sixties of the past century and, recently, has gained strong attention due main to energy needs and environmental issues. MHD pumps and generators, nuclear reactors cooling and reduction cells in aluminium industries are some examples of its applications (Shercliff, 1965; Davidson, 2001 and Sutton and Sherman, 2006). Just to mention the importance of the theme here studied, recently, important structural and CFD commercial packages, like ANSYS/CFX, are introducing in their numerical kernels some Maxwell equation solvers as beta versions that are automatically coupled to the structural and/or CFD solvers (ANSYS/CFX, 2009).

Normally, flow of an electrically conducting fluid inside channels is present in those applications and, therefore, has been considered by several researchers for typical situations. Initially, the interest was focused only on flow dynamics (Tao, 1960; Malashetty and Leela, 1992). Later, thermal effects were taken into account by studying the thermally developing flow under Hartman fully developed velocity profile and constant thermal properties (Nigam and Singh, 1960; Alpher, 1961). Finally, since some MHD devices (such as those in nuclear reactors) generally operate at high temperatures, the main focus on thermally developing flow moved to the study of the effects of variable transport properties on flow (Heywood, 1965; Rosa, 1971; Attia and Kob, 1996; Attia, 1999 and Lima *et al*., 2007).

Although a substantial improvement had been done in the understanding of the governing physics in a MHD heat and fluid flow channel problem through those previous works, it is known that flow within such MHD devices is seldom fully developed over its entire length, and large heat fluxes may occur at their entrance regions, quite apart from the variable thermal properties influence. Consequently, studies on hydrodynamic developing and imultaneously developing of MHD flows became the subject of many investigations for many years.

Shohet (1961) and Shohet *et al*. (1962) obtained numerical solutions of the hydrodynamic and thermal entry problems on a parallel-plate channel based on the finite difference scheme developed by Bodoia and Osterle (1961). Later, Hwang (1962) applied a similar numerical procedure (Hwang and Fan, 1963) and solved the simultaneously entry problem by considering a uniform velocity profile at the inlet channel.

Manohar (1966) developed an "exact" analysis of the same problem, based on a modified numerical method originally developed by Hartree (1949), where derivatives in the *x*-direction are replaced by finite-differences, while the other quantities are replaced by their averages. According to the author, the method employed is more accurate than the previous ones as they roughly correspond to the first iteration of its scheme.

Taking accounting of a parabolic velocity profile at the entry of the channel, Hwang *et al*. (1966) studied the hydrodynamic entry problem and compared their results with those of Maciulaitis and Loeffler Jr. (1964) that employed the approximate Karman-Pohlhausen integral method.

So far, all previous works on entry problem were effectuated by considering constant transport properties. Recently, Setayesh and Sahai (1990) extended the simultaneously developing flow and heat transfer problems, showing that, under certain circumstances, variation on transport properties with temperature have a significant influence on the development of both velocity and temperature profiles.

As a result of the necessity of analytical analysis on this theme, the main goal of the present work is to develop hybrid solutions, through application of the so-called Generalized Integral Transform Technique - GITT (Cotta, 1993; Cotta and Mikhailov, 1997; Cotta, 1998 and Santos *et al*., 2001), for the simultaneously developing MHD flow and heat transfer in a channel of plane-parallel plates. At light of its hybrid nature, it is expected the GITT approach is going to be a proper benchmarking methodology in this field of research, as will be clear later in the results discussion.

2. MATHEMATICAL FORMULATION

It is considered here the steady-state simultaneously developing of the MHD flow and heat transfer, of a Newtonian, electrically conducting, incompressible fluid, within a parallel-plate channel of height $h = 2b$. Fluid, at uniform inlet temperature, T_e, enters the channel under uniform and parallel flow condition or under a parabolic profile, which is the velocity profile of a non-MHD fully developed laminar flow. The two semi-infinite plates could be at different temperatures, T_{w1} and T_{w2} (this is not done in this work, i.e., $T_{w1} = T_{w2}$). By considering viscous dissipation, and analyzing the problem from wall to wall, the governing boundary layer equations are written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad ; \qquad 0<y<1, \quad x>0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left[\mu\frac{\partial u}{\partial y}\right]-Ha^2\sigma\left(E_z+u\right) \quad ; \qquad 0<y<1, \quad x>0 \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right]+Ec\left(\mu\frac{\partial u}{\partial y}\right)^2+EcHa^2\sigma\left(E_z+u\right)^2 \quad ; \qquad 0<y<1, \quad x>0 \tag{3}$$

These equations are submitted to the inlet and boundary conditions:

$$\left.\begin{array}{l} u(0,y)=u_e(\mathrm{y}) \\ v(0,y)=0 \\ T(0,y)=T_e(\mathrm{y}) \end{array}\right\} \quad ; \quad x=0, \quad 0<y<1 \tag{4-6}$$

$$\left.\begin{array}{l} u(x,0)=0 \\ v(x,0)=0 \\ T(x,0)=0 \end{array}\right\}, \quad y=0, \quad x>0 \quad ; \qquad \left.\begin{array}{l} u(x,1)=0 \\ v(x,1)=0 \\ T(x,1)=\theta_{w2}=0 \end{array}\right\}, \quad y=1, \quad x>0 \tag{7-12}$$

In the above formulation, the following dimensionless groups were employed:

$$x=\frac{\mu_{w1}}{h^2\rho\bar{u}_0}x^*, \quad y=\frac{y^*}{h}, \quad u=\frac{u^*}{\bar{u}_0}, \quad v=\frac{h\rho}{\mu_{w1}}v^*, \quad p=\frac{p^*-p_0}{\rho\bar{u}_0^2},$$
$$\mu=\frac{\mu^*}{\mu_{w1}}, \quad \sigma=\frac{\sigma^*}{\sigma_{w1}}, \quad k=\frac{k^*}{k_{w1}}, \quad T=\frac{T^*-T_{w1}}{T_e-T_{w1}}, \quad \theta_{w2}=\frac{T_{w2}-T_{w1}}{T_e-T_{w1}} \tag{13}$$
$$E_z=\frac{E_z^*}{\bar{u}_0B_0}, \quad \Pr=\frac{\mu_{w1}c_p}{k_{w1}}, \quad E_c=\frac{\bar{u}_0^2}{c_p\left(T_e-T_{w1}\right)}, \quad Ha=B_0h\left(\frac{\sigma_{w1}}{\mu_{w1}}\right)^{1/2}$$

3. SOLUTION METHODOLOGY

3.1. Streamfunction-only Formulation

For two-dimensional channels, and following orientation of previous works that employed the GITT approach (Santos *et al.*, 2001), it is numerically more advantageous use the streamfunction as the solution variable. Therefore, from its definition, Eqs. (1) to (12) are re-written in the streamfunction-only formulation as:

$$\frac{\partial\psi}{\partial y}\frac{\partial^3\psi}{\partial x\partial y^2}-\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial y^3}=\frac{\partial^4\psi}{\partial y^4}-Ha^2\frac{\partial^2\psi}{\partial y^2} \quad ; \qquad 0<y<1, \quad x>0 \tag{14}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}+E_c\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+E_cHa^2\left(Ez+\frac{\partial\psi}{\partial y}\right)^2 \quad ; \qquad 0<y<1, \quad x>0 \tag{15}$$

These equations are submitted to the inlet and boundary conditions:

$$\left.\begin{array}{l}\psi(0,y)=\psi_0(y)\\ \left.\dfrac{\partial\psi}{\partial x}\right|_{x=0}=0\\ T(0,y)=T_e(y)\end{array}\right\} \quad ; \quad x=0, \quad 0<y<1 \tag{16-18}$$

$$\left.\begin{array}{l}\psi(x,0)=0\\ \left.\dfrac{\partial\psi}{\partial y}\right|_{y=0}=0\\ T(x,0)=0\end{array}\right\}, \quad y=0, \quad x>0 \quad ; \qquad \left.\begin{array}{l}\psi(x,1)=1\\ \left.\dfrac{\partial\psi}{\partial y}\right|_{y=1}=0\\ T(x,1)=0\end{array}\right\}, \quad y=1, \quad x>0 \tag{19-24}$$

To improve the GITT performance further, the boundary condition at the upper plate, Eq. (22), should be made homogeneous. This is done through a filtering process, where the original potential, $\psi(x,y)$, is splited up in a filtered potential, $\phi(x,y)$, plus a filtering potential, $\psi_F(y)$, which could be any fully developed profile related to the original problem. In this work, it was employed the exact fully developed flow profile. These can be written as:

$$\psi(x,y)=\phi(x,y)+\psi_F(y) \quad ; \qquad \psi_F(y)=\begin{cases}3y^2-2y^3 & ;\ Ha=0\\ \dfrac{Ha\,\cosh\left[\dfrac{Ha}{2}y\right]-\sinh\left[\dfrac{Ha}{2}\right]+\sinh\left[\dfrac{Ha}{2}(1-2y)\right]}{Ha\,\cosh\left[\dfrac{Ha}{2}\right]-2\sinh\left[\dfrac{Ha}{2}\right]} & ;\ Ha\neq 0\end{cases} \tag{25, 26}$$

3.2. Eigenvalue Problems

Now, after establishing a proper formulation, it must be chosen auxiliar eigenvalue problems that form the basis for the integral transformation process. Such eigenvalue problems are obtained from homogeneous versions of the original problems (Perez-Guerrero, 1993; Santos *et al.*, 2001). The following eigenvalue problems were chosen,

a) For streafunction field:

$$\frac{d^4\tilde{Y}_i(y)}{dy^4}=\mu_i^4\,\tilde{Y}_i(y) \ , \qquad 0<y<1 \tag{27}$$

$$\left\{\begin{array}{l}\tilde{Y}_i(0)=0\\ \left.\dfrac{d\tilde{Y}_i}{dy}\right|_{y=0}=0\end{array}\right. \ ; \qquad \left\{\begin{array}{l}\tilde{Y}_i(1)=0\\ \left.\dfrac{d\tilde{Y}_i}{dy}\right|_{y=1}=0\end{array}\right. \tag{28-31}$$

The solution of the eigenproblem, Eqs. (27) to (31) is specified as:

$$\tilde{Y}_i(y)=\begin{cases}\dfrac{\sin\left(\mu_i\left[y-\frac{1}{2}\right]\right)}{\sin\left(\frac{\mu_i}{2}\right)}-\dfrac{\sinh\left(\mu_i\left[y-\frac{1}{2}\right]\right)}{\sinh\left(\frac{\mu_i}{2}\right)}\ ; & i=2,\ 4,\ 6,\ \dots\\[2ex] \dfrac{\cos\left(\mu_i\left[y-\frac{1}{2}\right]\right)}{\cos\left(\frac{\mu_i}{2}\right)}-\dfrac{\cosh\left(\mu_i\left[y-\frac{1}{2}\right]\right)}{\cosh\left(\frac{\mu_i}{2}\right)}\ ; & i=1,\ 3,\ 5,\ \dots\end{cases}$$ Eigenfunctions (32)

$$\begin{cases}\cosh(\mu_i)\cos(\mu_i)=1\\ N_i=\int_0^1\tilde{Y}_i^2(y)=1\end{cases}$$ Eigenvalues/norm (33, 34)

$$\begin{cases}\bar{\phi}_i(x)=\int_0^1\tilde{Y}_i(y)\,\phi(x,y)\,dy\\ \phi(x,y)=\sum_{i=1}^{\infty}\tilde{Y}_i(y)\,\bar{\phi}_i(x)\end{cases}$$ Integral transform/inverse pair (35, 36)

b) For temperature field:

$$\frac{d^2C_i(y)}{dy^2}=-\beta_i^2\,C_i(y)\ ,\qquad 0<y<1 \tag{37}$$

$$C_i(0)=0\ ;\qquad C_i(1)=0 \tag{38, 39}$$

Its solution is given as:

$$\tilde{C}_i(y)=\frac{C_i(y)}{M_i^{1/2}}=\frac{1}{M_i^{1/2}}\sin(\beta_i y)$$ Eigenfunctions (40)

$$\begin{cases}\beta_i=i\pi\\ M_i=\int_0^1 C_i^2(y)=\frac{1}{2}\end{cases}$$ Eigenvalues/norm (41, 42)

$$\begin{cases}\bar{T}_i(x)=\int_0^1\tilde{C}_i(y)\,T(x,y)\,dy\\ T(x,y)=\sum_{i=1}^{\infty}\tilde{C}_i(y)\,\bar{T}_i(x)\end{cases}$$ Integral transform/inverse pair (43, 44)

Finally, by considering the filtering process and the eigenfunctions orthogonality properties, integration of Eqs. (14) and (15), according to integral transform and inverse formulae, yields the following coupled system of infinity ordinary differential equations, in *x* direction:

$$\sum_{k=1}^{\infty}A_{ik}(x)\frac{d\bar{\phi}_k(x)}{dx}\ =\ B_{i\phi}(x)\ ,\qquad i=1,2,3,\dots,\infty \tag{45}$$

$$\sum_{k=1}^{\infty}G_{ik}(x)\frac{d\bar{T}_k(x)}{dx}\ -\ \sum_{j=1}^{\infty}H_{ij}(x)\frac{d\bar{\phi}_j(x)}{dx}\ =\ B_{iT}(x)\ ,\qquad i=1,2,3,\dots,\infty \tag{46}$$

This coupled system is submitted to the following integral transformed initial conditions:

$$\bar{\phi}_i(0) = \int_0^1 \tilde{Y}_i(y)\left[\psi_0(y) - \psi_F(y)\right] dy \ , \qquad \text{i = 1, 2, 3, ..., } \infty \tag{47}$$

$$\bar{T}_i(0) = \int_0^1 \tilde{C}_i(y) T_e(y)\, dy \ , \qquad \text{i = 1, 2, 3, ..., } \infty \tag{48}$$

The above coefficients, resulting from the integral transformation process, are straightforward defined as:

$$A_{ik}(x) = \int_0^1 \tilde{\mathrm{Y}}_{\mathrm{i}}(y)\left\{\tilde{\mathrm{Y}}_{\mathrm{k}}''(y)\frac{\partial\psi(x,y)}{\partial y} - \tilde{\mathrm{Y}}_k(y)\frac{\partial^3\psi(x,y)}{\partial y^3}\right\} dy \tag{49}$$

$$B_{i\phi}(x) = \int_0^1 \left\{\tilde{\mathrm{Y}}_{\mathrm{i}}''(y) - \tilde{\mathrm{Y}}_{\mathrm{i}}(y)\, Ha^2 \frac{\partial^2\psi(x,y)}{\partial y^2}\right\} dy \tag{50}$$

$$G_{ik}(x) = \int_0^1 \tilde{\mathrm{C}}_{\mathrm{i}}(y)\tilde{\mathrm{C}}_{\mathrm{k}}(y)\frac{\partial\psi(x,y)}{\partial y} dy \tag{51}$$

$$H_{ij}(x) = \int_0^1 \tilde{\mathrm{C}}_{\mathrm{i}}(y)\,\tilde{\mathrm{Y}}_{\mathrm{j}}(y)\frac{\partial T(x,y)}{\partial y} dy \tag{52}$$

$$B_{iT}(x) = \int_0^1 \left\{-\frac{1}{\mathrm{Pr}}\tilde{\mathrm{C}}_i'(y)\frac{\partial T(x,y)}{\partial \mathrm{y}} + E_c\,\tilde{\mathrm{C}}_{\mathrm{i}}(y)\left[\left(\frac{\partial^2\psi(x,y)}{\partial y^2}\right)^2 + Ha^2\left(E_z + \frac{\partial\psi(x,y)}{\partial y}\right)^2\right]\right\} dy \tag{53}$$

As a first attempt to progress towards a full numerical resolution approach, and in this way, looking for a hybrid methodology in a generalized and unified way (for unified code purposes), none of the above integral coefficients, which could be in the present problem, were analytically obtained. Those coefficients are obtained through subroutine DFEJER from IMSL (1991), which uses Fejer quadratures for numerical integration.

The main aim is to make available a unified GITT approach that can be employed for all kinds of problems in a straightforward way. The robustness of this procedure will be verified in problems where non-linearities are going increasing more and more. Depending on the non-linearity degree, the number of points of quadratures, *NQR*, should be further verified.

4. RESULTS

To solve the coupled system given by Eqs. (45) to (48), a Fortran 90 program was written and implemented on a two-processor 3.0 GHz Intel Xeon computer. In order to obtain numerical results, the expansions were truncated to finite orders $N\phi$ and NT, and a relative error criterion target of 10^{-8} was imposed to subroutine DIVPAG from IMSL (1991), which is appropriate to solve stiff systems of ordinary differential equations like that. Otherwise explicitly specified, all results are computed by using $N = N\phi = NT = 300$. Also, for the present problem, it was adopted a large number of quadrature points, $NQR = 1000$, to guarantee a full convergence of the integral coefficients for all cases simulated, independently of the main governing parameter, the Hartman number, although with just $NQR = 300$ all results were already sufficiently converged.

Results for the main potentials, as longitudinal velocity component, wall velocity gradient (friction factor), bulk temperature and Nusselt number are illustrated in graphical and tabulated forms, for different entry conditions and different governing parameters, namely Hartman, Prandtl, Eckert and electric field numbers.

First of all, a convergence study should be done for some difficult to converge case, in order to qualify the hybrid methodology adopted as a benchmarking tool. Consequently, Tabs. (1) and (2) bring convergence rate behaviors for the centerline velocity, wall velocity gradient, bulk temperature, and mean Nusselt number, at different axial positions, considering uniform and parabolic hydrodynamic inlet conditions, respectively. For Fig. (1), results are illustrated by considering a case where $Ha = 50$, $Pr = 1.0$, $E_c = 0.0$ and $E_z = 0.0$ (no viscous dissipation and open-circuited channel), while for Fig. (2), it was employed $Ha = 20$, $Pr = 0.75$, $E_c = 0.1$ and $E_z = -0.5$ (with viscous dissipation, short-circuited channel). These values are taken according to Wang *et al*. (1966) and Setayesh and Sahai (1990), respectively. Though only hydrodynamic simulations were done by Wang *et al*. (1966), the thermal results are included since the present code was developed in a coupled form.

Table 1. Convergence behavior of the main flow and heat transfer potentials, for different axial positions.
(Ha = 8, Pr = 1.0, E_c = 0.0, E_z = 0.0 and non-MHD parabolic inlet velocity profile)

N		10	100	200	250	300	10	100	200	250	300
x	x_{DH}	$u_c(x)$					$\left.\frac{\partial u(x,y)}{\partial y}\right\|_{y=0}$				
0.001	*0.0375*	1.4648	1.4649	1.4649	1.4649	**1.4649**	8.09824	8.12571	8.12506	8.12497	**8.12492**
0.01	*0.3750*	1.3368	1.3368	1.3368	1.3368	**1.3368**	10.2582	10.2531	10.2530	10.2530	**10.2530**
0.02	0.7500	1.2977	1.2977	1.2977	1.2977	**1.2977**	10.5696	10.5684	10.5684	10.5684	**10.5684**
0.025	0.9375	1.2910	1.2910	1.2910	1.2910	**1.2910**	10.6144	10.6138	10.6138	10.6138	**10.6138**
0.05	1.8750	1.2844	1.2844	1.2844	1.2844	**1.2844**	10.6558	10.6558	10.6558	10.6558	**10.6558**
0.075	2.8125	1.2842	1.2842	1.2842	1.2842	**1.2842**	10.6571	10.6571	10.6571	10.6571	**10.6571**
0.1	3.7500	1.2842	1.2842	1.2842	1.2842	**1.2842**	10.6571	10.6571	10.6571	10.6571	**10.6571**
2.0	75.000	1.2842	1.2842	1.2842	1.2842	**1.2842**[a]	10.6571	10.6571	10.6571	10.6571	**10.6571**[b]
x	x_{DH}	$T_b(x)$					$Nu_m(x)$				
0.001	*0.0375*	0.97241	0.96977	0.96977	0.96977	**0.96977**	31.349	31.763	33.246	34.006	34.766
0.01	*0.3750*	0.85745	0.85685	0.85685	0.85685	**0.85685**	16.064	15.563	15.721	15.802	15.882
0.02	0.7500	0.77484	0.77437	0.77437	0.77437	**0.77437**	13.134	12.846	12.929	12.971	13.014
0.025	0.9375	0.73990	0.73948	0.73948	0.73948	**0.73948**	12.363	12.122	12.190	12.224	12.259
0.05	1.8750	0.59727	0.59695	0.59695	0.59695	**0.59695**	10.488	10.347	10.384	10.403	10.423
0.075	2.8125	0.48614	0.48589	0.48589	0.48589	**0.48589**	9.7515	9.6451	9.6724	9.6864	9.7004
0.1	3.7500	0.39619	0.39598	0.39598	0.39598	**0.39598**	9.3714	9.2822	9.3046	9.3160	9.3275
2.0	75.000	0.00000	0.00000	0.00000	0.00000	**0.00000**	8.2998	8.2286	8.5035	8.1654	8.3541

[a] Exact values from fully developed velocity equation: $u_c(x) = 1.2842$, $\left.\frac{\partial u(x,y)}{\partial y}\right|_{y=0} = 2*5.32855 = 10.6571$

[b] Hwang *et al.* (1960), $u_c(x) = 1.2862$, $\left.\frac{\partial u(x,y)}{\partial y}\right|_{y=0} = 2*5.2563 = 10.5126$

Table 2. Convergence behavior of the main flow and heat transfer potentials, for different axial positions.
(Ha = 20, Pr = 0.75, E_c = 0.1 and E_z = - 0.5 and uniform inlet velocity profile)

N		10	100	200	250	300	10	100	200	250	300
x	x_{DH}	$u_c(x)$					$\left.\frac{\partial u(x,y)}{\partial y}\right\|_{y=0}$				
0.001	*0.0375*	1.0568	1.0784	1.0789	1.0790	**1.0790**	27.4783	24.0650	24.0224	24.0163	**24.0127**
0.01	*0.3750*	1.1103	1.1106	1.1106	1.1106	**1.1106**	22.2267	22.2251	22.2251	22.2251	**22.2251**
0.02	0.7500	1.1110	1.1110	1.1110	1.1110	**1.1110**	22.2223	22.2222	22.2222	22.2222	**22.2222**
0.025	0.9375	1.1110	1.1110	1.1110	1.1110	**1.1110**	22.2222	22.2222	22.2222	22.2222	**22.2222**
0.05	1.8750	1.1110	1.1110	1.1110	1.1110	**1.1110**	22.2222	22.2222	22.2222	22.2222	**22.2222**
0.075	2.8125	1.1110	1.1110	1.1110	1.1110	**1.1110**	22.2222	22.2222	22.2222	22.2222	**22.2222**
0.1	3.7500	1.1110	1.1110	1.1110	1.1110	**1.1110**	22.2222	22.2222	22.2222	22.2222	**22.2222**
2.0	75.000	1.1110	1.1110	1.1110	1.1110	**1.1110**[a]	22.2222	22.2222	22.2222	22.2222	**22.2222**[b]
x	x_{DH}	$T_b(x)$					$Nu_m(x)$				
0.001	*0.0375*	0.95430	0.95585	0.95600	0.95604	**0.95606**	32.207	46.599	47.011	47.757	48.481
0.01	*0.3750*	0.90047	0.90214	0.90224	0.90226	**0.90227**	16.985	19.129	19.260	19.355	19.442
0.02	0.7500	0.89358	0.89515	0.89522	0.89524	**0.89525**	13.947	15.507	15.620	15.678	15.728
0.025	0.9375	0.89458	0.89608	0.89615	0.89617	**0.89617**	13.174	14.619	14.729	14.778	14.821
0.05	1.8750	0.90822	0.90936	0.90941	0.90942	**0.90943**	11.421	12.633	12.734	12.769	12.796
0.075	2.8125	0.92105	0.92191	0.92195	0.92195	**0.92196**	10.801	11.933	12.031	12.060	12.082
0.1	3.7500	0.93073	0.93138	0.93140	0.93141	**0.93141**	10.501	11.590	11.687	11.713	11.732
2.0	75.000	0.95857	0.95862	0.95862	0.95862	**0.95862**	9.9333	10.898	10.989	11.007	11.020

[a] Exact values from fully developed velocity equation: $u_c(x) = 1.1110$, $\left.\frac{\partial u(x,y)}{\partial y}\right|_{y=0} = 2*11.1111 = 22.2222$

[b] Hwang *et al.* (1960), $u_c(x) = 1.1123$, $\left.\frac{\partial u(x,y)}{\partial y}\right|_{y=0} = 2*10.3810 = 20.7620$

As one can easily see, from the lowest truncation order columns on Tabs. (1) and (2), the filtering process is an extremely efficient and interesting analytical tool, as the limiting fully developed velocity and gradient velocity profiles are automatically recovered for axial position far from the entry channel. Clearly, at least three significant digits are already converged for N = 10, even for the parabolic velocity profile at axial positions very near the channel inlet. The worst convergence rates are those of Nusselt number, since it does depend on velocity potential, which should be already fully converged.

At first glance, it appears that, at least for the range of parameters illustrated, the velocity inlet condition exerts little influence on convergence rates. However, it could exert some influence on heat transfer behavior. Consequently, to make a proper comparison, the same situation illustrated at Tab. (1) is re-evaluated by considering an uniform velocity profile at the channel inlet. Figures (1a) and (1b) bring comparisons for the bulk temperature and mean Nusselt number, respectively, for both uniform and parabolic inlet profiles.

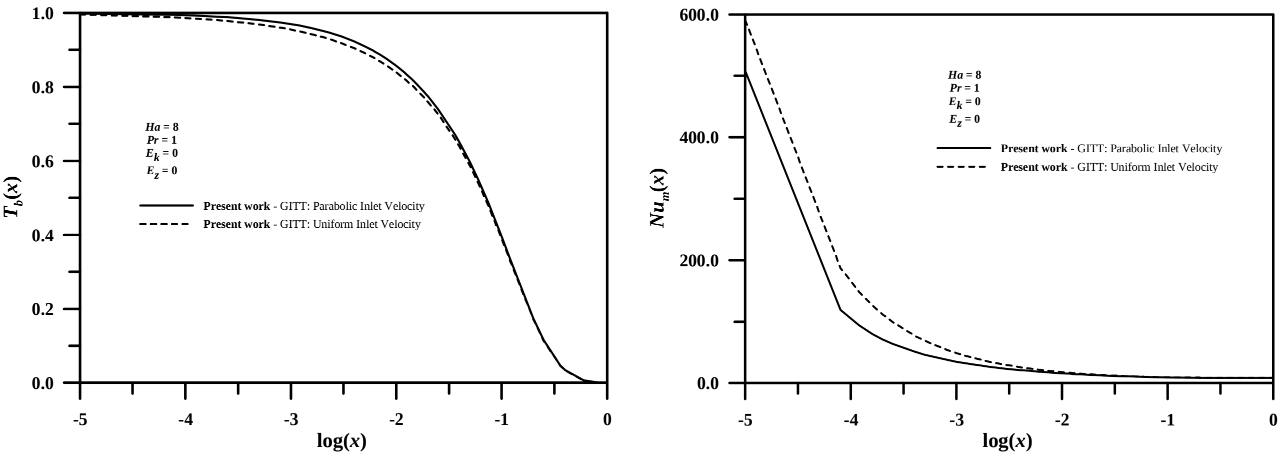

Figure 1. Effect of inlet velocity profile on: (a) Bulk mean temperature and (b) Average Nusselt number, for Ha = 8

As it was expected, one of the effects of a magnetic field (Hartman number) on flow is to flatten the velocity profile. Therefore, flow with a non-MHD fully developed profile (parabolic) at the entry, in general, requires a few longer entrance regions than flow with uniform velocity profile at the entry, so, the inlet condition exert some influence on heat transfer.

Figure (2a) and (2b) bring the development of the longitudinal velocity component, for various values of the transversal coordinate y, illustrating comparisons with numerical finite difference results due to Wang *et al*. (1966), for Hartman numbers Ha = 8 and 20, respectively. A parabolic velocity profile is employed in these cases.

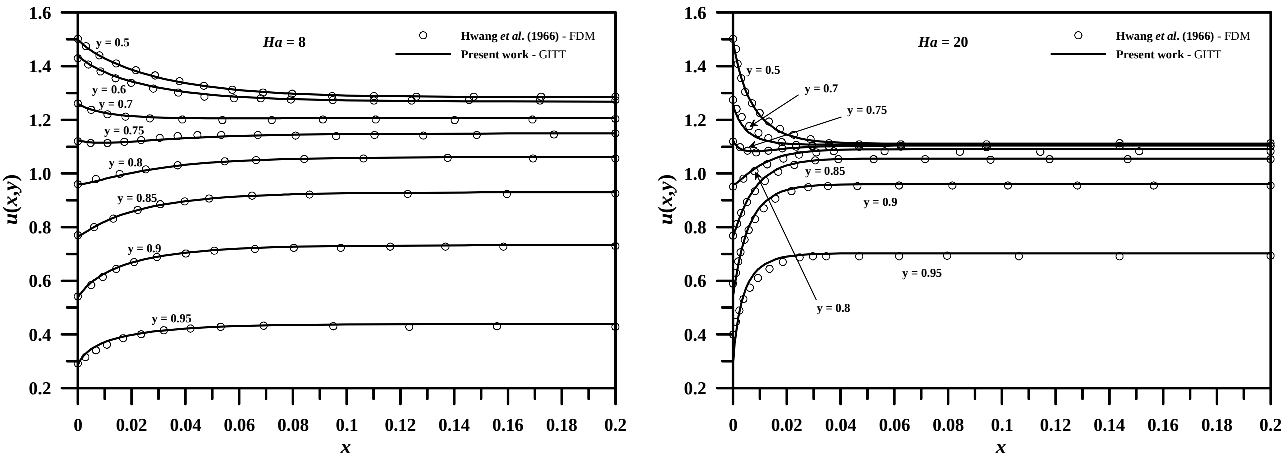

Figure 2. Effect of magnetic field on flow development: comparison with numerical results for different transversal positions: (a) Ha = 8, (b) Ha = 20

To give a better insight on this behavior, Fig. (3) illustrates the development of the longitudinal velocity component profile, for various values of the longitudinal *x*, bringing comparisons with numerical finite difference results due to Hwang *et al*. (1966), for Hartman numbers *Ha* = 0, 8, 20 and 100. Results for *Ha* = 0 are showed to verify the stability of the hybrid approach even in situations where no numerical evaluation needs to be performed.

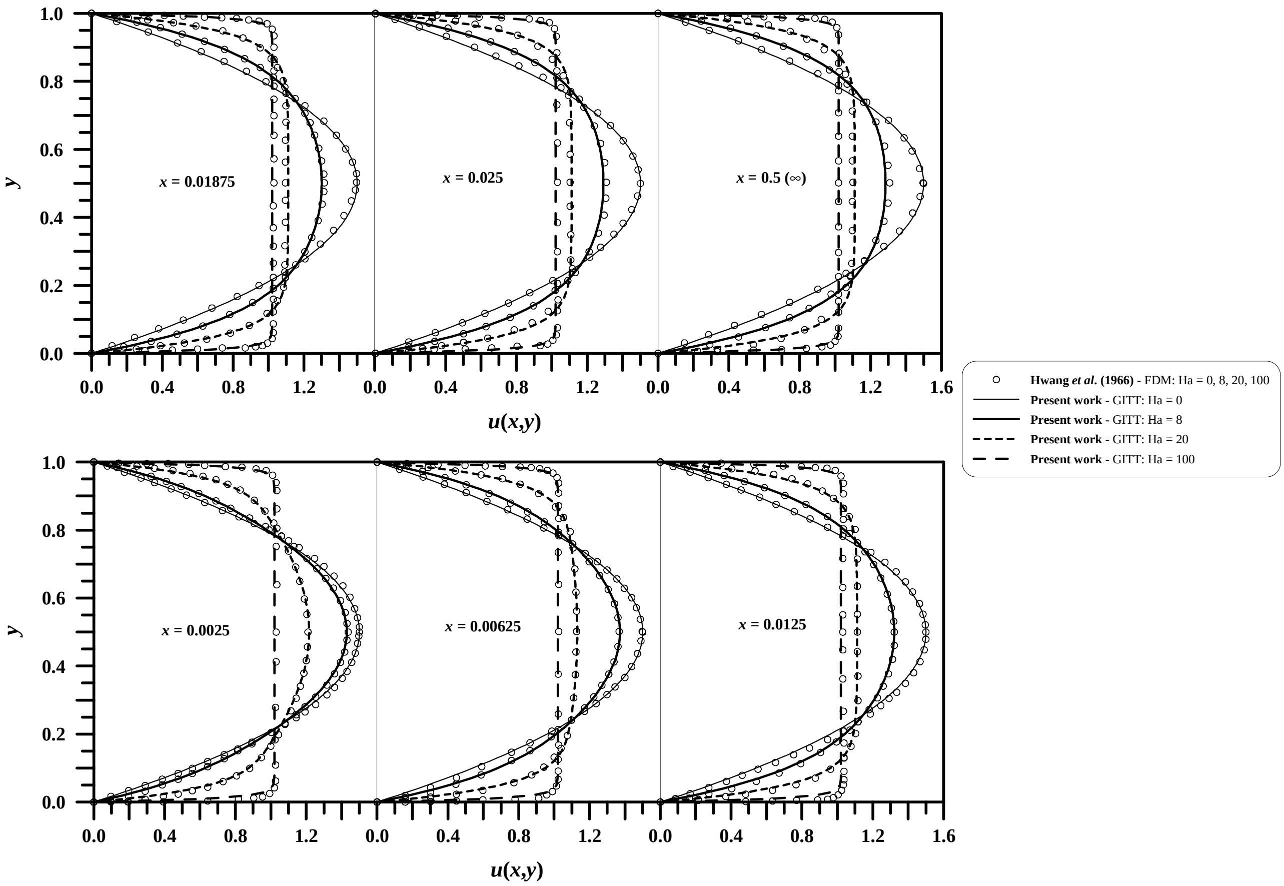

Figure 3. Effect of magnetic field on longitudinal velocity profile development: comparisons with numerical results for Ha = 0, 8, 20 and 100

Results illustrated in Figs. (2) and (3) confirm the flatness effect of the magnetic field (Hartman number) on flow development. Also, the present results are in excellent agreement with those of Hwang *et al*. (1966). As *Ha* is increased, the velocity in the center portion of the channel (y = 0.5) decreases. Also, as commented before, the entrance length becomes shorter. This is due to the overall retarding effect of the Lorentz force ($\mathbf{J} \times \mathbf{B}$).

Similar numerical computations, now accounting for the simultaneously hydrodynamic and thermal development, were developed by Shohet *et al*. (1962) and Setayesh and Sahai (1990).

After redefining the dimensionless groups of Shohet *et al*. (1962), mainly those related to the temperature field, the longitudinal velocity and temperature developments are illustrated, and additionally compared, on the following Figs. (4a) and (4b) for Ha = 8, Pr = 0.1, E_c = 1.0 and E_z = 0. The hydrodynamic results are also the same as those presented by Manohar (1966).

These figures show an excellent agreement between numerical and hybrid results. A closer look at these figures also shows a slightly better agreement of the present results with the numerical results of Manohar (1966), who used a higher order finite difference scheme in the *x*-coordinate than those of Shohet *et al*. (1962).

Figures (5a) and (5b) make a comparison between the present results and the most recent numerical results of Setayesh and Sahai (1990. Different field parameter and Eckert number are considered, and their influence on flow and heat transfer developments is analyzed.

Figure (5a) brings a comparison for the bulk mean temperature and the average Nusselt number, considering a situation where, Ha = 20, Pr = 0.75, E_c = 1 and E_z = - 0.5. To make a deeper study, Fig. (5b) illustrates the average Nusselt number behavior development for two different electric field parameters, (E_z = 0.0 and -1,0) and Eckert number (E_c = 0.1 and 1.0), maintaining constant the Hartmann, Ha = 20, and Prantl, Pr = 1.0, numbers.

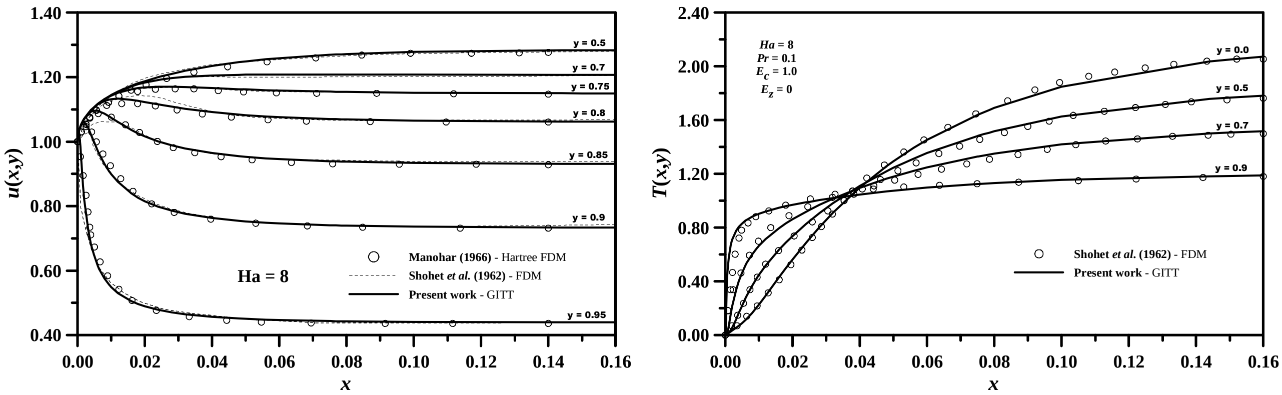

Figure 4. Development behavior of the (a) Longitudinal velocity and (b) Temperature, for different transversal coordinates. Comparisons with other numerical results for Ha = 8, Pr = 0.1, E_c = 1.0 and E_z = 0.

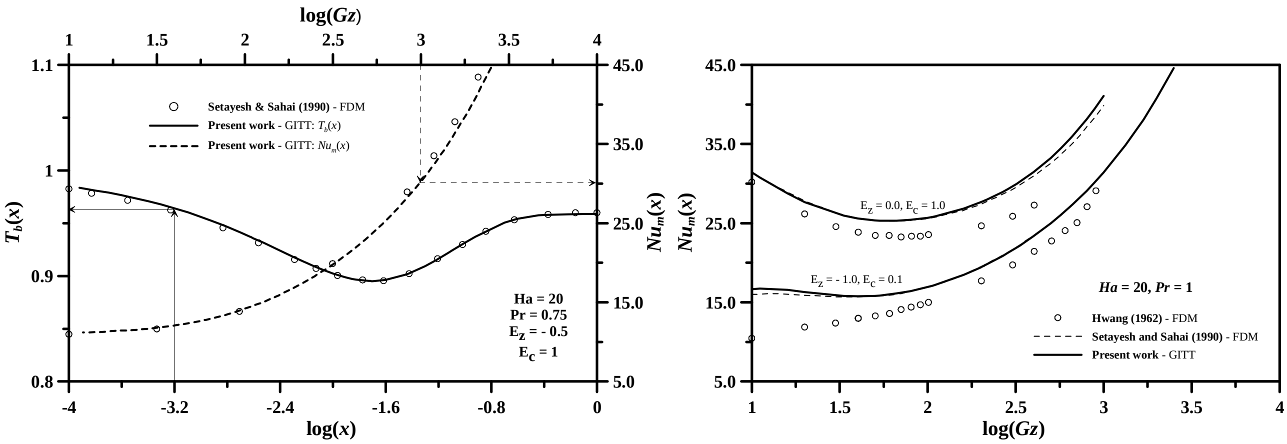

Figure 5. Development behaviors of the mean bulk temperature and average Nusselt number for Ha = 20.
(a) Pr = 0.75, E_c = 1.0 and E_z = - 0.5, (b) Pr = 1.0, E_c = (1.0, 0.1) and E_z = (0.0, - 0.5).

Observe the non-asymptotic developing of the bulk mean temperature along channel. Viscous dissipation, electric and magnetic fields interfere strongly altogether on flow and heat transfer rates.

Numerically, some small discrepancies are observed for the average Nusselt number on Fig. (5b), when comparisons are effectuated with the numerical findings of Hwang (1962) and Shohet *et al*. (1962). Certainly, some kinds of numerical errors were carried in those works, since a good agreement is observed when the present results are compared with those obtained with a higher order finite difference method by Manohar (1966) and with those more recent numerical results of Setayesh and Sahai (1990).

Finally, according to the accurate results presented in tabular and graphical forms, the GITT approach can be used as an efficient numerical tool for benchmarking purposes on developing MHD flow with constant properties, due mainly to its analytical character and the automatic global error control, provided by the IMSL (1990) routines. Indeed, as one can see from the transformed system, its numerical implementation is an extremely easy task.

Following the trends in this field of research, analysis with temperature-dependent properties will be analyzed in future works, as well as problems with variable magnetic field.

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