SUPPRESSING CHAOS IN A DOUBLE-WELL OSCILLATOR WITH LIMITED POWER SUPPLY USING ELECTROMECHANICAL DAMPED DEVICE

Juliano G. Iossaqui. **e-mail: jiossaqui@yahoo.com.br**
UNESP – Sao Paulo State, Department of Mechanical Engineering, CP 473, 17033-360, Bauru, SP, Brazil.

José M. Balthazar. e-mail: jmbaltha@rc.unesp.br
UNESP – Sao Paulo State, Department of Statistics, Applied Mathematical and Computation, CP 178, 13500-230, Rio Claro, SP,

Jorge L.P. Felix. E-mail: jorgelpfelix@yahoo.com.br
UNESP – Sao Paulo State, Department of Statistics, Applied Mathematical and Computation, CP 178, 13500-230, Rio Claro, SP

Bento R. Pontes Jr. e-mail: brpontes@feb.unesp.br
UNESP – Sao Paulo State, Department of Mechanical Engineering, CP 473, 17033-360, Bauru, SP, Brazil.

Abstract. *In this paper, we analyze the chaotic dynamics of an electromechanical damped Duffing oscillator coupled to a rotor. The electromechanical damped device or electromechanical vibration absorber consists of an electrical system coupled magnetically to a mechanical structure (represented by the Duffing oscillator with double-well potential), and that works by transferring the vibrational energy of the mechanical system to the electrical system. A Duffing oscillator with double-well potential it is considered. Numerical simulations results are presented to demonstrate the effectiveness of the electromechanical vibration absorber.*

Keywords: *Electromechanical Absorber, Nonlinear Dynamic, Non-Ideal Vibration, Chaos.*

1. INTRODUCTION

For the symmetric Duffing oscillator, the potential function can be expressed as follows (Youngiae et al., 2000):

$$V(x) = \frac{a}{2}x^2 + \frac{b}{4}x^4 \tag{1}$$

where a and b are constants. Here we consider only the bounded case with positive b $(b > 0)$. Then, depending on the sign of a, the potential function becomes one of two different types: Double-well potential for a $(a < 0)$ or Single-well potential for a $(a > 0)$ For the Duffing oscillator with a double-well potential there are two stable Equilibrium point at $x = \pm\sqrt{-a/b}$ and one unstable equilibrium point at $x = 0$. On the contrary, the Duffing oscillator with a single well has only a stable equilibrium point at $x = 0$.

The Duffing equation with a double-well potential (with a negative linear stiffness) it is an important model. One physical realization of such a Duffing oscillator model is a mass particle moving in a symmetric double well potential. This form of the equation also appears in the transverse vibrations of a beam when the transverse and longitudinal deflections are coupled(Sun and Lim, 2007). The Duffing equation with negative linear stiffness also describes the dynamics of a buckled beam as well as a plasma oscillator(Sang and Kim, 2000).

The damped and forced double-well Duffing equation has been a subject of intensive study over the last few decades as a landmark chaotic system.

(Venkatesan and Lakshmanan , 1997) showed numerically and analytically the existence of bifurcations and chaos in a double-well Duffing oscillator. The stability and bifurcation of a van der Pol-Duffing oscillator with the delay feedback are investigated by (Suqi et al., 2008). New methods have been used to suppress chaos by various authors, which considered the double-well Duffing equation (Tereshko et al., 2004), (Alvarez-Ramirez and Espinoza-Paredes, 2003 and (Sun et al, 2006).

These methods have been applied for systems whose energy sources are described by a harmonic function. However, in several mechanical experiments the oscillator cannot be driven by systems whose amplitude and frequency are arbitrarily chosen, since the forcing source has a limited available energy supply. Such energy sources have been called ***non-ideal***, and the corresponding system a non-ideal oscillator. For this kind of oscillator, the driven system cannot be considered as given a priori, but it must be taken as a consequence of the dynamics of the whole system (oscillator and motor). For non-ideal dynamical systems, one must add an equation that describes how the energy source supplies the energy to the equations that govern the corresponding ideal dynamical system.

We remark that in non-ideal systems the so-called Sommerfeld effect is often present: steady state frequencies of the DC motor will usually increase as more power (voltage) is given to it in a step-by-step fashion. When a resonance condition with the structure is reached, the better part of this energy is consumed to generate large amplitude vibrations of the foundation without sensible change of the motor frequency. Eventually, enough power is supplied to the motor to cause a jump: the operating frequency increases and the foundation amplitude decreases, resulting in lower power consumption by the motor. For a complete review of different approaches see (Balthazar et al. , 2003). We announced that Souza et al., 2007) proposed a simple feedback control method to suppress chaotic behavior in oscillators with limited power supply(Non-ideal vibrations).

In this work, we study the dynamics behavior of the double-well oscillator with limited power supply coupled to an electromechanical damped device. A single-well oscillator with limited power supply coupled to an electromechanical damping vibration absorber was studied by (Souza et al., 2007). The purpose of this paper is to consider the dynamics of an electromechanical damping device, and that works by transferring the vibration energy of the mechanical system to the electrical system, consists of an electrical part coupled magnetically to a mechanical structure (modeled by a double-well Duffing oscillator). A linear electrical system it is applied for suppressing the chaotic vibrations which limit the performance of the motion in the mechanical structure.

This paper is structured as follows: in Section 2 we describe the model equations for the oscillator with limited power supply. Section 3 explores some aspects of the model dynamics from numerical simulations, emphasizing the performance of the electromechanical damped device on a suppressing the chaotic vibrations. Our conclusion is presented in Section 4.

2. THEORETICAL MODEL

The model shown in Fig. 1 it is a mechanical structure described by the Duffing oscillator with double-well potential coupled to an electromechanical vibration absorber. The structure consists of a mass m_1 , a viscous damping coefficient b , a linear spring constant k_1 and a nonlinear spring constant k_2 . The structure is excited to a source of limited power supply, with mass unbalanced m_0 and eccentricity r . This system is called a main system.

The vibration absorber consists by an electromechanical transducer and a RCL electrical circuit in series. The simplest transducer constant model is given by $S = 2\pi nlB$, where n is the number of turns in the coil, l is the radius of the coil and B is the uniform radial magnetic field strength in the annular gap. The transducer constant S also relates the electrical potential e , across to the terminals of the coil to the velocity of the coil, with respect to the permanent magnet. The electrical circuit consists by a linear inductor L , a linear capacitor C and a linear resistor R .

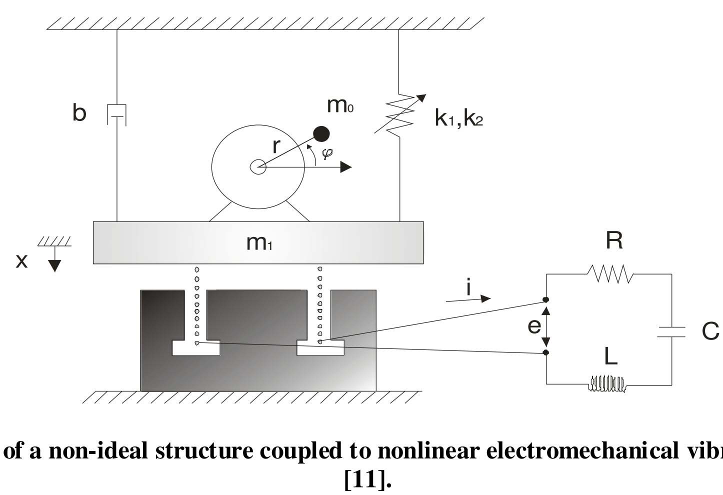

Figure 1 - Schematic of a non-ideal structure coupled to nonlinear electromechanical vibration absorber device [11].

The kinetic energy(Felix et al, 2009) and potential energy(Iosaqui , 2009)) [12] of the Duffing oscillator with double-well potential are described as:

$$K = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_0\left(\dot{x} - r\dot{\varphi}\cos\varphi\right)^2 + \frac{1}{2}m_0\left(r\dot{\varphi}sen\varphi\right)^2 + \frac{1}{2}I_0\dot{\varphi}^2 \tag{2}$$

$$V = -\frac{1}{2}k_1x^2 + \frac{1}{4}k_2x^4 \tag{3}$$

where x is the displacement of the main structure, φ is the angular displacement of the rotor, I_0 is the moment of inertia of the rotor.

The motion of the DC motor is governed by the following equations:

$$M(\phi') = Q(\phi') - H(\phi') \tag{4}$$

where the function $Q(\dot{\varphi})$ is the driving torque of the source of energy, the function $H(\dot{\varphi})$ is the resistive torque that is applied to the motor.

Note that, usually, the inductance taken to be as much smaller than the mechanical constant time of the vibrating system and, then in the stationary regime, we can take $Q(\dot{\varphi})$ as (linear) $Q(\dot{\varphi}) = u_1 - u_2\dot{\varphi}$, where u_1 is related to the voltage applied across to the armature of the motor and u_2 is a constant for each model of motor considered. We considered the resistive torque nulls.

The motion of the system is governed by the following governing differential equations, modified from (Felix et al, 2009)

$$\begin{aligned}
&(m_0 + m_1)\ddot{x} + b\dot{x} + k_1x + k_2x^3 - S\dot{q} = m_0r\left(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi\right)\\
&L\ddot{q} + R\dot{q} + \frac{1}{C}q + S\dot{x} = 0\\
&(I_0 + m_0r^2)\ddot{\varphi} - m_0r\ddot{x}\cos\varphi = u_1 - u_2\dot{\varphi}
\end{aligned} \tag{5}$$

It is convenient to rewrite (5), in terms o dimensionless variable:

$$\begin{aligned}
&\tau = \omega_m t\,,\\
&x = r\chi_1\,,\ \dot{x} = \omega_m r\chi_1'\,,\ \ddot{x} = \omega_m^2 r\chi_1''\,,\\
&q = q_0\chi_2\,,\ \dot{q} = \omega_m q_0\chi_2'\,,\ \ddot{q} = \omega_m^2 q_0\chi''\,,\\
&\varphi = \phi\,,\ \dot{\varphi} = \omega_m\phi' \text{ e } \ddot{\varphi} = \omega_m^2\phi''
\end{aligned} \tag{6a}$$

Then, the governing equations of motion (5), itself reduce to the following non-dimensional equations:

$$\begin{aligned}
&\chi_1'' - \omega_1^2\chi_1 = \varepsilon\left(\phi''\cos\phi - \phi'^2 sen\phi - \alpha_1\chi_1' - \beta_1\chi_1^3 + \gamma_2\chi_2'\right)\\
&\chi_2'' + \omega_2^2\chi_2 = \varepsilon\left(-\alpha_2\chi_2' - \gamma_1\chi_1'\right)\\
&\phi'' = \varepsilon\left(\lambda\chi_1''\cos\phi + \mu_1 - \mu_2\phi'\right)
\end{aligned} \tag{6b}$$

where

$$\varepsilon\alpha_1 = \frac{b}{\sqrt{k_1(m_0+m_1)}},\ \varepsilon\alpha_2 = \frac{R}{L\omega_m^2},$$
$$\varepsilon\beta_1 = \frac{k_2 r^2}{(m_0+m_1)\omega_m^2},\ \varepsilon = \frac{m_0}{m_0+m_1},$$
$$\varepsilon\lambda = \frac{m_0 r^2}{(I_0+m_0 r^2)},\ \varepsilon\gamma_1 = \frac{Sr}{L\omega_m q_0},$$
$$\varepsilon\gamma_2 = \frac{Sq_0}{(m_0+m_1)\omega_m r},$$
$$\varepsilon\mu_1 = \frac{u_1}{(I_0+m_0 r^2)\omega_m^2},\ \varepsilon\mu_2 = \frac{u_2}{(I_0+m_0 r^2)\omega_m},$$
$$\omega_1 = 1,\ \omega_2 = \frac{\omega_e}{\omega_m},$$
$$\omega_m^2 = \frac{k_1}{m_0+m_1} \text{ e} \tag{7}$$

2. DYNAMICS ANALYSIS OF THE SYSTEM

Equation (6b) it is numerically integrated, by using the fourth order Runge-Kutta algorithm with variable step-time. The numerical simulations were done in Simulink of MATLAB©.

In the following numerical calculation, the values of system parameters are given in such a way that the local natural frequency of the main system ω_1 is equal to the local frequency of the absorber ω_2. The initial conditions are taken as being nulls.

The responses are characterized by tracing the time evolutions, phase portrait and Lyapunov exponents.

The main aim of the electromechanical damping device in this section, it is to eliminate or suppress the mechanical chaotic vibrations or instability issue of the mechanical Duffing oscillator. Due to the characteristics of the electromechanical damping device, it is not possible to stop the motion of the mechanical Duffing model, but we have to determine the parameter band in which the electrical system transforms the chaotic vibrations of the mechanical Duffing oscillator to steady state or to another type of periodic vibrations, but not to another chaotic vibration.

Figure 2 shows the frequency response curves in amplitude for the system with an electromechanical damping vibration absorber. The values of the system parameters are

$$\alpha_1 = 0.2,\ \alpha_2 = 10,\ \beta_1 = 3,\ \gamma_1 = 20,\ \gamma_2 = 10,\ \lambda = 5,\ \varepsilon = 0.1 \text{ e } \mu_2 = 15 \text{ [10, 11].}$$

For the system without an electromechanical absorber the parameters of coupling γ_1 and γ_2 are taken as being nulls. The symbol '*' represents the system without absorber and the symbol '.' represents the system with absorber.

Figures 2(a) and 2(b) show the amplitude χ_1 of the harmonic oscillation in the main system and the amplitude χ_2 of the harmonic oscillation in the electromechanical damped vibration absorber, respectively. This graph it is estimated, by numerical simulations, defining the amplitude for angular velocity of the DC motor shaft or excitation frequency as being the mean value of the oscillations, and the amplitude for the foundation oscillation as absolute maximum value on the stationary state motion from equation (6). When is used the electromechanical absorber, we may observe that the amplitude of the Duffing oscillator is then weakly reduced.

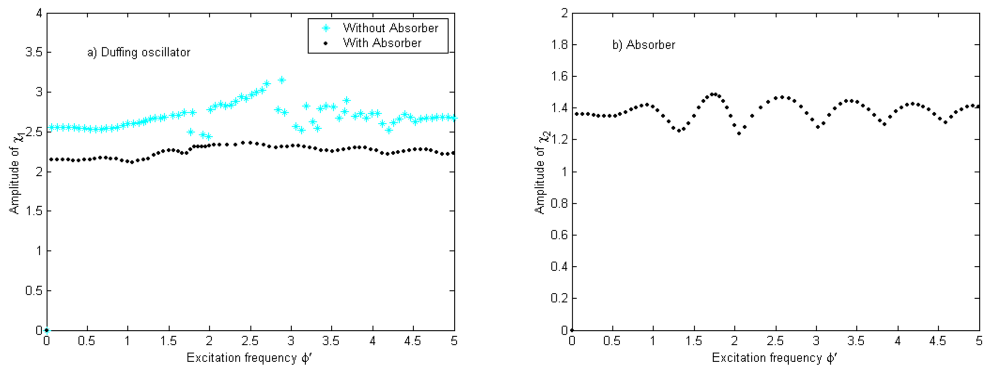

Figure 2 – a) Frequency response curve for the double-well Duffing oscillator without and with absorber and b) Frequency response curve for the electromechanical absorber.

The dynamic behavior of the electromechanical vibration absorber in the system (double-well Duffing oscillator) was observed from time histories of the displacements χ_1 and its respective phase portrait for distinct control parameter μ_1 (figures 3 through 7). Considering the other parameters fixed, we have: for $\mu_1 \in [0{,}14]$ the dynamic behavior is of stable periodic motion kind for the both systems without and with absorber (figure 3). The both systems are in the positive well. For $\mu_1 \in [15{,}18]$ the dynamic behavior is of the stable periodic motion type, but the system without absorber is in the negative well and the system with absorber is in the positive well (figure 4).

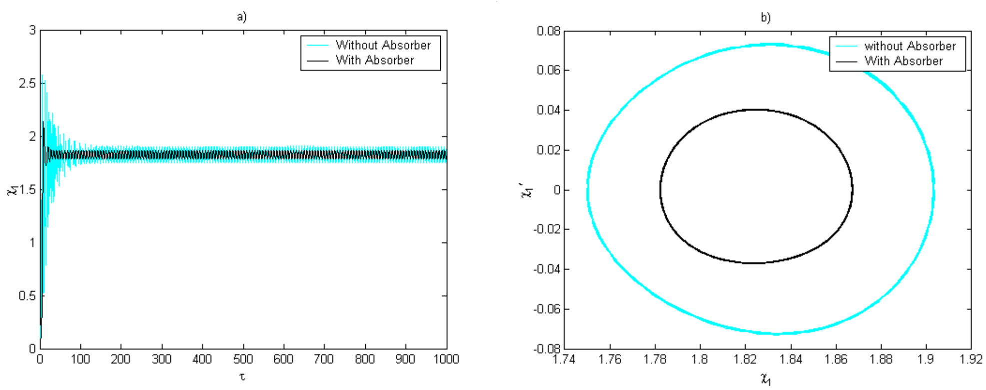

Figure 3 – a) Time history of the displacement of the mechanical Duffing oscillator, b) Phase portrait. For $\mu_1 = 14$.

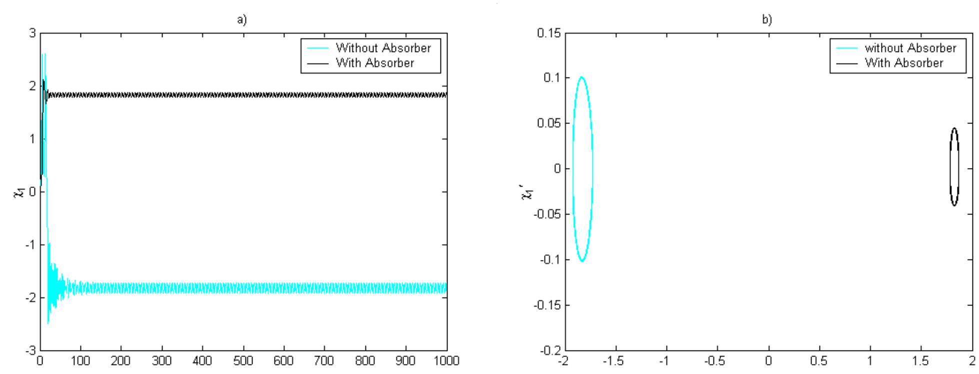

Figure 4 **– a) Time history of the displacement of the mechanical Duffing oscillator, b) Phase portrait. For $\mu_1 = 15$.**

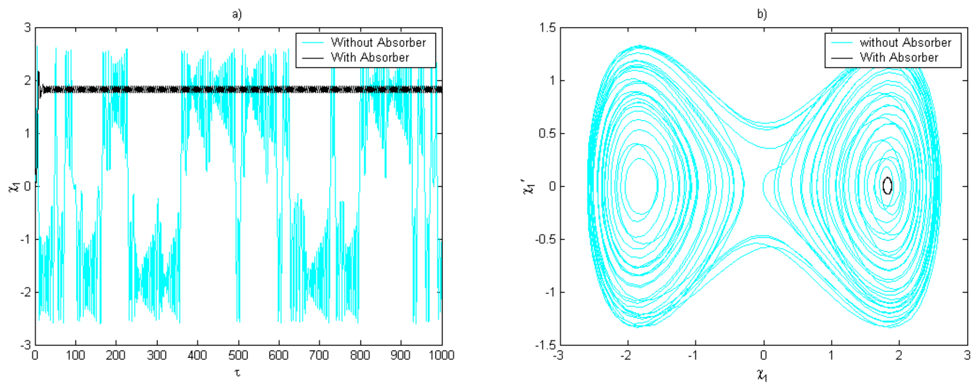

Figure 5 – a) Time history of the displacement of the mechanical Duffing oscillator, b) Phase portrait. For $\mu_1 = 19$.

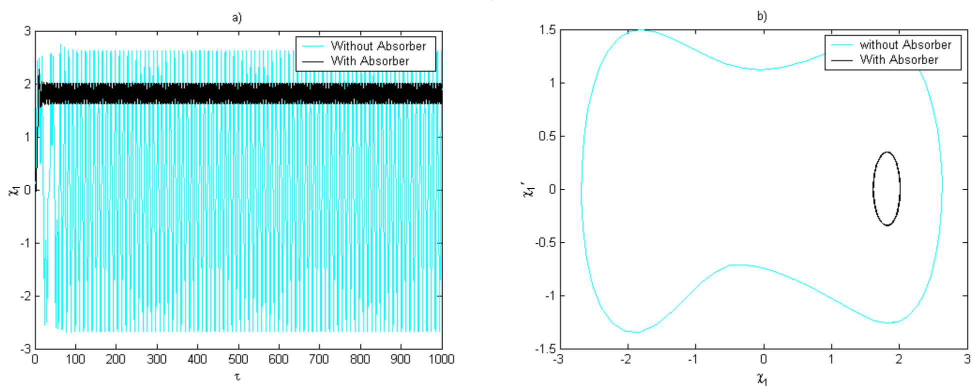

Figure 6 – a) Time history of the displacement of the mechanical Duffing oscillator, b) Phase portrait. For $\mu_1 = 28$.

For $\mu_1 \in [19,27]$, the system without absorber is chaotic while the system with absorber it is stable and periodic. The system without absorber goes back and forth over the two well, sampling one well and then the other (figure 5). For $\mu_1 \in [28,42]$, the system without absorber is stable and periodic, and its orbits go through of the two wells. The system with absorber is stable and periodic, but its orbits belong to the positive well (figure 6). For $\mu_1 > 43$, the system without absorber has behavior chaotic while the system with absorber is stable and periodic.

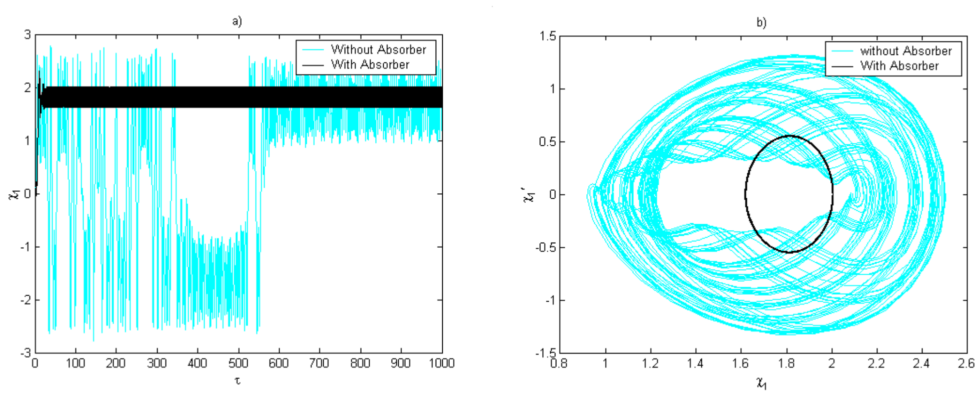

Figure 7 – a) Time history of the displacement of the mechanical Duffing oscillator, b) Phase portrait. For $\mu_1 = 43$.

In figure 8, we note the alteration of characteristic curve of the energy source (DC motor) due to electromechanical absorber. Figure 8(a) and (b) show the Duffing oscillator in a periodic ($\mu_1 = 14$) and chaotic region ($\mu_1 = 19$), respectively.

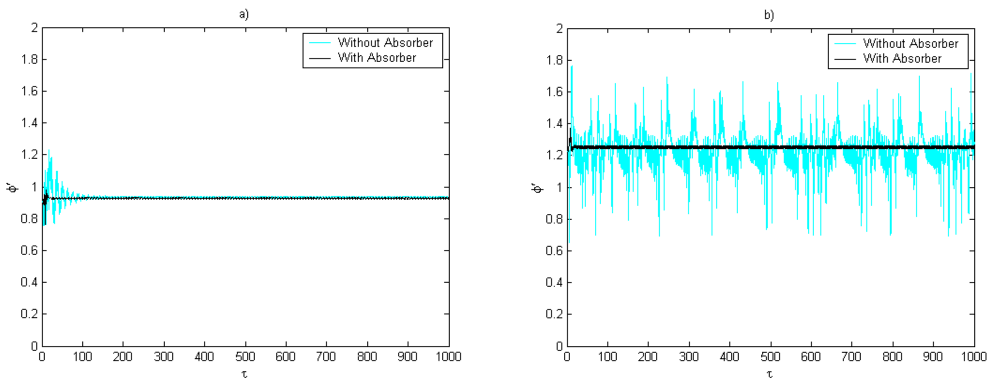

Figure 8 – Time evolution of the motor velocity: a) for $\mu_1 = 14$ and b) for $\mu_1 = 19$.

The Lyapunov exponents are calculated to prove the occurrence of a chaotic vibration and the suppressing of the chaotic motion by using the method of Wolf et al. [13].

Figure 9 shows the largest three Lyapunov exponents for the control parameter corresponding to figure 2. Figure 9(a) and (b) show the Lyapunov exponents of the system without absorber and Lyapunov exponents of the system with absorber, respectively.

The fact that the largest Lyapunov exponent it is positive at $\mu_1 \in [19,27]$ and $\mu_1 > 43$ proves the occurrence of a chaotic vibration in the system without absorber (figure 9(a)).

The suppressing chaos it is verified in figure 9(b) which shows all Lyapunov exponents negative.

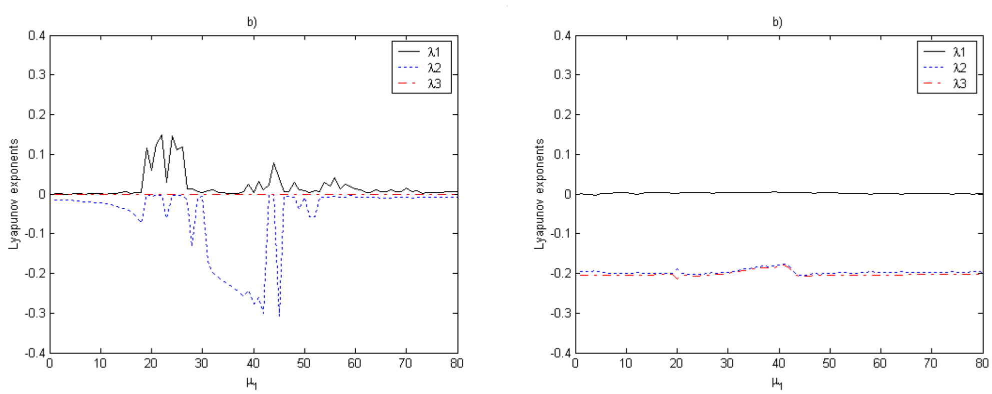

Figure 9 – Three largest Lyapunov exponents of the system: a) without absorber and b) with absorber.

3. CONCLUSIONS

In this paper, we have considered the dynamics of a double-well Duffing oscillator coupled to a rotor (a source of limited power supply) and an electromechanical damped device (vibration absorber device). The main aim was analyze the behavior of an electromechanical absorber in a chaotic oscillator. The reduction of amplitude of vibration was considered .

We have also found that the chaotic motion of the mechanical Duffing oscillator has been transformed to periodic motion; see as a quenching of chaotic vibrations. The main conclusion is that the electrical system eliminates the mechanical chaotic vibrations.

Future works will deals with experimental works.

4.ACKNOWLEDGEMENTS

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