UNCERTAINTY ANALYSIS IN THE DESIGN OF MODAL FILTERS USING PIEZOELECTRIC SENSOR ARRAYS

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Abstract. *Modal transducers allow independent sensing, actuation and control of individual vibration modes. Shaped piezoelectric layers were initially proposed to this end but an array of independent piezoelectric transducers with weighted actuation/sensing signals were shown to be easier to implement and allows reconfigurable modal filters since the weighting can be done via software. Several methodologies to determine optimal weights for a modal filter based on a given array of sensors were proposed in the literature. In a previous work, a methodology for the topology optimization of piezoelectric sensor arrays in order to maximize the effectiveness of a set of selected modal filters was presented. This was done using a genetic algorithm optimization for the selection of twelve piezoceramic sensors, from an array of thirty-six piezoceramic sensors regularly distributed over an aluminum plate, which maximize the frequency-band of a set of modal filters, each one aiming at one of the first vibration modes. It was shown that it is possible to improve the effectiveness and frequency-band of a set of modal filters with a reduced number of sensors by optimizing the topology of the sensor array. However, this optimization may also lead to a higher sensitivity of modal filters performance on design parameters. Therefore, this work presents a robustness analysis of modal filters using a topology optimized array design with a reduced number of sensors subjected to uncertainties in the weighting coefficients. This is done using stochastic modeling tools to build a probabilistic model of the uncertain parameters and Monte Carlo method to evaluate the realizations of modal filters performance indices.*

Keywords: *modal filters, piezoelectric sensor arrays, stochastic modeling, uncertainty analysis*

1. INTRODUCTION

Smart structures are integrated systems, composed of a host structure, sensors and actuators which are able to monitor and act to ensure both structural integrity and adaptability to changes in operational conditions. The development of materials having special functional properties has made possible the development of new generation of both sensors and actuators. In the concept of modern science, the actuator effect is defined according to the material's capability of generating mechanical energy from electric, magnetic or thermal energy, while the sensor effect, on the other hand, is defined by the converse energy conversion (Chopra, 2002).

The use of piezoelectric materials (specially piezoceramics) as sensing and actuating elements has been extensively studied due to the possibility of building them as lightweight and compact devices in several geometric configurations, since they are relatively inexpensive and present the necessary electromechanical coupling. In terms of applications, integrated piezoelectric sensors and actuators have been most often applied to the active control of mechanical vibration and noise in structures subjected to several types of excitation, or even self-excited structures, especially for aeronautic and aerospace applications (Chopra, 2002).

On the other hand, the performance of integrated systems applied to active vibration and noise control can be substantially improved by the use of high quality modal filters (Chen and Shen, 1997, Sun et al., 2001). In this context, the development of active control strategies with optimal performance using modal sensors and actuators has been the object of intensive research. Modal sensors and actuators working in closed loop enable to observe and control independently specific vibration modes, reducing the apparent dynamical complexity of the system and the necessary energy to control them (Fripp and Atalla, 2001; Preumont et al., 2003; Friswell, 2001). The high performance of modal controllers depends on the several parameters. The size, form and also the quality of piezoelectric material's effective eletromechanical coupling coefficient must be considered to the development of modal sensors and actuators. Though pioneer projects have considered the development of continuous modal sensors and actuators, the evolution of modal filter techniques and its applications to active vibration control indicates several advantages in the use of an array of discrete sensors instead (Shelley, 1991).

The high-performance of discrete sensors array depends on the convenient weighting of the sensors signals, in order to achieve optimal modal isolation (Fripp and Atalla, 2001; Preumont et al., 2003). Several numerical methods have been used for the evaluation of the weighting coefficients for the signals measured by the array of sensors (Meirovitch and Baruh, 1982; Shelley, 1991; Chen and Shen, 1997). These techniques may lead to high-performance modal filters, but generally within a limited frequency band.

Preumont et al. (2003) have suggested that the frequency band of high-performance filtering depends on the relation between the number of vibration modes to be filtered, in that frequency band, and the number of sensors in the array. They conclude that the number of sensors in the array should be larger than the number of vibration modes to be filtered. Although this is true for an arbitrarily distributed array of sensors, it is possible to show that the location of the sensors, that is the array topology, has a significant effect on the observability of the vibration modes and, thus, on the filtering performance of modal filters derived from it. Therefore, it should be possible to optimize the array topology and, consequently, increase the number of filtered vibration modes, thus the frequency band, for a given number of sensors available.

Topology optimization techniques are common in advanced structural design, for instance the simultaneous design of actuated mechanical devices, and often present a multiobjective character (Frecker, 2003). Techniques for topology optimization include but are not limited to genetic algorithms search methods. Genetic algorithm (GA) methods are search algorithms based on the survival of the fittest theory applied for a structured set of parameters Goldberg (1989). GA-based optimization methods have also been used for the design optimization of controlled structures (Han and Lee, 1999; Trindade, 2007).

Previous studies focused on designing and validating a methodology for the topology optimization of a sensor array with the objective of improving the performance of modal filters derived from it (Pagani and Trindade, 2008a, 2008b, 2009). This was done combining a standard technique for the evaluation of the coefficients for weighting the sensors signals with a proposed strategy for the optimization of the sensor array topology. The methodology was applied to a free plate with an array of thirty-six bonded piezoceramic patch sensors. A set of modal filters, aiming at isolating the first five vibration modes of the plate, were evaluated using only selected twelve of the thirty-six sensors of the array. An optimization strategy based on GA was proposed to find a sensor array topology that minimizes the norm of the filtering residue along a wider frequency band. It was shown that it is possible to improve the effectiveness and frequency-band of a set of modal filters with a reduced number of sensors by optimizing the topology of the sensor array. However, this optimization may also lead to a higher sensitivity of modal filters performance on design parameters. Therefore, this work presents a robustness analysis of modal filters using a topology optimized array design with a reduced number of sensors subjected to uncertainties in the weighting coefficients. This is done using stochastic modeling tools to build a probabilistic model of the uncertain parameters and Monte Carlo method to evaluate the realizations of modal filters performance indices.

2. DESIGN OF MODAL FILTERS

The design of a modal filter from an array of sensors requires the output signals of each sensor to be weighted and summed such that: i) the observability of the target vibration modes are maximized; and ii) the observability of the undesired vibration modes are minimized. Therefore, it is possible to consider the FRF of an equivalent single degree of freedom system with natural frequency ω_i and damping factor ζ_i, corresponding to the target i-th vibration mode, as the desired FRF of the weighted signal of the modal filter, which can be written as

$$g_i(\omega) = \frac{2\zeta_i\omega_i^2}{\omega_i^2 - \omega^2 + 2j\zeta_i\omega_i\omega}. \tag{1}$$

Whenever the vibration modes are weakly damped and relatively well spaced, the resonance peaks are well defined and, thus, (1) represents a realistic objective for the filtered FRF signal. Let $\mathbf{Y}$ be a matrix with columns that represent the FRFs of the n selected sensors in the array and discretized in a frequency domain $[\omega_1, \ldots, \omega_m]$. Let $\mathbf{G}_i = [g_i(\omega_1), \ldots, g_i(\omega_m)]$ be the vector representing (1) in the discrete frequency domain. The vector of coefficients $\boldsymbol{\alpha}_i$ which equates the filtered output (weighted sum of sensors outputs) to the one defined by the vector $\mathbf{G}_i$ is the solution of the following system

$$\begin{bmatrix} Y_1(\omega_1) & \cdots & Y_n(\omega_1) \\ \vdots & \ddots & \vdots \\ Y_1(\omega_m) & \cdots & Y_n(\omega_m) \end{bmatrix} \begin{bmatrix} \alpha_{i1} \\ \vdots \\ \alpha_{in} \end{bmatrix} = \begin{bmatrix} g_i(\omega_1) \\ \vdots \\ g_i(\omega_m) \end{bmatrix}. \tag{2}$$

In general, the linear system defined by (2) admits only approximate solutions, which will be denoted $\boldsymbol{\alpha}_i^\dagger$. The vector of weighting coefficients $\boldsymbol{\alpha}_i^\dagger$ represents the best solution, in a least squares sense, for the design of a modal filter which isolates the i-th vibration mode response. If several vibration modes are to be considered simultaneously as target modes for the filter design, it is necessary to define $\mathbf{G}$ as the matrix of target FRFs with dimension $m \times p$, where p denotes the number of target modes. Consequently, the approximate solution of (2), $\boldsymbol{\alpha}^\dagger$, is a matrix of dimension $n \times p$, that is one column vector of weighting coefficients for each one of the target modes. This may be written in a compact form as

$$\mathbf{Y}\boldsymbol{\alpha}^\dagger = \mathbf{G}. \tag{3}$$

Actually, $\mathbf{Y}\boldsymbol{\alpha}^\dagger$ approximates $\mathbf{G}^\dagger$, a matrix with columns that are the orthogonal projection of the columns of $\mathbf{G}$ onto the space spanned by the columns of $\mathbf{Y}$. The traditional Moore-Penrose pseudo-inverse solution of (3) for a full column rank $\mathbf{Y}$ matrix (with columns that are linearly independent) may be obtained by pre-multiplying (3) by $\mathbf{Y}^{\mathrm{H}}$, where the symbol H denotes the hermitian,

$$\mathbf{Y}^{\mathrm{H}}\mathbf{Y}\boldsymbol{\alpha}^{\dagger} = \mathbf{Y}^{\mathrm{H}}\mathbf{G}, \tag{4}$$

such that

$$\boldsymbol{\alpha}^{\dagger} = \left(\mathbf{Y}^{\mathrm{H}}\mathbf{Y}\right)^{-1}\mathbf{Y}^{\mathrm{H}}\mathbf{G}. \tag{5}$$

$\mathbf{Y}^{\dagger} = \left(\mathbf{Y}^{\mathrm{H}}\mathbf{Y}\right)^{-1}\mathbf{Y}^{\mathrm{H}}$ is then the pseudo-inverse of $\mathbf{Y}$. This approximate solution is known to be the best fit in a least square sense to the linear system (3) since it minimizes the Euclidean norm of the column-wise residuals. This result can be obtained by writing the residual norm for column i as

$$\|\mathbf{G}_i - \mathbf{Y}\boldsymbol{\alpha}_i\|_2^2 = (\mathbf{G}_i - \mathbf{Y}\boldsymbol{\alpha}_i)^{\mathrm{H}}(\mathbf{G}_i - \mathbf{Y}\boldsymbol{\alpha}_i) = \boldsymbol{\alpha}_i^{\mathrm{H}}\mathbf{Y}^{\mathrm{H}}\mathbf{Y}\boldsymbol{\alpha}_i - 2\boldsymbol{\alpha}_i^{\mathrm{H}}\mathbf{Y}^{\mathrm{H}}\mathbf{G}_i + \mathbf{G}_i^{\mathrm{H}}\mathbf{G}_i, \tag{6}$$

which has the following gradient

$$\nabla\left[\|\mathbf{G}_i - \mathbf{Y}\boldsymbol{\alpha}_i\|_2^2\right] = 2\mathbf{Y}^{\mathrm{H}}\mathbf{Y}\boldsymbol{\alpha}_i - 2\mathbf{Y}^{\mathrm{H}}\mathbf{G}_i. \tag{7}$$

The least square solution $\boldsymbol{\alpha}_i^*$ can be found by imposing that the gradient in (7) vanishes and solving the resulting equation for $\boldsymbol{\alpha}_i^*$, which leads to

$$\boldsymbol{\alpha}_i^* = \left(\mathbf{Y}^{\mathrm{H}}\mathbf{Y}\right)^{-1}\mathbf{Y}^{\mathrm{H}}\mathbf{G}_i. \tag{8}$$

Thus, $\boldsymbol{\alpha}^* = \boldsymbol{\alpha}^{\dagger}$ and, consequently, the pseudo-inverse method provides the least square solution. On the other hand, for a full column rank matrix, the inversion of $\mathbf{Y}^{\mathrm{H}}\mathbf{Y}$ is unnecessary and computationally inefficient, since $\mathbf{Y}$ may be decomposed through QR decomposition, where $\mathbf{Q}$ is an orthonormal matrix and $\mathbf{R}$ is upper triangular, such that $\mathbf{Y} = \mathbf{QR}$ and (5) can be rewritten as

$$\boldsymbol{\alpha}^{\dagger} = \left[(\mathbf{QR})^{\mathrm{H}}\mathbf{QR}\right]^{-1}(\mathbf{QR})^{\mathrm{H}}\mathbf{G}, \tag{9}$$

which, after expansion and accounting for $\mathbf{Q}^{\mathrm{H}}\mathbf{Q} = \mathbf{I}$, reads

$$\boldsymbol{\alpha}^{\dagger} = \mathbf{R}^{-1}\mathbf{Q}^{\mathrm{H}}\mathbf{G}. \tag{10}$$

Notice that the inverse of $\mathbf{R}$ does not need to be evaluated, instead the upper triangular linear system, $\mathbf{R}\boldsymbol{\alpha}^{\dagger} = \mathbf{Q}^{\mathrm{H}}\mathbf{G}$, is solved through back substitution, which is more computationally efficient. For all the cases studied in the present work, the solution through QR decomposition was always convenient, since the FRF matrix has had full column rank. If at least two columns of the FRF matrix are linearly dependent, this means that two sensors outputs are equivalent so that one of them is dispensable and the array formed by these sensors is equivalent to one with one sensor less, thus, it should present lower performance. Nevertheless, if this is the case, the singular value decomposition (SVD) is the suitable method to approximate the least square solution.

In practice, the truncation of matrix $\mathbf{Y}$ over a given frequency range will affect its QR decomposition and, thus, the approximate solution of the linear system (3). Let $\mathbf{Y}$ be the FRF matrix truncated at frequency $\omega_t \leq \omega_m$. Recent works have shown that there is a value for $\omega_t = \omega_l$ such that all vibration modes inside the frequency range $[\omega \leq \omega_l]$ are perfectly filtered, except the target ones, whereas vibration modes with natural frequency larger than ω_t are not filtered (Preumont et al., 2003; Pagani and Trindade, 2008a). In order to filter the higher frequency modes, higher values for ω_t have to be considered in the filter design (FRF truncation), but in this case, only a partial filtering can be assured for all modes, including those in the lower frequency range.

3. APPLICATION TO A PLATE WITH AN ARRAY OF BONDED PIEZOCERAMIC PATCHES

In this section, the modal filter design technique presented in the previous section is applied to a plate with bonded piezoceramic patches, acting as sensors, to analyze its effectiveness and evidence its limitations.

3.1 Finite element modeling

The host structure considered is a free rectangular aluminum plate, of dimensions 320×280×3 mm, and has thirty-six identical thickness-poled PZT-5H piezoceramic patches bonded to its upper surface. The piezoceramic patches have dimensions 25×25×0.5 mm. Figure 1 presents a geometric description of model. The material properties are: i) Aluminum – Young's modulus 70 GPa, Poisson's ratio 0.33, mass density 2700 kg/m^3; and ii) PZT-5H – mass density 7500 kg/m^3, and elastic $c^E_{11} = c^E_{22} = 127$ GPa, $c^E_{33} = 117$ GPa, $c^E_{12} = 80.2$ GPa, $c^E_{13} = 84.7$ GPa, $c^E_{44} = c^E_{55} = 23.0$ GPa, $c^E_{66} = 23.5$ GPa, piezoelectric $d_{31} = d_{32} = -274$ pCN^{-1}, $d_{33} = 593$ pCN^{-1}, $d_{15} = d_{24} = 741$ pCN^{-1} and dielectric $\epsilon^T_{11} = \epsilon^T_{22} = 27.7$ nFm^{-1}, $\epsilon^T_{33} = 30.1$ nFm^{-1} constants. The model was built and simulated in ANSYS commercial software. The structural element SHELL99, with a single layer, has been used to model the aluminum plate, while the element SOLID226 has been considered to model the piezoelectric patches. The element SOLID226 presents nodal degrees of freedom, for displacements in x, y and z directions and electric voltage, and eletromechanical coupling properties required to model the sensor and actuator effects. This element has been used in the cubic form, with 20 nodes, eight in each face (with commons nodes at vertices). For the plate, 3584 SHELL99 elements were used, while 50 SOLID226 elements were considered for each piezoceramic patch.

To ensure an ideal perfect bonding between the piezoceramic patches and the plate, the nodes on the bottom surface of the patches are mechanically coupled to the ones on the top surface of the plate. To this end, the nodes of the SHELL99 element must be offset to the contact surface with the SOLID226 element and the finite element meshes for both elements must be coherent. This is the geometric condition for an ideal (perfect) coupling between the aluminum plate and the piezoceramic patches.

All nodes of the piezoelectric patches surfaces bonded to the plate are considered electrically grounded. The dielectric properties of PZT-5H prevent the homogeneous distribution of the induced electrical charges on the free surface of the patches. Therefore, measurement of the electric potential in a specific node on the free surface will correspond to local information on induced strain. In practice, the free surface of each patch is covered with an electrode which ensures a uniform level of induced electric potential (equipotential) in the free surface of each patch.

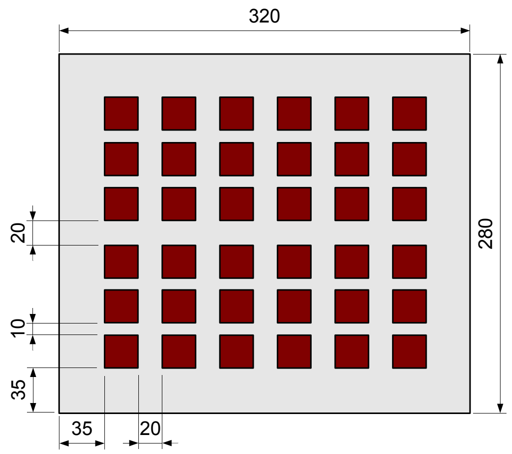

Figure 1. Aluminum plate with thirty-six PZT-5H piezoceramic patches bonded to one of its surfaces (dimensions in mm).

The analyses presented in this section indicate that although, in average, the number of modes, and thus the frequency range of the modal filter, is limited by the number of sensors considered, properly selected topologies could increase the frequency range, for a given number of sensors, or reduce the number of sensors, for a given frequency range. This suggests that, for a given number of sensors in an array, its topology could be optimized to enhance its performance.

4. OPTIMIZATION OF THE SENSORS ARRAYS TOPOLOGIES

This section presents a methodology for improving filtering performance through the optimization of the sensors arrays topologies.

4.1 Optimization strategy

After some numerical simulations with straightforward topologies derived from the base array with thirty-six sensors presented previously, it becomes clear that the relation between the array topology and the filtering performance is quite complex, even when the mode shapes are known, and, hence, optimal solutions require a more advanced strategy. An extensive search of the possible combinations of twelve sensors from the thirty-six available would lead to an impracticable computational cost, since around one billion ($C_{36,12}$) combinations would have to be evaluated. Extensive search could however be considered using selected subspaces to identify rationales for the parameters' setup of another search strategy (Pagani and Trindade, 2008b; Pagani and Trindade, 2009).

GAs are more suitable search methods in these cases when the research space is too large, strongly multimodal and non-linear. It is chosen here to setup a GA search by defining a random initial population formed by so-called individuals with chromosomes that are composed of twelve genes. Each gene is an integer number from 1 to 36 representing the sensor index. Therefore, one individual represents a sensors array topology formed by the twelve sensors defined by its genes.

Following the standard GA evolutive process, the initial population is considered to evolve along a set of generations through reproduction (crossover), mutation and selection operations. While reproduction and mutation operations aim to provide diversity to the population, the selection operation aims to rank individuals with respect to a fitness or objective function. Since this is a random search algorithm, the optimal results are dependent on the initial population and on the reproduction, mutation and selection parameters. However, it is expected that for a sufficiently large number of generations or size of the initial population, the algorithm will converge to the global optimum. More details on convergence and selection of operations parameters can be found in (Pagani and Trindade, 2008b).

Since any individual of the population is composed by twelve different integer numbers in the domain [1,...,36], a specific routine was written to build the initial population. For each individual, the routine scrambles randomly a vector of integers from 1 to 36 and, then, the first 12 elements of the scrambled vector define the corresponding individual. This procedure is repeated for all individuals in the initial population. The selection of the first 12 elements in the scrambled vector does not imply in a tendency since the distribution of the sensor indices in the vector is equiprobable.

The mutation operation, considered in this work, consists in replacing one of the 12 genes (sensors), selected randomly, of an individual by another one, selected randomly from the complementary group of sensors, that is, from the 24 remaining sensors not present in the individual. This procedure prevents the generation of an individual with repeated genes. The reproduction (crossover) operation combines the initial and final sections of two individuals (parents) to form a new individual (child), where the breaking position of the parents' sequences (chromosomes) is defined randomly. In this case, the generation of an individual with repeated genes is possible and, when this is the case, the fitness function of this individual is not evaluated to save computational time; instead a small fitness value is attributed to it, such that its selection probability is also small. The selection operation is based on a stochastic universal sampling algorithm, where the expectation of individuals in the population is evaluated from a fitness ranking.

Besides the choice of reproduction, mutation and selection operators, it is necessary to define the size of the initial population (N), the number of best individuals (elite) which are kept unmodified from one generation to another (Σ), the percentage of the population in each generation which are generated by crossover (T_c) and the total number of generations the population evolves (N_p). Once defined T_c, the remaining part of population is generated by either the previous elite or mutation operation. Crossover percentages of [30, 40, 50, 60, 70]% lead to genic mutation rates of [5.8, 5.0, 4.2, 3.3, 2.5]%. Apart from the procedures proposed for the construction of the initial populations, the mutation operation and the parameters' definition, the optimization was performed using operators and algorithms of MATLAB Genetic Algorithm and Direct Search (GADS) Toolbox.

4.2 Objective function to rank the modal filters performance

The objective of the present optimization is to find the topology of an array with twelve sensors that maximizes the filtering quality, over a given frequency range, of modal filters designed to isolate a given set of resonances of the structure. In the present work, a particular case of interest was studied using the first three vibration modes of the free plate as target vibration modes to be isolate by the modal filters. The target frequency range is [0, 1000] Hz, which is higher than the limit frequency $\omega_l = 800$ Hz for the present case and contains fourteen resonances (four after ω_l). Therefore, the FRF truncation frequency is defined as $\omega_t = 1000$ Hz such that, for an arbitrary array topology, no filtering quality can be guaranteed along the frequency range, while an optimal topology can maximize this quality. For implementation purposes, the objective function to be minimized is then defined as the residual error norm

$$J = \left\| |\mathbf{G}_t| - |\mathbf{Y}_t \boldsymbol{\alpha}^\dagger| \right\|_2 . \tag{11}$$

where $\mathbf{G}_t$ and $\mathbf{Y}_t$ are the target and measured, by each sensor, FRFs truncated at frequency ω_t and $\boldsymbol{\alpha}^\dagger$ is the vector of weighting coefficients, evaluated using $\mathbf{G}_t$ and the QR decomposition of matrix $\mathbf{Y}_t$ in (10).

Another possible strategy that was presented in a previous work consists on maximizing the frequency range for a given filtering quality (Pagani and Trindade, 2008a).

4.3 Results for the optimal topology

In this section, the results obtained for the modal filters with optimal sensors array topologies are presented. Based on previous studies (Pagani and Trindade, 2008b), the following parameters were set for the GA optimization: initial population of $N = 1500$ individuals, crossover rate at $T_c = 45\%$, genic mutation rate at $T_g = 4.6\%$, elite population at $\Sigma = 2$ individuals and termination criteria at $N_p = 35$ generations. To minimize the dependence of GA optimization on the initial population, fifty simulations with different initial populations were performed for each case and the best results from these simulations are saved.

Figure 2 presents the normalized filter output, such that the amplitude at target resonances is unitary, and the corresponding optimal topology, in which the twelve selected sensors are highlighted from the original thirty-six sensors array. It shows that topology optimization for isolation of the first three vibration modes has provided excellent performance up to 1000 Hz. Therefore, the modal filter was effective up to the fourteenth mode and, thus, four additional resonant modes were effectively filtered compared to an arbitrary topology.

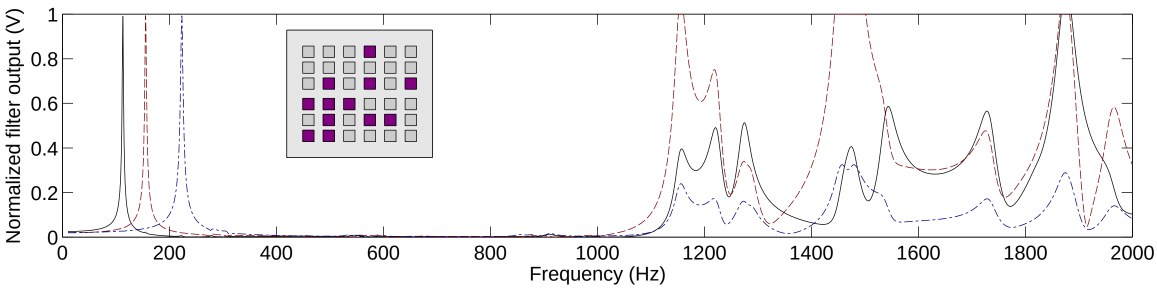

Figure 2. Normalized output of the modal filter designed for the isolation of the first three vibration modes.

As it was shown, it is possible to improve the effectiveness and frequency-band of a set of modal filters with a reduced number of sensors by optimizing the topology of the sensor array. However, this optimization may also lead to a higher sensitivity of modal filters performance on design parameters. Therefore, the next section presents a robustness analysis of these modal filters using an optimal topology subjected to uncertainties in the weighting coefficients.

5. UNCERTAINTIES ANALYSIS

This section presents an approach for analyzing random uncertainties in the weighting coefficients. A Gaussian probability density function is assumed for each weighting coefficient α_j, for which the mean values are based on the nominal ones designed in the previous section and the standard deviations are estimated from experiments, such that

$$p(\alpha_j) = \frac{1}{\sqrt{2\pi}\sigma_\alpha} \exp\left\{ -\frac{1}{2\sigma_\alpha^2} (\alpha_j - \bar{\alpha}_j)^2 \right\} \tag{12}$$

where $\bar{\alpha}_j$ are the real part of the weighting coefficients, normalized to the maximum weight of 0.35 allowed by the voltage divider circuit (Figure 3) used for the measurements. σ_α is an estimation of the standard deviation based on 12 sample experiments, each consisting of a manual setup of the potentiometer in one of the 16 similar circuits constructed in the laboratory (Figure 3). Since the level of precision in the manual setup is much more dependent on the sensibility of each potentiometer, the measurement technique for setup verification and the user's experience, than the nominal value of the weighting coefficient, the standard deviation σ_α was considered to be constant for all weighting coefficients. Three values of σ_α based on the laboratory experiments were considered 0.0003, 0.0008 and 0.0030.

Based on these assumptions, N random realizations were generated for each weighting coefficient with MATLAB function *normrnd* and, then, combined to form N random realizations of the vector of weighting coefficients $\boldsymbol{\alpha}(\theta_i)$. Each realization $\boldsymbol{\alpha}(\theta_i)$ was then used to evaluate a realization of the filter output $\tilde{\mathbf{G}}(\theta_i) = \mathbf{Y}\boldsymbol{\alpha}(\theta_i)$. The mean-square convergence analysis with respect to the independent realizations $\tilde{\mathbf{G}}(\theta_i)$ was carried out considering the function

$$conv(n_s) = \frac{1}{n_s} \sum_{i=1}^{n_s} \|\tilde{\mathbf{G}}(\theta_i) - \tilde{\mathbf{G}}^N\|^2, \tag{13}$$

where n_s is the number of simulations, or the number of sets of weighting coefficients considered, and $\tilde{\mathbf{G}}^N$ is the response calculated using the corresponding mean model. Figure 4 shows the mean-square convergence analysis. It is possible to observe that 2000 simulations are enough to assure convergence. Despite that, the statistical analyses presented in the following sections consider all $N = 4000$ simulations performed.

The statistical analyses of the FRFs were performed from a Gamma distribution fit to their amplitudes at each frequency to calculate maximum likelihood estimates of the distribution parameters using MATLAB function *gamfit*. Then, these parameters were used to calculate the 95% confidence intervals for the FRF amplitudes, with MATLAB function *gaminv*. More details on the stochastic modeling methodology used here can be found in (Cataldo et al., 2008; Soize, 2001; Santos and Trindade, 2009).

Figures 5, 6 and 7 show the difference between the normalized filter outputs when using the gaussian model for α and when using the nominal model (nominal α). The normalized filter output using the gaussian model for the weighting coefficients vector is represented by its mean value and 95% confidence interval. For reference, the normalized filter output using the nominal model is shown in Figure 2. From Figures 5, 6 and 7, it is possible to notice that the uncertainty in the weighting coefficients may yield variations in comparison with the nominal model of the order of 6%, 16% and 60% for growing values of standard deviation σ_α. Notice also that, despite the mode to be isolated, the error seems to be concentrated in the third resonance and it is much less important outside the frequency band of interest. However, for the filter designed to isolate the third mode, the mean model yields small errors in the response at the third resonance despite the standard deviation of the weighting coefficients.

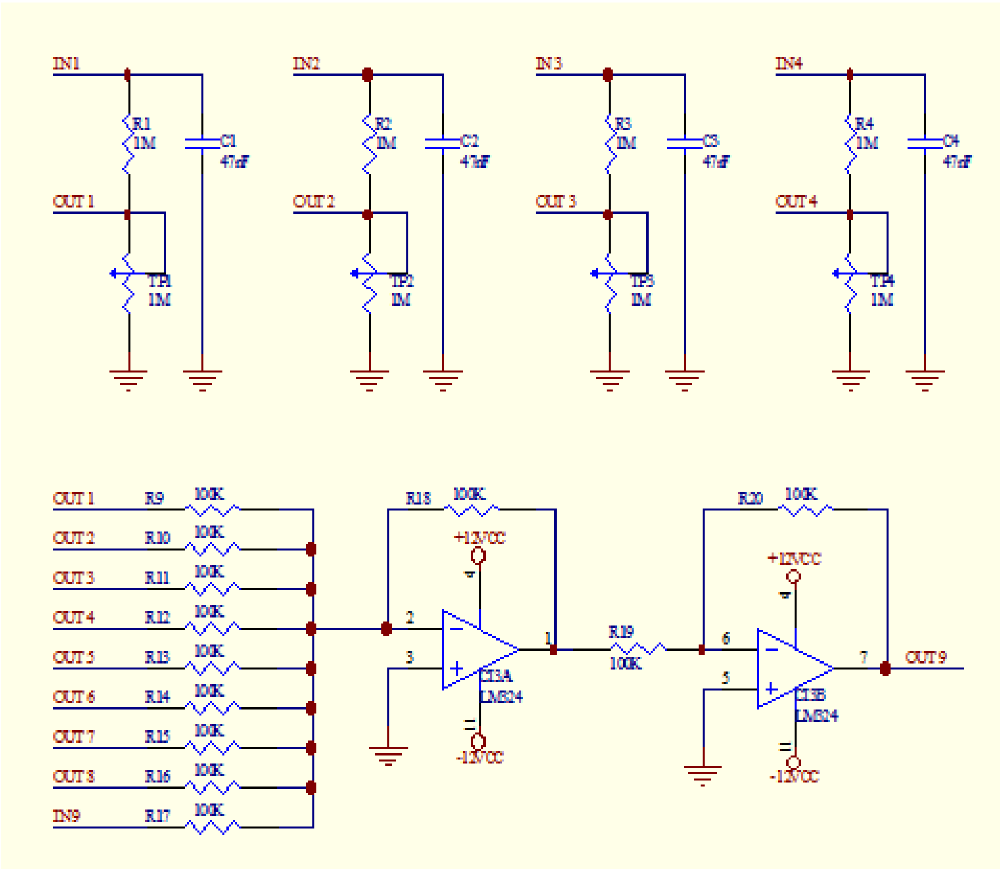

Figure 3. Electric circuit designed for weighting and summing sensor signals.

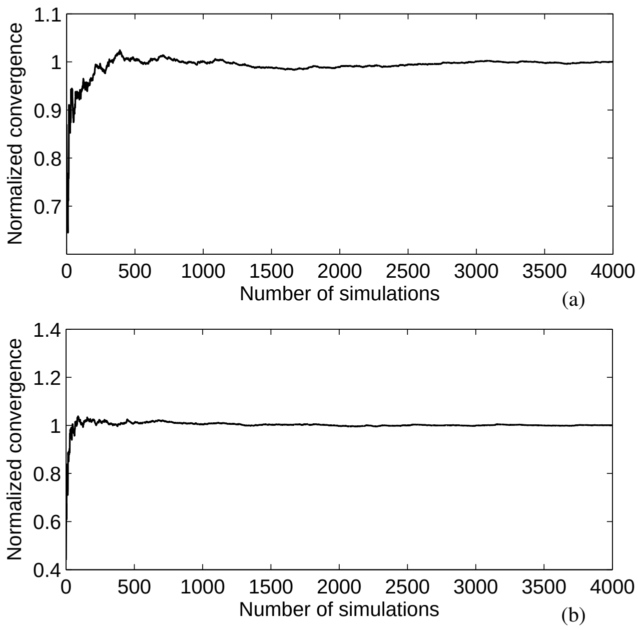

Figure 4. Mean square convergence of Monte Carlo simulation using $\sigma_\alpha = 0.0003$ (a) and $\sigma_\alpha = 0.0030$ (b).

6. CONCLUSIONS

This work presented a robustness analysis of modal filters using a topology optimized array design with a reduced number of sensors subjected to uncertainties in the weighting coefficients. This was done using stochastic modeling tools to build a probabilistic model of the uncertain parameters and Monte Carlo method to evaluate the realizations of modal filters performance indices. It was shown that optimal filter output is somewhat sensitive to the weighting coefficients but, as long as the standard deviation from nominal values are kept around 0.0003, the performance predicted with the

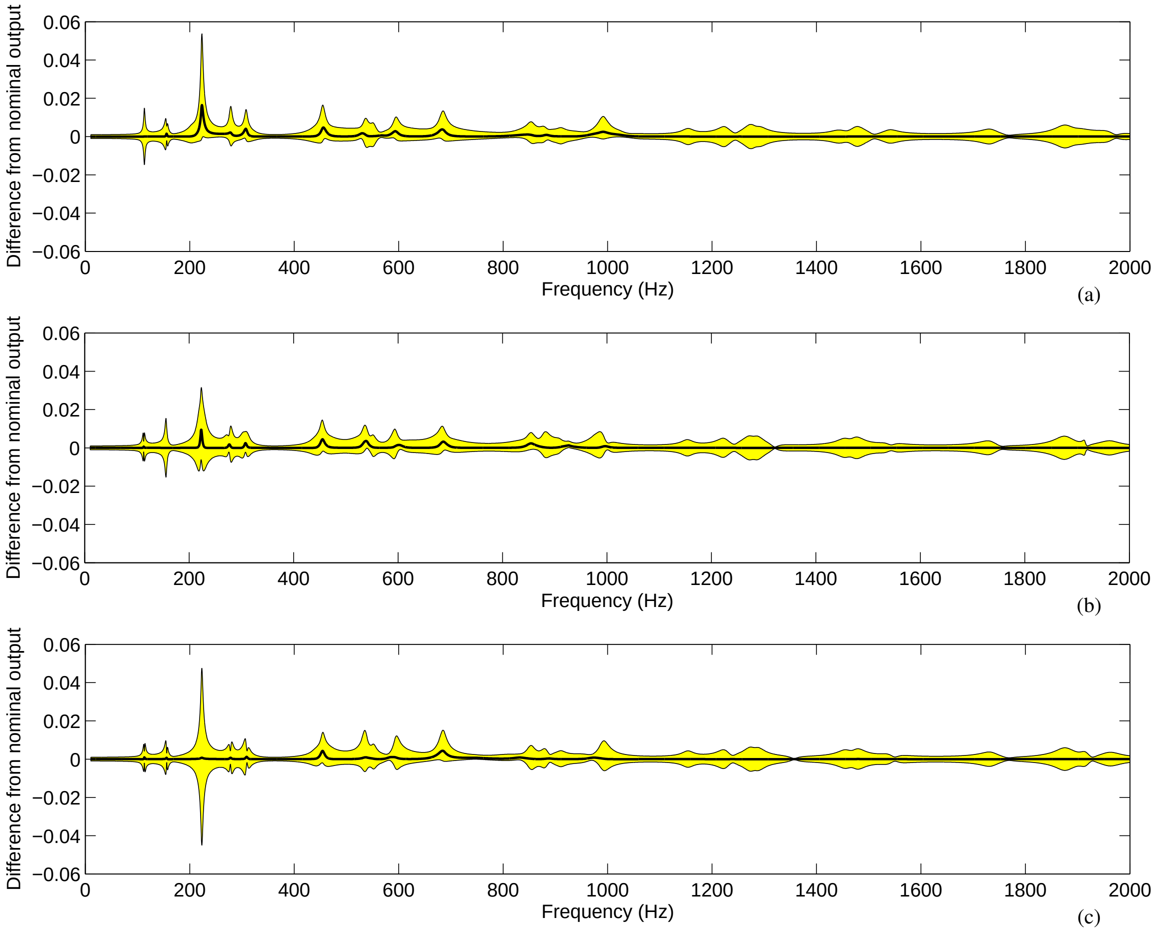

Figure 5. Difference from nominal model of filter output mean and confidence intervals for the isolation of first (a), second (b) and third (c) modes.

nominal model is satisfactorily reliable. Future works will be directed to the inclusion of sensors positioning as uncertain parameters, experimental validation of the stochastic modeling and application of the proposed modal filters for active vibration control.

7. ACKNOWLEDGEMENTS

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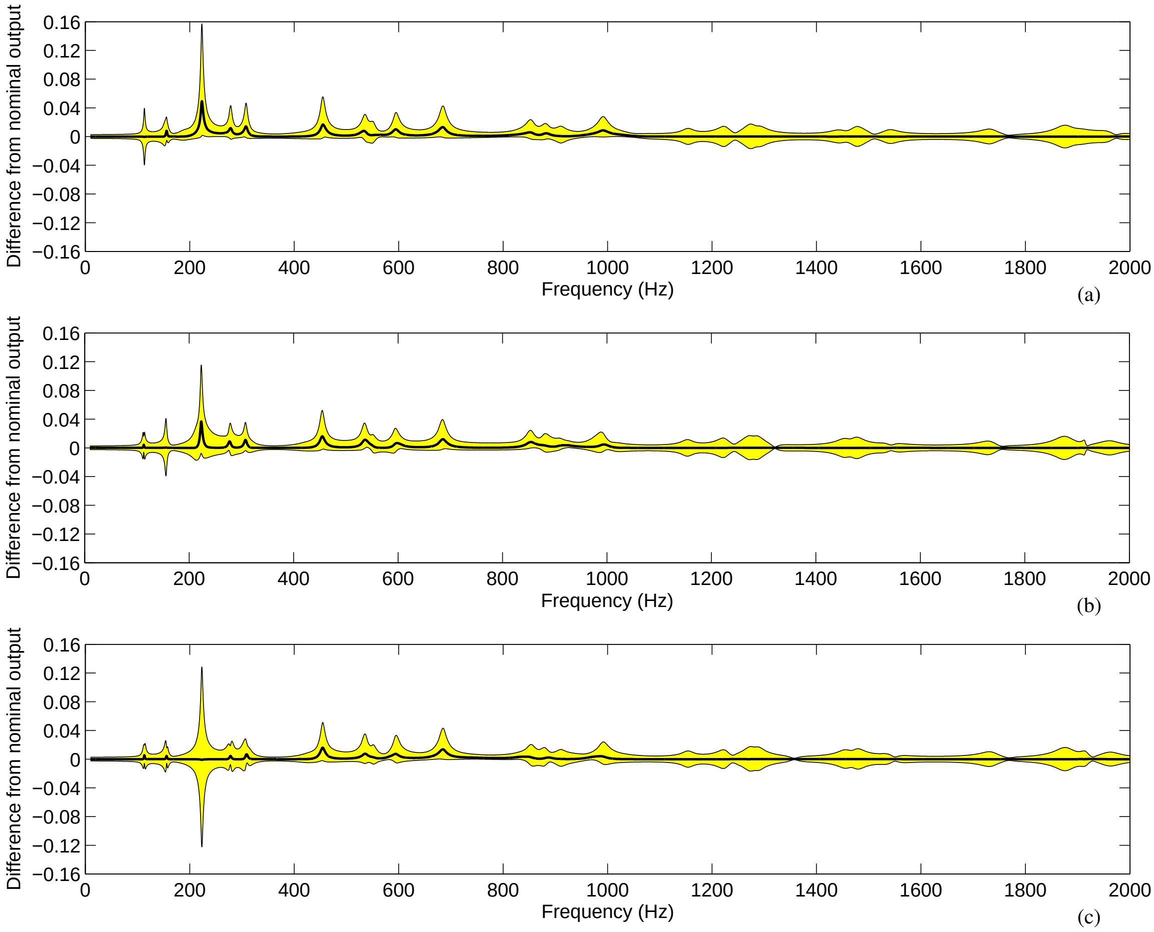

Figure 6. Difference from nominal model of filter output mean and confidence intervals for the isolation of first (a), second (b) and third (c) modes.

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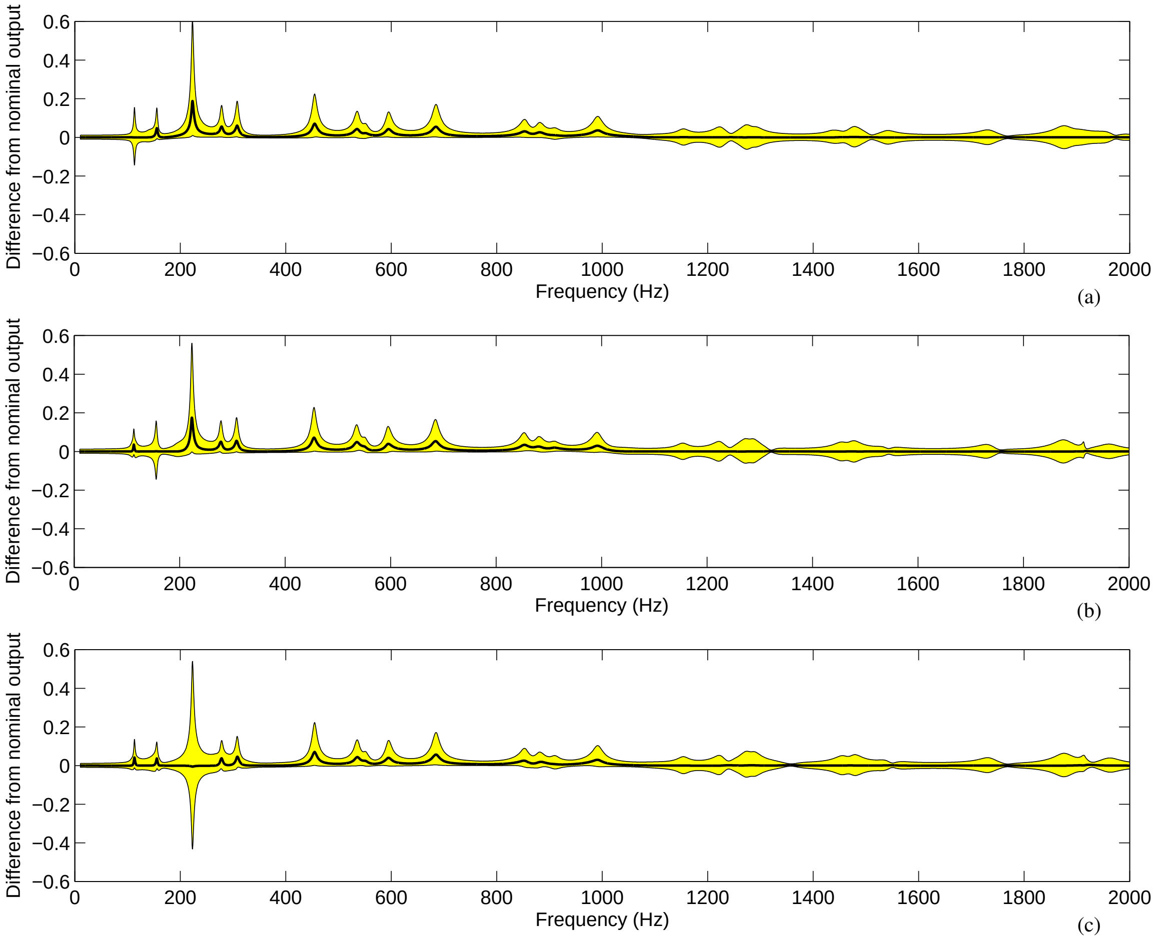

Figure 7. Difference from nominal model of filter output mean and confidence intervals for the isolation of first (a), second (b) and third (c) modes.

9. Responsibility notice

The authors are the only responsible for the printed material included in this paper.