SIMULATION OF LAMINAR TRANSIENT THERMAL ENTRANCE OF NON-NEWTONIAN FLUIDS

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Abstract. *In the present work the transient laminar forced convection, hidrodinamically developed and thermally developing, it is studied in channels of flat plates and the circular duct for non-Newtonian fluids considering the power-low model. The Generalized Integral Transform Technique is used to eliminate the variables in which the spatial diffusion is prevalent, it follows a system of partial differential equations coupled, which is solved numerically using the Method of Lines. The presented solution allows an accurate analysis for the problem, in all dominion, for different times.*

Keywords*: Thermal Entrance, Non-Newtonian fluids, Hybrid Solution.*

1. INTRODUCTION

The study of phenomena related to heat transfer in the internal forced convection has been done essentially to meet a practical necessity, once the majority solutions for this type of phenomenon, especially in area of the thermal entrance and simultaneous development, can be useful in the design of thermal devices with better performance. Therefore, electronic components, condensers, evaporators, heat exchangers, among others, are examples of possible use. With the increasing miniaturization and the necessity of optimization of these equipment, this study has become a global necessity. Therefore, the motivation of the research is not only a purely academic exercise anymore, due to the increase of its practical importance in several industrial processes and other segments of the economy, where the fluids have Non-Newtonian behavior. Typical examples of substances with this behavior are: suspension of solids in liquids, polymers, plastics, petroleum, pharmaceutical and biological fluids. Notably in several areas of engineering, such as nuclear engineering, spatial, petrochemical, among others**.**

Using Integral Transform associated with the Method of Lines. According Wouwer *et al* (2005), Method of Lines is one of the most popular schemes to solve partial differential equations. First the spatial variables are approximated by using the method of finite differences, finite elements or finite volume. Second, the ordinary system resulting is numerically solved. The Integral Transform eliminates the spatial variables in which ones the diffusion is predominant and Method of lines treats hyperbolic part resulting, where the convection is predominant, the system is truncated in a number of required terms for convergence and numerically solved. Results are presented to the average temperature throughout the duct. This work can be inserted in the context of the forced convection problems transient, being considered an extension of the work of Cotta & Gerk (1994), Gondim (1997), Castellões (2004) and others, in resolution of energy equation.

2. PROOBLEM FORMULATION

To illustrate the application of the solution, this procedure will be applied to the problem of thermal entrance, in the laminar flow hydrodynamically developed inside of ducts, such as the case of the channel of flat plates and circular tubes. The temperature of entrance will be taken over as a time function. The effects of axial conduction, the viscous dissipation and free convection on the wall will be negligible, and the physical properties are considered constant. According to Cotta & Gerk (1994), the energy equation is given by:

$$r^n \frac{\partial \theta(r,z,\tau)}{\partial \tau} + r^n \frac{u(r)}{2^{4-2n}} \frac{\partial \theta(r,z,\tau)}{\partial z} = \frac{\partial}{\partial r}\left(r^n \frac{\partial \theta(r,z,\tau)}{\partial r} \right) \quad 0 < r < 1, \quad z > 0, \quad \tau > 0 \tag{1.a}$$

With the following inlet and boundary conditions:

$$\theta(r,z,0)=\theta_0(r,z), \qquad 0<r<1, \quad z>0 \tag{1.b}$$

$$\theta(r,0,\tau)=\theta_e(r,\tau), \qquad 0\le r\le 1, \quad \tau>0 \tag{1.c}$$

$$\left.\frac{\partial\theta(r,z,\tau)}{\partial r}\right|_{r=0}=0,\ \theta(1,z,\tau)=0,\ z>0,\tau>0 \tag{1.d-e}$$

According to Mikhailov & Özisik (1984, p.343) $u(r)=\frac{1+(n+2)c}{1+c}(1-r^{\frac{1+c}{c}})$ (1.f)

Where c is a parameter of the model, according Özisik & Mikhailov (1984), c is equal to 1 for Newtonian fluid.
Dimensionless groups:

$$r=\frac{R}{b},\ z=\frac{\alpha.Z}{\overline{U}D_h^2},\ D_h=2^{2-n}b,\ \tau=\frac{\alpha.t}{b^2},\ \ u(r)=\frac{U(R)}{\overline{U}},\ \theta(r,z,\tau)=\frac{T(R,Z,t)-T_w}{\Delta T_0},\ \Omega=\frac{\omega b^2}{\alpha}$$

$$W(r)=r^n\,\frac{u(r)}{2^{4-2n}} \tag{2}$$

3. METHODOLOGY OF SOLUTION

3.1. Eigenvalue problem

$$-\frac{d}{dr}\left(r^n\frac{d\psi_i(r)}{dr}\right)=\mu_i^2W(r)\psi_i(r),\ \left.\frac{d\psi_i(r)}{dr}\right|_{r=0}=0, \quad \psi_i(1)=0 \tag{3.a-c}$$

Wich normalized eigenfunctions $\tilde{\psi}_i(r)$ and the eigenvalue μ_i are obtained of the eigenvalue problem solution Eqs. (3.a-c), through the own Integral Transform Technique, presented in Cotta (1993).

3.2. Transformed pair

$$\overline{\theta}_i(z,\tau)=\int_0^1\tilde{\psi}_i(r)W(r)\theta(r,z,\tau)dr\ ,\ \ \theta(r,z,\tau)=\sum_{i=1}^{\infty}\tilde{\psi}_i(r)\overline{\theta}_i(z,\tau) \tag{4.a-b}$$

3.3. Transformed problem

Applying the integral transformation in the problem 1.a-f, are obtained:

$$\sum_{j=1}^{nt}A_{ij}\frac{\partial\overline{\theta}_j(z,\tau)}{\partial\tau}+\frac{\partial\overline{\theta}_i(z,\tau)}{\partial z}=-\mu_i^2\overline{\theta}_i(z,\tau) \tag{5.a}$$

$$\overline{\theta}_i(z,0)=0,\ \overline{\theta}_i(0,\tau)=\int_0^1\tilde{\psi}_i(r)W(r)\theta_e(r,\tau)dr,\ A_{ij}=\int_0^1\tilde{\psi}_i(r)\tilde{\psi}_j(r)dr \tag{5.b-c}$$

3.4. Transformed sistem solution

The transformed system Eqs. (5.ac) is solved using Method of Lines, where the spatial variable is discretized generating an ordinary differential system of first order, which is numerically solved with automatic control on the error. The Eq. (4b) is used to recover the original potential.

3.5. Average temperature and Nusselt number

The average temperature and Nusselt number are given, according to Cotta (1998, p. 267 seq.), by:

$$\theta_m(z,\tau) = 2^{4-2n}(n+1)\int_0^1 r^n W(r)\theta(r,z,\tau)dr\,, \quad Nu(z,\tau) = 2^{2-n}\frac{\left.\frac{\partial\theta(r,z,\tau)}{\partial r}\right|_{r=1}}{\theta(1,z,\tau)-\theta_m(z,\tau)} \qquad \text{(6.a-b)}$$

4. RESULTS

4.1. Computational program

Table 1. Analysis of average Temperature Convergence along z in Flat Plates, to $\theta_e(r,\tau)=1$ with nt=20.

Time	z	Gondim 1997	Castellões 2004	Δz 0.001	0.0005	0.00025	0.00010	0.00005
τ=0.005	0.0000375	0.98641	0.98988	0.99191	0.99175	0.99175	0.99175	0.99175
	0.0001500	0.92320	0.94306	0.95591	0.95228	0.95225	0.95212	0.95177
	0.0002625	0.75544	0.82154	0.83946	0.84662	0.84472	0.84466	0.84469
	0.0003750	0.45177	0.56967	0.57095	0.60460	0.62242	0.62164	0.62163
τ=0.01	0.0000542	0.98983	0.99035	0.98949	0.98949	0.98949	0.98949	0.98949
	0.0002708	0.94570	0.95162	0.95566	0.95559	0.95560	0.95560	0.95559
	0.0004875	0.83059	0.85631	0.86571	0.86420	0.86417	0.86417	0.86417
	0.0007042	0.57192	0.64416	0.66948	0.67897	0.67748	0.67747	0.67747
τ=0.03	0.0001667	0.98084	0.97999	0.97787	0.97787	0.97787	0.97787	-
	0.0008333	0.93003	0.93126	0.93255	0.93255	0.93255	0.93263	-
	0.0015000	0.82670	0.83695	0.83915	0.83915	0.83915	0.83914	-
	0.0021667	0.58926	0.62451	0.63807	0.63751	0.63751	0.63750	-
τ=0.05	0.0002292	0.97537	0.97485	0.97268	0.97268	0.97268	0.97268	-
	0.0011458	0.92193	0.92202	0.92090	0.92090	0.92094	0.92092	-
	0.0020625	0.86255	0.86526	0.86662	0.86662	0.86662	0.86662	-
	0.0029792	0.73887	0.75049	0.75279	0.75279	0.75279	0.75278	-

The Tab. 1. presents a study of the average temperature convergence along z to the channel of flat plates, where Δz represents the interval in discreet mesh along z. Is possible to see that there is an improvement in the convergence to increase the refining of the mesh. Is also made a comparison with data from Gondim (1997) and Castellões (2004), a difference is observed in the results due the withdrawal of the term of axial diffusion in this model. In the table 2 is performed the same study of convergence for the circular duct.

The transient thermal entrance in the plan plates channel and the circular duct is studied through Integral Transform Technique associated with the Method of Lines. Results were presented for the transient average temperature along z, the results are compared with the data presented by Gondim (1997) and Castellões (2004). The present solution proved to be useful in the study of transient thermal entrance inside channels of flat plates and the circular duct, showing a good agreement with the data literature, representing an alternative to strictly numerical methods, and offers larger mathematical freedom in the solution, due of its hybrid nature, analytical-numerical. Tables 3. and 4. show results for different values of the parameter c to the channel of flat plates Tab. 3 and the to the circular duct Tab.4, where c varied from 0.5 to 3, representing the behavior of the average temperature for different types of Non-Newtonian fluids. It is observed that the average temperature varied with the increase of c index. However there was a relatively small variation for the range of the studied values.

Table 2. Analysis of average Temperature Convergence along z in Circular Duct, to $\theta_e(r,\tau)=1$ with nt=20.

Time	z	Δz 0.001	0.0005	0.00025	0.00010	0.00005
τ=0.01	0.0005000	0.96179	0.96174	0.96174	0.96174	0.96174
	0.0023000	0.76845	0.76809	0.76806	0.76806	0.76806
	0.0034000	0.51735	0.52033	0.52019	0.52018	0.52018
	0.0037000	0.42986	0.43572	0.43581	0.43578	0.43578
	0.0041000	0.30425	0.31004	0.31210	0.31205	0.31205
τ=0.03	0.0005000	0.96175	0.96174	0.96174	0.96174	-
	0.0058000	0.78626	0.78625	0.78625	0.78625	-
	0.0087000	0.60847	0.60846	0.60846	0.60846	-
	0.0101000	0.49525	0.49523	0.49523	0.49523	-
	0.0121000	0.30308	0.30324	0.30323	0.30323	-
τ=0.05	0.0005000	0.96175	0.96174	0.96174	-	-
	0.0062000	0.81307	0.81307	0.81307	-	-
	0.0121000	0.66774	0.66774	0.66774	-	-
	0.0162000	0.49458	0.49458	0.49458	-	-
	0.0205000	0.25361	0.25360	0.25360	-	-
τ=0.06	0.0005000	0.96175	0.96174	0.96174	-	-
	0.0092000	0.76204	0.76204	0.76204	-	-
	0.0181000	0.53064	0.53063	0.53063	-	-
	0.0203000	0.44292	0.44292	0.44292	-	-
	0.0235000	0.35831	0.35831	0.35831	-	-

Table 3. Analysis of average Temperature along z in Flat Plates to : $\theta_e(r,\tau)=1$, with nt=20, $\Delta z = 0.00025$.

Time	z	*C* 0.5	1	2	3
τ=0.005	0.0000375	0.9910	0.9917	0.9922	0.9924
	0.0001500	0.9591	0.9522	0.9463	0.9439
	0.0002625	0.8634	0.8447	0.8333	0.8291
	0.0003750	0.5740	0.6224	0.6302	0.6323
τ=0.01	0.0000542	0.9885	0.9895	0.9901	0.9903
	0.0002708	0.9599	0.9556	0.9520	0.9504
	0.0004875	0.8805	0.8642	0.8537	0.8498
	0.0007042	0.6792	0.6775	0.6766	0.6763
τ=0.03	0.0001667	0.9759	0.9779	0.9791	0.9795
	0.0008333	0.9300	0.9325	0.9323	0.9318
	0.0015000	0.8521	0.8391	0.8305	0.8272
	0.0021667	0.6261	0.6375	0.6418	0.6432
τ=0.05	0.0002292	0.9703	0.9727	0.9741	0.9747
	0.0011458	0.9147	0.9209	0.9245	0.9256
	0.0020625	0.8682	0.8666	0.8632	0.8616
	0.0029792	0.7637	0.7528	0.7463	0.7441

Table 4. Analysis of average Temperature along z in Circular Duct, to: $\theta_e(r,\tau)=1$, with nt=20, $\Delta z = 0.00025$.

Time	z	*C* 0.5	1	2	3
τ=0.01	0.0005000	0.9590	0.9617	0.9658	0.9639
	0.0023000	0.7772	0.7681	0.7641	0.7618
	0.0034000	0.4803	0.5202	0.5358	0.5431
	0.0037000	0.3508	0.4358	0.4642	0.4765
	0.0041000	0.0994	0.3121	0.3644	0.3855
τ=0.03	0.0005000	0.9590	0.9617	0.9658	0.9639
	0.0058000	0.7873	0.7862	0.7861	0.7843
	0.0087000	0.5996	0.6085	0.6141	0.6154
	0.0101000	0.4554	0.4952	0.5127	0.5189
	0.0121000	0.1170	0.3032	0.3536	0.3716
τ=0.05	0.0005000	0.9590	0.9617	0.9658	0.9639
	0.0062000	0.8015	0.8131	0.8212	0.8220
	0.0121000	0.6640	0.6677	0.6708	0.6708
	0.0162000	0.4633	0.4946	0.5098	0.5149
	0.0205000	0.0418	0.2536	0.3096	0.3286
τ=0.06	0.0005000	0.9590	0.9617	0.9658	0.9639
	0.0092000	0.7486	0.7620	0.7708	0.7719
	0.0181000	0.5117	0.5306	0.5413	0.5446
	0.0203000	0.3982	0.4429	0.4631	0.4701
	0.0235000	0.2708	0.3583	0.3912	0.4027

5. CONCLUSION

A solution was formally presented to the problem of thermal transient entrance of Non-Newtonian fluids inside ducts. The solution was obtained through Integral Transform associated with the Method of Lines. Results were presented for the average temperature transient along z, to the channel and flat plates and the duct circular. In this work were tested two situations. The results are compared with the data presented by Gondim (1997) and Castellões (2004). The present solution proved to be useful in the study of transient thermal entrance, presenting a good agreement with the literature, representing an alternative to strictly numerical methods, and offers greater mathematical freedom in the solution due its hybrid nature, analytical-numerical.

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