NONLINEAR DYNAMIC MODEL FOR TRANSIENT ELASTOHIDRODYNAMIC CONTACT FORCES

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Abstract. *With the growing market urge for new and improved products, the development time becomes shorter every day. It already has become common sense that engineers have to deal with more efficient simulation models to avoid using trial and error techniques, decreasing the number of prototypes produced and test proceduress. Dealing with complex dynamic systems is a common procedure in the industries nowadays. Consequently, the use of rough approaches for components, such as the stiff rolling elements bearings, diverges from the appropriate means of numerical simulation. In order to fully understand the complexities of the dynamic behavior of rolling elements bearings, its basic functionalities must be studied, i.e., the mechanical contacts linking its elements and the raceways. Those contacts are the only vibration transmission points between the axis inside the bearing and its housing. Thus, the need for more efficient simulation tools greatly increase the demand on the numerical models and approximation methods used in the mechanical area. Hence, a non-stiff model is needed in order to fully understand the behavior of the bearing in a dynamic environment. To comprehend the dynamic behavior of the whole system, each contact has to be examined. Using a Multi-Level method to simulate the elastohydrodynamically lubricated (EHL) contacts; the dynamic behavior of the oil film was analyzed under free vibration conditions. Some key influence factors on the dynamic behavior were also analyzed. As a result, the oil film could be characterized as a nonlinear spring-damper group and the stiff contact unconstrained, making the EHL contacts flexible linking elements on the bearings. Nonetheless other approaches as the linear spring-damper model were investigated, but the accuracy of the linear results was shown to be poor.*

Keywords*: contact forces, elastohydrodynamic lubrication, multi-level methods*

1. INTRODUCTION

With the growing urge of the markets for new and improved products, each year the development time becomes shorter. It already has become common sense that engineers have to deal with more efficient simulation models to avoid the use of trial and error techniques, decreasing the number of prototypes produced and test procedures.

Dealing with complex dynamic systems is a common procedure in the industries nowadays. So, the use of rough approximations for components, such as the rolling elements bearings, diverges from the appropriate means of numerical simulation.

In order to fully understand the complexities of the dynamic behavior of rolling elements bearings, its basic function characteristic must be studied, the mechanical contacts linking its elements and the raceways. Those contacts are the only vibration transmission points between the shaft inside the bearing and the housing of it. The first study on the properties of these contacts were made by H. R. Hertz and published on the work "Über die Berührung fester elasticher Körper". Due to this work, the general contact mechanics of elastic bodies was named after Hertz. Direct from his work the nonlinear behavior of the contact can be attained.

The direct use of the dry contact stiffness, as presented in Villa (2007), can be a useful approximation to the dynamics of the full bearing, but, doing so, the lubricant effects are neglected. Since the first studies on the lubrication of highly loaded contacts, the damping and stiffness of the oil film are known to be effective over the contact. Due to the influence of the elastic deformation on the oil film thickness this kind of lubrication was entitled Elastohydrodynamic (EHL).

The first satisfactory numerical results for the point EHL contact were presented by Hamrock (1976). In his work, a finite difference method was used for the steady state lubricated problem. But there was not until great improvement on the computational power and the use of advanced methods that the transient EHL contact could be analyzed. In 1991 Venner introduced the Multi-Level method for the EHL point contact, using the Multi-Level Multi-Integration, MLMI, to evaluate the elastic deformation due to the high contact pressure (Venner and Ludbrecht, 2000).

Based on a set of meshes with different grid sizes, this method can greatly reduce computational time by operating the different error frequencies components on different discretization grids. Anyhow, a finite difference method is used to evaluate the Reynolds equation on those grids.

Most of the developments on the transient EHL contacts afterwards were in the surface discontinuities field. In Venner and Ludbrecht (1994, 1996) the effect of surface topology was evaluated as a moving transverse ridge through the contact or as waviness of the surface.

Using this new method, Wijnant (1998) first demonstrate the transient contact response due to harmonic excitation and free vibration. On his work, the influence of the transient response is observed over the film thickness and most of all the first linear fit of the dynamic response is introduced, achieving the first simulated values for the oil damping on EHL contacts.

In possession of the fitted values of damping and stiffness, Wensing (1998) observed the influence of the rolling element bearing on a simple rotor system. Also in Wijnant and Wesing (1999) some comments on the contact dynamics can be found. Some improvements on the transient EHL algorithm were also proposed by Goodyer (2001), focused on algorithm optimization and studying some surface topology problems.

However none of those presented works deal with the nonlinearity of the elastic contact as presented by Hertz. In this work, the authors want to introduce a nonlinear fit method for the contact force, in order to obtain the non linear stiffness coefficients of the contact, despite of the linear viscous damper. The results will be compared with the linear fit model, applied on the free vibration response.

2. EHL TRANSIENT MODEL

A Multi-Level algorithm, as presented in Venner and Ludbrecht (2000) and adapted to the transient elliptic load in Wijnant (1998), was used to model the problem. The dimensionless form of the Reynolds equation used is shown in Eq. 1.

$$\frac{\partial}{\partial X}\left(\frac{\overline{\rho}H^3}{\overline{\eta}\overline{\lambda}}\frac{\partial P}{\partial X}\right)+\kappa^2\frac{\partial}{\partial Y}\left(\frac{\overline{\rho}H^3}{\overline{\eta}\overline{\lambda}}\frac{\partial P}{\partial Y}\right)-\frac{\partial(\overline{\rho}H)}{\partial X}-\frac{\partial(\overline{\rho}H)}{\partial T}=0 \tag{1}$$

In this case, H and P are respectively the dimensionless film thickness and pressure, $\overline{\eta}$ and $\overline{\rho}$ are the fluid properties viscosity and density, κ is the contact elliptic ratio, given by the contact geometry, and $\overline{\lambda}$ is a dimensionless group given by:

$$\overline{\lambda}=\frac{6u_m\eta_0(2R)^2}{a^3 p_h}\left(\frac{\mathcal{E}}{\mathcal{K}}\right)^2 \tag{2}$$

In this equation u_m is the sum of the contacting surface velocities, η_0 is the viscosity at ambient pressure, R is the sum of curvatures, $\mathcal{E}$ and $\mathcal{K}$ are the first and second elliptic integrals according to Harris(1991), a is the contact ellipse minor axis and p_h is the maximum pressure over the contact area.

Along with the Reynolds equation, the elastic integral has to be solved for the contact strain. The thickness equation reads:

$$H(x,y)=H_0+SX^2+(1-S)Y^2+\frac{1}{\mathrm{K}\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{P(X',Y')dX'dY'}{\sqrt{(Y-Y')^2+\kappa^2(X-X')^2}} \tag{3}$$

Where the fourth term on the right hand side is the strain integral, the first term is the mutual approach of the bodies, the second and third terms are the geometrical approximation for the elliptic body, where:

$$S=\frac{\mathcal{E}-\kappa^2\mathcal{K}}{\mathcal{K}-\kappa^2\mathcal{K}} \tag{4}$$

In opposition to the surface topology evaluations, the principal variable is the mutual approach. Thus, the dynamic behavior of the rolling element against a fixed raceway can be evaluated. The mutual approach, in the static condition, is obtained from the force balance equation as the integral constant.

$$\frac{3}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}P(X,Y)dXdY=1 \tag{5}$$

For the transient case a equation of motion is needed. So, as presented in Wijnant (1998) and reproduced by Goodyer (2001), the inertia term, related to the mass of the rolling element, is introduced in a dimensionless natural frequency approach, as in Eq. 6.

$$\frac{1}{\Omega_n^2}\frac{d^2H_0}{dT^2}+\frac{3}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}P(X,Y)dXdY=1 \tag{6}$$

Where Ω_n is the dimensionless natural frequency of the system. The Eq. 6 represents the free vibration mode of the system. The influence of the inertia term tends to be small, due to the $1/\Omega_n^2$ factor. Thus, assuming a greater influence of the load term in a harmonic loading condition, the static equilibrium equation can be rewritten as in Wijnant (1998):

$$\frac{3}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}P(X,Y)dXdY=1+A_h\sin(\Omega_eT) \tag{7}$$

Where Ω_e is the excitation frequency and A_h the amplitude. The Eq. 7 is applicable mostly in a quasi-static approach, assuming that the speed of changes in the contact is greater than the speed of the changes in the load, i.e. $\Omega_n >> \Omega_e$. In other words, the changes in the oil film stabilize fast enough not to influence the harmonic response.

Also the pressure dependent relations of the fluid properties have to be evaluated with the transient model. The Dowson and Higgins (1977) density-pressure relation was used. The Roelands equation for the viscosity-pressure relation is applied, as presented in Larsson (2000).

2.1. Linear Dynamic Model

Considering the oil film a set of linear spring and damper, the EHL contacts becomes a linear dynamic system. Figure 1 shows this simplified model of the problem, as initially proposed by Wijnant and Wensing (1999).

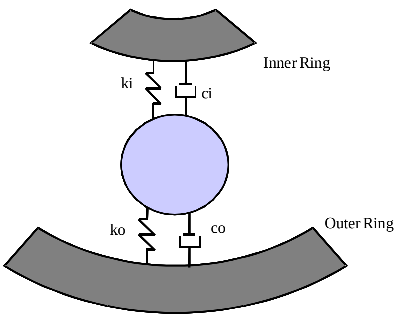


Figure 1 – Approximated spring and damper linear model;

In this case, the equation of motion can be rewritten as:

$$\frac{1}{\Omega_n^2}\frac{d^2H_0}{dT^2}+C_1\frac{d\ H_0}{dT}+K_1H_0=1 \tag{8}$$

The contact forces generated at the inner and outer raceways contacts are adjusted to stiffness and damping constants, given by K_1 and C_1. For the sake of simplicity, the mutual approach H_0 will be replaced by u as the principal variable. For a dimensionless contact force, one might have:

$$f_d=K_1u+C_1\dot{u}=\frac{3}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}P(X,Y,T)dXdY \tag{9}$$

As the dimensionless contact force is given by the integral of the pressure over the contact area, a least-square method can be use to approximate the evaluated forces to the dynamic model. Defining the squared quantity q^2, one might have:

$$q^2 = \sum_{w=0}^{N} \left((f_d)_w - K_1 u_w - C_1 \dot{u}_w \right)^2 \tag{10}$$

Minimizing the relation for K_1 and C_1 the following linear system can be achieved:

$$\begin{bmatrix} \sum_{w=0}^{N}(u_w)^2 & \sum_{w=0}^{N}\dot{u}_w u_w \\ \sum_{w=0}^{N}\dot{u}_w u_w & \sum_{w=0}^{N}(\dot{u}_w)^2 \end{bmatrix} \cdot \begin{bmatrix} K_1 \\ C_1 \end{bmatrix} = \begin{bmatrix} \sum_{w=0}^{N}(f_d)_w u_w \\ \sum_{w=0}^{N}(f_d)_w \dot{u}_w \end{bmatrix} \tag{11}$$

The approach proposed by Wijnant (1998) uses not the motion equation, but the harmonic vibration equation. In the same way, the least square method results in a linear system of equation, as Eq. 11, substituting f_d by $1 + A_h \sin\left(\Omega_e T\right)$. In order to assimilate the influence of the inertia term, the first method is going to be used.

The least-square fitting method for the lubricant forces is commonly used in experimental methods for hydrodynamic bearings, as presented in Zhao (2005) and Zhou (2004). However the nonlinear form is used. The next section introduces the nonlinear fitting process for the EHL case.

2.2. Nonlinear Dynamic Model

In the same way that the dynamic force in Eq. 9 can be adjusted to a linear spring and damper model, it can be adjusted to a nonlinear model, as proposed in Zhao (2005) and Zhou (2004). In this case, only one direction is considered, i.e., the only possible motion is normal to the contact plane. Thus, the dynamic force is written as:

$$f_d = K_1 u + C_1 \dot{u} + K_2 u^2 + C_2 \dot{u}^2 + K_3 u^3 + C_3 \dot{u}^3 + B_4 u \dot{u} \tag{12}$$

Where B_4 is the mixed dynamic coefficient. In the same way as the linear method, a linear system can be found, Eq. 13. Solving the linear system leads to a solution vector with all coefficients. As seen on Eq. 11 and Eq. 13, the system is symmetrical and can be easily evaluated by straightforward computational methods.

$$\begin{bmatrix}
\sum_{w=0}^{N}(u_w)^2 & \sum_{w=0}^{N}\dot{u}_w u_w & \sum_{w=0}^{N}(u_w)^3 & \sum_{w=0}^{N}(\dot{u}_w)^2 u_w & \sum_{w=0}^{N}(u_w)^4 & \sum_{w=0}^{N}(\dot{u}_w)^3 u_w & \sum_{w=0}^{N}(u_w)^2 \dot{u}_w \\
 & \sum_{w=0}^{N}(\dot{u}_w)^2 & \sum_{w=0}^{N}(u_w)^2 \dot{u}_w & \sum_{w=0}^{N}(\dot{u}_w)^3 & \sum_{w=0}^{N}(u_w)^3 \dot{u}_w & \sum_{w=0}^{N}(\dot{u}_w)^4 & \sum_{w=0}^{N}(\dot{u}_w)^2 u_w \\
 & & \sum_{w=0}^{N}(u_w)^4 & \sum_{w=0}^{N}(u_w)^2(\dot{u}_w)^2 & \sum_{w=0}^{N}(u_w)^5 & \sum_{w=0}^{N}(u_w)^2(\dot{u}_w)^3 & \sum_{w=0}^{N}(u_w)^3 \dot{u}_w \\
 & & & \sum_{w=0}^{N}(\dot{u}_w)^4 & \sum_{w=0}^{N}(u_w)^3(\dot{u}_w)^2 & \sum_{w=0}^{N}(\dot{u}_w)^5 & \sum_{w=0}^{N}(\dot{u}_w)^3 u_w \\
 & \text{sym} & & & \sum_{w=0}^{N}(u_w)^6 & \sum_{w=0}^{N}(u_w)^3(\dot{u}_w)^3 & \sum_{w=0}^{N}(u_w)^4 \dot{u}_w \\
 & & & & & \sum_{w=0}^{N}(\dot{u}_w)^6 & \sum_{w=0}^{N}(\dot{u}_w)^4 u_w \\
 & & & & & & \sum_{w=0}^{N}(u_w \dot{u}_w)^2
\end{bmatrix} \cdot \begin{bmatrix} K_1 \\ C_1 \\ K_2 \\ C_2 \\ K_3 \\ C_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} \sum_{w=0}^{N}(f_d)_w u_w \\ \sum_{w=0}^{N}(f_d)_w \dot{u}_w \\ \sum_{w=0}^{N}(f_d)_w (u_w)^2 \\ \sum_{w=0}^{N}(f_d)_w (\dot{u}_w)^2 \\ \sum_{w=0}^{N}(f_d)_w (u_w)^3 \\ \sum_{w=0}^{N}(f_d)_w (\dot{u}_w)^3 \\ \sum_{w=0}^{N}(f_d)_w \dot{u}_w u_w \end{bmatrix} \tag{13}$$

3. NUMERICAL SIMULATION

Along with the finite difference Multi-Level method for the evaluation of the Reynolds equation a hybrid relaxation method was used as presented in Venner and Ludbrecht (2000). Both Gauss-Sidel and Jacobi models were used for the discretization of the problem. Even though the mesh dependent relaxation triggering value proposed for choosing between models produced fine values, making this a fixed value improves convergence on finer grids, as shown in Nonato and Cavalca (2008).

The need for two relaxation procedures comes from the dual behavior of the problem. As can be seen in Eq.1, when the pressure is high, i.e., inside the contact area, the viscosity ratio becomes extremely high and the equation is governed mostly by the advection operator. On the other hand, when the pressure is low, the Pouisseuille terms are more effective over the oil film flow.

As given in Wijnant (1998), the dimensionless natural frequency of the EHL contact is related to the one in the dry hertzian contact. In that case, the period of oscillation is given by $T_n = 5.13/\Omega_n$, so the values for the constant Ω_n was chosen to be *0.5*, *1* and *2* times the dry contact frequency of *5.13*, for most of the simulations.

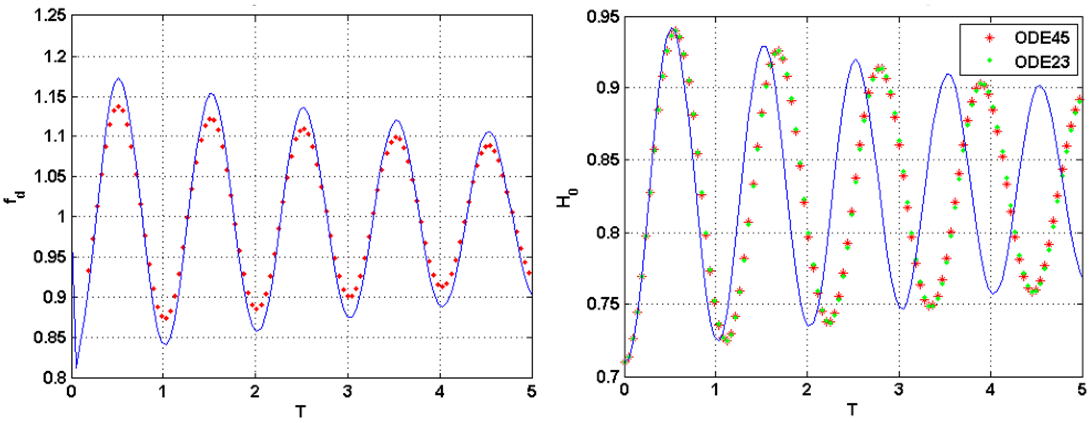
In order to characterize the EHL contact, the dimensionless parameters *M* and *L* introduced by Moes (1992) were used. They are related to the imposed load and the lubricant parameters respectively. Also is necessary to specify the elliptic ratio of the contact. As shown in Wijnant (1998), as well as in Nonato (2009), the effect of the elliptic ratio tends to decouple both directions in the Reynolds and thickness equations, leading to a result similar to one of lower load cases. Thus, all contacts presented in this paper will be evaluated using the elliptic ratio 1.

The motion equation was evaluated using a Newmark-ß method. Because the pressure is fully dependent on the integrated variable H_0, the discrete equation has to be analyzed during the relaxation of the Reynolds equation. To initiate the motion of the system a disturbance of *90%* of the static equilibrium was induced in the suspended mass position, i.e., the mutual approach.

4. DYNAMIC FIT RESULTS

From the numerical simulation, both position and velocity of the mutual approach were obtained for each step on the time integration. The dynamic force is also obtained in each step by integrating the pressure over the contact area. Using the linear system from Eq. 11, one might obtain the values of K_1 e C_1 for the linear model.

Figure 2 shows the transient dynamic forces over the contact and the mutual approach with both corresponding linear dynamic fit. The modeled contact was $M = 100$, $L = 15$, $\kappa = 1.0$ and $\Omega_n = 5.13$, or the same dry contact frequency. The solid blue lines are the numerical simulation results and the dotted lines are the fitting results. To evaluate the mutual approach using the fit data, two different Runge-Kutta implicit integrators were use, one of the fourth and fifth orders, ode45, and one of the second and third, ode23, from the MatLab® commercial package.



a) b)

Figure 2 – a) Dynamic force and b) mutual approach linear dynamic fit for $M = 100$, $L = 15$, $\kappa = 1.0$ and $\Omega_n = 5.13$;

Even being close to the simulated results, the least square linear dynamic model was not sufficient to represent the mutual approach. Despite the fair frequency adjust on the force, the same behavior was not found on the mutual approach. In order to quantify discrepancies of the fitting process, an error measuring quantity has to be introduced. Using the mean value of the punctual difference of both results, the force error, e_f is defined. For this case $e_f = 1.4607 \times 10^{-3}$, $C_1 = 0.00594$ and $K_1 = 1.2070$.

The linear error is even greater in minor load cases or when the fluid compressibility has more influence than the applied load. For instance, when $M = 5$ and $L = 15$; due to the viscous effects of the lubricant the predicted mutual approach is less negative. In that case the contacting bodies are lifted apart by the lubricant and the dynamic behavior has an inverse response. The static equilibrium occurs at $H_0 = -0.3807$. The obtained dynamic coefficients were

$C_1 = -0.01028$ and $K_1 = -2.5792$. It is clear that with a negative stiffness and damping the dynamic system is unstable.

To avoid incorrect fitting and improve the adjusted model the nonlinear dynamic terms were introduced. Using the linear system of Eq. 6, the same case shown in figure 2 can be evaluated. To avoid using an excessive number of parameters a sensitivity analysis was made. The table 1 shows the influence on the force error when removing one of the coefficients at a time compared to the analysis performed with all of them.

Furthermore an error for the acceleration of the mutual approach was included. Using the fitted values and the dimensionless frequencies, the acceleration can be checked with the one calculated throughout the numerical analysis. Given by $e_{\ddot{u}}$, this error is also evaluated by the mean value of the punctual differences.

Table 1 – Sensitivity analysis of the nonlinear model, effect over the error when removing each of the coefficients;

Coef.	**All**	$-K_1$	$-C_1$	$-K_2$	$-C_2$	$-K_3$	$-C_3$	$-B_4$
e_f	4,0598E-08	3,1971E-05	4,4172E-08	6,8039E-07	5,9783E-08	1,9215E-06	4,3791E-08	4,6021E-08
$e_{\ddot{u}}$	2,7114E-05	1,6217E-02	2,9566E-05	4,4901E-04	3,8068E-05	1,2183E-03	2,9376E-05	3,1213E-05

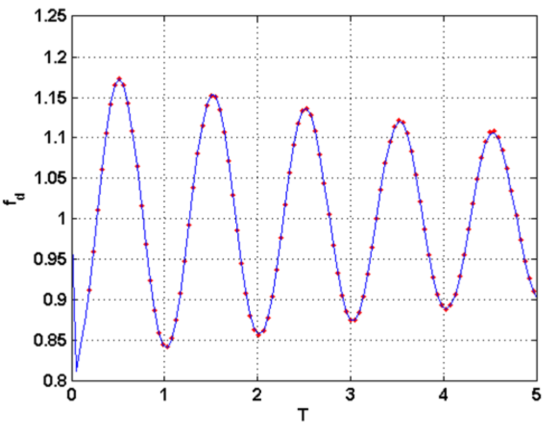
The only effective influencing coefficients over the force error, when removed from the analysis, are K_1, K_2 and K_3. So, as predicted by Wijnant (1998), the oil film would have only a viscous damping. In this work, only the linear damping coefficient is used.

Hence, the same case as presented in figure 2, using the nonlinear model results shown in table 2. Also for two different cases of natural frequencies are present.

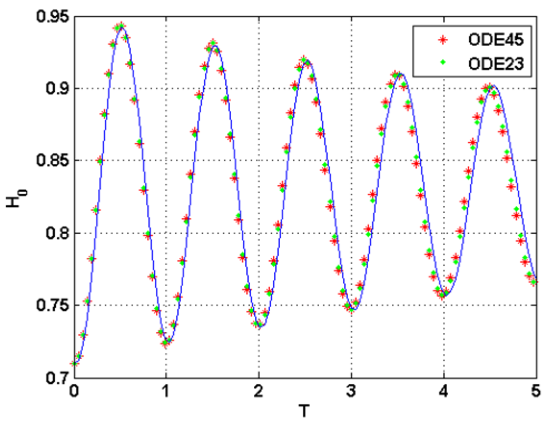
Table 2– Nonlinear fitting model results for $M = 100$, $L = 15$, $\kappa = 1{,}0$;

Ω_n	K_1	D_1	K_2	K_3	e_f	$e_{\ddot{u}}$
2,65	0,9108	0,0112	0,3330	0,0169	3,2120E-08	9,7913E-06
5,13	0,9293	0,0093	0,2858	0,0514	5,9053E-08	3,8751E-05
10,26	0,8933	0,0081	0,3396	0,0377	5,3282E-09	5,0883E-06

The results for the three frequencies had a fairly good relation, which indicates enough precision on the fitting method. Some of the divergence comes from the time discretization of the numerical results, where the fixed timestepping procedure can jeopardize the convergence. Figure 3 shows dynamic force and mutual approach for the nonlinear model.



a)



b)

Figure 3 – a) Dynamic force and b) mutual approach nonlinear dynamic fit for $M = 100$, $L = 15$, $\kappa = 1.0$ and $\Omega_n = 5.13$;

Also the influence of the initial condition has to be evaluated. Using three different initial conditions, *90%*, *80%* and *60%* of the static equilibrium, the dynamic coefficients were obtained and listed on table 3.

It can be seen that even with great variation of the initial condition, the dynamic coefficients tend to close values and the error are kept small. Once more the time-step is a critical factor, with greater deviation comes greater gradients on the responses and worst discretization.

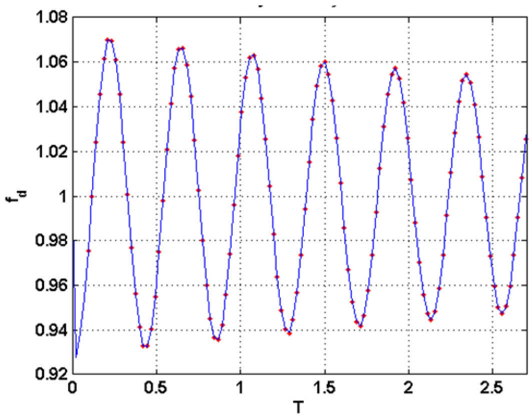
However, for all the cases the viscous damping had very similar values. It shows that the linear approximation to the damping is very precise, but in the other hand, the third degree polynomial is not as accurate to represent the EHL contact stiffness. This behavior can be expected as the dry contact stiffness does not behave as a polynomial and, furthermore, can be modeled differently, as used in Fukata (1985) to describe the rolling element bearings orbits.

Table 3 – Nonlinear fit data for tree different initial conditions;

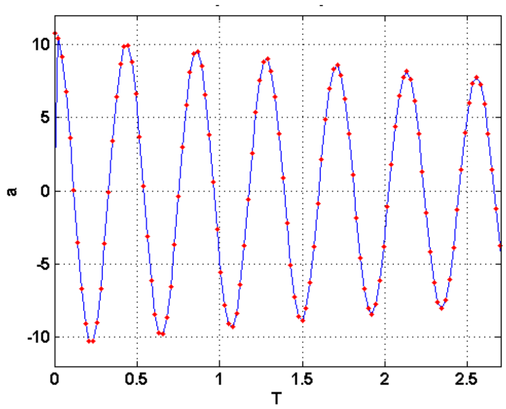
Initial Condition	K_1	D_1	K_2	K_3	e_f	$e_{\ddot{u}}$
90%	0,6694	0,0132	0,5387	-0,0475	8,7696E-09	6,6995E-06
80%	0,6833	0,0136	0,5200	-0,0393	1,1525E-07	4,6498E-05
60%	0,7311	0,0138	0,4568	-0,0119	3,9656E-07	1,2609E-04

Using the proposed model, the contacts present on a real rolling element bearing can be analyzed. Assuming a pure radial load of *1500N* on a deep groove ball bearing, which has *36mm* of pitch diameter, a set of reasonable values for the input parameters are $M = 9912$, $L = 11{,}75$, $\kappa = 0{,}1092$, for the main loaded contact with the inner ring.

With a smaller simulation time and larger number of points on the time domain, to avoid miss representativeness of the results, the real load case was evaluated. Figure 4 shows the nonlinear fit for the dynamic force and the acceleration of the contact. Table 4 shows the fitted values and errors of the problem.



a)



b)

Figure 4 –Nonlinear fit of the a) dynamic contact force and b) acceleration for an inner ring-element contact on a real deep groove ball bearing;

Also the mutual approach fit can be evaluated. Figure 5 shows a zoomed result for the deep groove ball bearing case. The integrator precision is heavily influenced by the numerical time step. Even thought, the error of the fit model is low, as in table 4, some divergence is noticeable.

Table 4 – Nonlinear fitted data for the deep groove ball bearing case;

K_1	D_1	K_2	K_3	e_f	$e_{\ddot{u}}$
0,3652	0,0017	0,8098	-0,1509	1,1208E-10	9,2454E-07

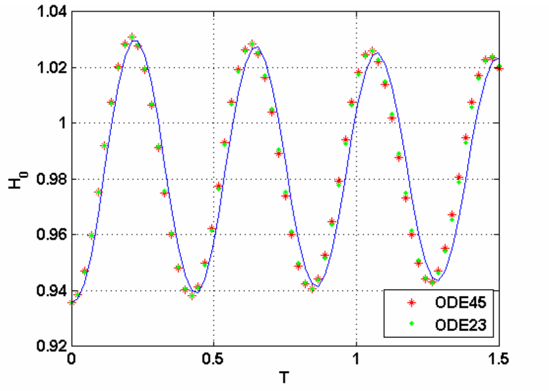


Figure 5 –Nonlinear fit of the mutual approach for both integrators;

5. CONCLUSION

With the proposed nonlinear model results, the characterization of the EHL contact forces is better depicted than the previous linear results and highly approximates the results obtained from the numerical simulation. Thus, a more reliable approach for the real EHL dynamic behavior on real contacts can be observed and, furthermore, quantified.

The aim of this work was completely fulfilled by the developed methods. The model achieved good convergence levels and its representativeness is quite satisfactory. Despite minor errors introduced by the fix discretization on the time domain, the fitted mutual approach, as the principal variable, corresponds closely to the numerical simulated results.

The nonlinear model, as it is, can also be applied to the harmonic excitation results, given a wider range of application on the dynamic estimation of EHL contacts. On any of the previous situation the contact is therefore modeled as a flexible dynamic linkage and further vibration studies can be performed over the whole mechanical system.

6. ACKNOWLEDGEMENTS

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