IDENTIFICATION OF MATERIALS AND PAVEMENT LAYER THICKNESS USING INVERSE PROBLEM

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Abstract. *Pavement can be defined as multiple semi-infinite layer structure destined whose function is to transmit the load traffic to the sub grade, beyond providing to the users good ride conditions as comfort and safety. The pavements degradation is quite fast when compared to other engineering structures (buildings or dams for instance); it is design to perform well from 10 to 20 years of traffic; after that are necessary periodic interventions (from 4 to 8 years). The knowledge of the existent structure (thickness and layers properties) is basic to be possible to plain maintenance strategies that can optimize the investments and maximizing the benefits for the society. In Brazil is usual open windows (or trenches) in the pavement to determine the layers thickness; this procedure besides onerous cause traffic jam and the studied site should be closed to the traffic temporarily. Non destructive techniques of thickness determination already are in use, as a GPR - Ground Penetration Radar, but its use is not practice in Brazil. The objective of this work is develop a method to identify the materials and thickness analysis on pavements structure through the thermal properties variation of each layer, minimizing the costs and the damages to the pavement and supplying to the engineering a structure profile. The data about temperature was monitoring in two pavements, (rigid and flexible), with materials, thickness and thermal properties known; it was accomplished for two months through a thermal couples sequence introduced in the pavement in a cylindrical hole of 0,015 m of diameter and 1 m of depth. Parallel, the thermal diffusivities of the materials used in the pavements were experimentally determined constituting a database of such properties. A mathematical model of one-dimensional heat transfer in semi-infinite medium was proposed to describe the temperature variations in function of the time and depth. The energy equation was applied successively, with first kind boundary conditions adapted for three layers of pavements, and one more soil layer with infinite thickness (sub grade). The objective of the resolution of the inverse problem, in this case, consists of estimate the thermal diffusivity of the materials and its respective thickness, whit base in the heat transfer data in the pavements, for subsequent identification of the material using the database. The methods for resolution of the inverse problem were: Net Research Modified and Genetic Algorithm. All these methods presented similar answers, evidencing the possibility to identify the proposed variables, however with times of execution different computational, with quick disadvantage for the Net Research Modified. The proposed method was efficient for identify the materials and estimate the thickness of the pavements, however the identification of the material through the thermal diffusivity, it depends on a database with necessary information on the intervals of variation of this thermal property in each material type.*

Keywords*: Pavement Maintenance, Layers Identification, Inverse Problem*

1. INTRODUCTION

Pavement can be defined as multiple semi-infinite layer structure whose function is to transmit the load traffic to the subgrade, beyond providing to the users good ride conditions as comfort and safety. The pavements degradation is quite fast when compared to other engineering structures (buildings or dams for instance); it is design to perform well from 10 to 20 years of traffic; after that are necessary periodic interventions (from 4 to 8 years).

The knowledge of the existent structure (thickness and layers properties) is basic information to be possible to plain maintenance/rehabilitation strategies that can optimize the investments and maximizing the benefits for the society. As the rule, there isn't a database about the pavement information on our country.

In Brazil is usual open windows (or trenches) in the pavement to determine the layers thickness; this procedure besides onerous cause traffic jam and the road need be closed to the traffic temporarily (Senço, 1997). The DNIT– Departamento Nacional de Infra-Estrutura de Transportes recomend the preliminar study thought to trenches to determine the pavement layer characteristics as wet content, density and CBR (California Bearing Ratio) besides the asphalt layer properties need to be done each *2 km,* or closer, in case of change of structure profile. It's consists in open a hole in the pavement surface with $1\ m^2$ and remove the material to laboratory characterization. This procedure are very expensive (involve people, special equipments) besides cause traffic interferences (DNIT, 2006).

Not always its necessary analyses all the variables mentioned previously. The layer thickness and the material identification are essential information and in some cases, enough to take a decisions. The aim of this paper is to propose a new method for identification thickness and materials type on pavements structures. The proposal method has as base the thickness and materials identification through the diffusivity estimation, and thickness of each layer through the heat transfer resolution through inverse problem using data of temperature variation in function of the time and of the depth.

2. HEAT TRANSFER EXPERIMENTS ON PAVEMENTS

The tests to measure the temperature in pavements profile are being accomplished at the UNIJUÍ on Laboratory of Civil Engineering. The temperature are being monitored in two pavements, a PCC – Portland Cement Concrete and a Flexible one through thermal probes (temperature sensor in several depths with automatic acquisition data), composed by *n* temperature sensor, put in holes of *0,015 m* of diameter by *0,9 m* depth in each one of the pavements. The Figure 1 presents a scheme about monitored pavements.

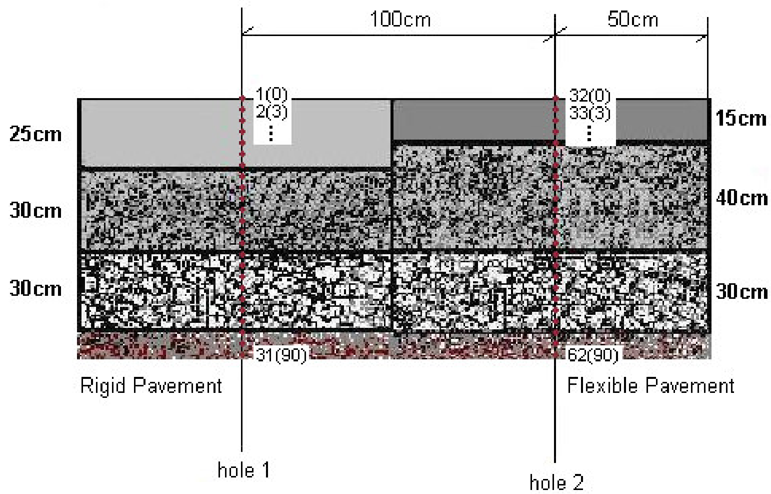

Figure 1. Pavement scheme (the first no. indicates the channel and the second the depth)

The pavements are characterized as following:

Subgrade: composed of natural clayed soil (typical of the Ijuí area);

Subgrade Reinforcement: granular base composed of quarry residue, with *30 cm* depth, responsible by water drainage in case there is infiltration. The quarry residue layer has a uniform grain size distribution, facilitating the water seepage.

Base: *30 cm* thickness of in the rigid pavement and *40 cm* in the flexible pavement, it is composed by granular base couse, when compacted there are a low void layer; it is responsible for the strength and stress distribution caused by the traffic.

There were two sealing layer: *Portland Cement Concrete (rigid pavement)*: with *25 cm* of thickness was made with Portland Cement, aggregate and water, and *Bituminous (flexible pavement)*: is compound by *15 cm* cold pre-mixed, made with emulsion (water, asphalt and emulsion) and aggregate; it was use a sealing coat too, to waterproof the bituminous surface.

3. DIRECT PROBLEM

The mathematical model that describes the heat transfer problem in pavements was obtained from Energy Equation (Ozisik, 1993). For the one-dimensional case, the Differential Equation of Heat Conduction, is given by Eq.1:

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha_i} \cdot \frac{\partial T}{\partial t} \quad \text{in } 0 < z < L,\ t > 0 \text{ for the layers } i = 1, 2 \text{ e } 3 \tag{1}$$

The boundary and initial conditions of the problem are given by Eqs. 2 to 4:

$$T(0,t) = T_S(t) \quad \text{for } t > 0 \tag{2}$$

$$T(L,t) = T_L(t) \quad \text{for } t > 0 \tag{3}$$

$$T(z,0) = T_{0i}(z) \quad \text{in } 0 < z < L \tag{4}$$

where T is the temperature (*ºC*),
z is the depth (*m*),
t is the time (*s*),
α_i is the thermal diffusivity of the layer i (*m²/s*),
T_0 is the initial condition (*ºC*),
T_S is the temperature in the surface (*ºC*),
T_L is the temperature in the end of the third layer (*ºC*),
L is the depth of the pavement (*m*).

The boundary conditions of the proposed problem are of the first kind and varied in function of the time. The thermal diffusivities are different for each layer. For those reasons, the Direct Problem (DP), given by Eqs. 1 to 4, was solved numerically by Central Finite Differences Method, using the explicit scheme. The Eq. (5) show the discretization of the Eq. 1, for this scheme. A grid with *380* points was used due to need of high precision in the determination of the thickness of the layers.

$$T_j^{t+1} = \left(\frac{\alpha\Delta t}{\Delta x^2}\right)\left(T_{j-1}^t + T_{j+1}^t\right) + \left(1 - \frac{2\alpha_i \Delta t}{\Delta x^2}\right)T_j^t \tag{5}$$

where Δx and Δt are the space and time intervals, respectively. The subscript j indicate the positions in the space grid and the superscript t indicate the temporary iterations.

4. INVERSE PROBLEM

The Inverse Problem (IP), of this work, it consists of the determination of the thermal diffusivities and thickness of the pavement layers, through the estimate of the values of these greatness; of the calculation of the temperature distribution in the space and in the time with DP and of the comparison of the distribution of calculated temperature, with the temperature distribution obtained experimentally.

For the resolution of the IP three methods of estimate of parameters were used: Method of Net Research Modified, Genetic and Memetic Algorithm.

The Method of Net Research Modified (NRM) it is described by Silva Neto and Moura Neto (2005) and Borges (2008) and it was adapted for the proposed problem, in the form of the following algorithm:

1. Estimate the intervals $I_p = [\beta_{p\min}, \beta_{p\max}]$ of values of each parameter β_p where *p=1,2,3, ...n* (number of parameters) that contain the optimal value of $\beta_p(\beta_{ot})$.

2. A partition of *s* points $\beta_{pk} = \beta_{p\min} + (k-1)\Delta\beta_p$ is built with *k=1,2,3,... ,s* and $\Delta\beta_p = (\beta_{p\max} - \beta_{p\min})/(s-1)$.

3. For each sequence of values $(\beta_{1k}, \beta_{2k}, ..., \beta_{nk})$ for *k=1,2,3,... ,s* the Direct Problem is solved using the numerical solution.

4. Calculate the differences d_i between the estimated solutions and the experimental data using Eq. (6).

$$d_i = \sum_{j=1}^{m}\sum_{t=0}^{tf}(T_{est}(j,t) - T_{exp}(j,t))^2 \tag{6}$$

Where , $i = 1,2,3,...,s^n$ e $T_{\exp}(j,t)$ they are the experimental data for each depth *j* and time *t*.

5. Identify the smallest value of $d_i(d_{\min})$. This difference corresponds to the set of parameters β_{ot} for the interval I_p.

6. Refinement of the solution. New interval $I_p = [\beta_{p\min}, \beta_{p\max}]$ is defined, such that $\beta_{p\min} = \beta_{pot} - \Delta\beta_p$ and $\beta_{p\max} = \beta_{pot} + \Delta\beta_p$.

7. Repeat the steps 2 to 6, esteeming so many intervals I_{pm}, m=1,2, ...nr (number of refinements) until that

$$\left|d_{\min}^{i+1} - d_{\min}^{i}\right| < \varepsilon$$

Where ε is the stopping criteria.

For the Genetic and Memetic algorithms was considered each set of parameters to be estimate like a individual or chromosome, in that each parameter is a gene and all the individuals a population. The difference among the Genetic Algorithm (GA) and the Memetic Algorithms (MA) it is that this makes a local search (box "Local Search", in Fig.2) of the parameters before restarting the whole process, while that GA doesn't make it. The Figure 2 presents the basic structure of the Memetic Algorithm, Britto (2007).

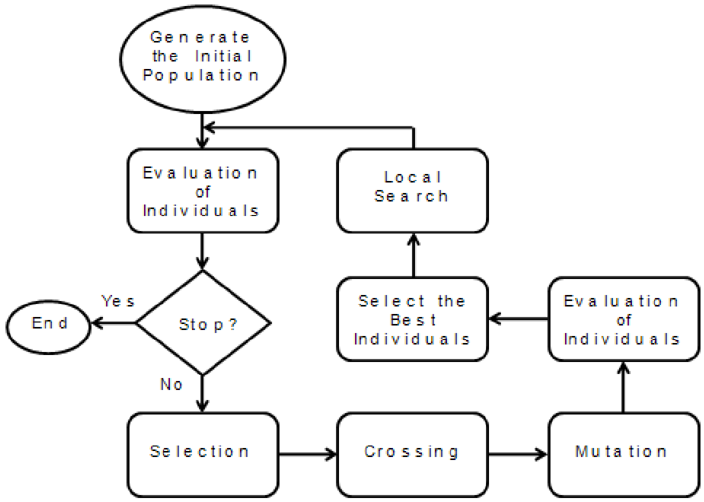

Figure 2. Fluxogram of the Genetic and Memetic algorithms.

The Genetic and Memetic algorithms were described by Britto (2007) and Leandro *et al*. (2008) and it was adapted for the proposed problem:

1. Consider each chromosome $Cro_i = [Gene1_i \quad Gene2_i \quad \cdots \quad GeneN_i]$ of N parameters. Generate the initial population being each gene generated by $Gene(n)_i = LI_i + r_i(LS_i - LI_i)$ where LI_i and LS_i are respectively the limits inferior and superior of the parameter *n*, and r_i is a random coefficient.
2. Make an evaluation of each individual using the determination coefficient $R_i^2 \geq R_{ot}$ where R_{ot} is the stopping criteria.
3. Order the population according with the determination coefficient of each individual's. Select the best individuals to begin the evolutionary process.
4. Select individuals for the crossings through a normalized geometric ranking, according with each individual's aptitude:

$$p(i) = \frac{q(1-q)^{rank(i)-1}}{1-(1-q)^P} \tag{7}$$

Where *q* represents the unitary rate of the chromosome with larger aptitude, *P* the size of the population, *rank(i)* the position of the individual when being classified in decreasing order by aptitude and *p(i)* it is the probability of selection of the chromosome *i*.

5. New individuals (sons) by arithmetic crossing:

$$Filho1 = r_j \cdot Cro1_i + (1-r_j)Cro2_i \tag{8}$$

$$Filho2 = r_k \cdot Cro2_i + (1-r_k)Cro1_i \tag{9}$$

Where *Cro1* and *Cro2* are the individuals selected for the crossing, r_j and r_k are random coefficients.

6. New individuals (sons) by arithmetic crossing with the extremes:

$$Filho3 = \begin{cases} r_j \cdot Cro1_i + (1-r_j)LI_i & se \quad r_k \leq 0{,}5 \\ r_j \cdot Cro1_i + (1-r_j)LS_i & se \quad r_k > 0{,}5 \end{cases} \tag{10}$$

Where r_j and r_k are the random coefficients, LI_i and LS_i are respectively the inferior and superior limits of the parameters.

7. New individuals (sons) by uniform mutation. The genes randomically selected, through a mutation probability *PM (%)* of the population, are substituted by other value any inside of the search space of the problem, as well as each gene of the initial population it was generated.

8. Make an evaluation of each new individual using the determination coefficient.

9. Order the population according with the determination coefficient of each individual's. Select the best individuals.

10. Make a Local Search (that step only exists for the Memetic Algorithm) to each number of initially defined generations. That local search can be a method of estimate of parameters like the Method of Net Research Modified, Levenberg-Marquadt Method or other. For the resolution of the proposed problem the Levenberg-Marquadt Method was used (Ozisik and Orlande, 2000).

11. If local search was done, make an evaluation of each individual again, order by aptitude and the select of the best.

12. Repeat the evolutionary process of the 4th. to the 11st. step until satisfying the stopping criteria.

5. RESULTS AND DISCUSSIONS

To test the methods of inverse problem, data were simulate (since the real experimental data are not still available) using values of the literature about the thermal diffusivity and the thickness of the three layers, being obtained a temperature distribution with the execution of the DP. Such data will become called in this work, just of experimental data.

The Figure 3 presents the temperature distributions calculated with the thermal diffusivities and great thickness obtained by the resolution of the IP with the three methods described previously and the experimental data. In this figure it is not possible to see whole the curves, since these they are put upon, being just more visible the green curves that represent GA. A correspondence is observed total practically between the distribution of calculated temperature and the experimental data, what can be confirmed by the close correlation coefficient of the unit, shown in the Table 1, indicating the efficiency of the used methods.

The obtained values, so much for the diffusivity as for the thickness of the layers for the three methods, they present some differences, mainly in the value of the thickness of the third layer, as it can be observed in Fig. 3. this difference is due to the small variation of the temperature in the beginning in that layer, what does with that several values of z satisfy the stopping criteria of the search methods. A solution for such problem is to provoke larger temperature gradients than those provoked by the surface conditions, using sources of heat in different depths.

Analyze it of the mistake made in the estimate of the incognito of the inverse problem, it was done in two ways: the use of the average ($\bar{x}$) of the values obtained by the three methods, as final result and the consideration of a standard deviation σ_x (Eq. 11) around that average and the comparison with the experimental data. The Table 1 exhibition that the largest deviation pattern in the variable thickness, was around 0,03 m, for the layer 3. For the definition of standard deviation, it means that the maximum mistake made in, approximately, 64% of the applications of the methods, it is of the order of 11,28%. Para the variable thermal diffusivity this mistake is of the order of 3% for the layer 2. The mistakes obtained for the other variables are smaller. Even so, everybody the results of the three methods were in the interval $(-\sigma_x, \sigma_x)$, what means a small dispersion around the average. Monday forms of analysis of the mistake it is the comparison of the data obtained by estimate with the experimental data, it indicates that those also meet in the interval $(-\sigma_x, \sigma_x)$, for the five calculated variables.

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}} \tag{11}$$

where x_i is the dice of the iteration *i*;

$\bar{x}$ is the average arithmetic of the data;

n is the number of data.

Considering the interval of a standard deviation between the average of each thickness and thermal diffusivity is noticed that the thickness and the experimental thermal diffusivities meet in this interval, evidencing the estimate quality.

The three methods present very close of 1 determination coefficient, reaffirming the efficiency of precision of the methods. It is possible this precision is due to the use of a stopping criteria very demanding in the simulations.

The method NRM has larger computational cost compared to the other methods, depending basically on the partition of points of the incognito ones. GA and MA depend on the initial population generated for the evolution of the population and to satisfy the stop approach, being like this, the time of execution is variable to each simulation, but smaller incomparably than the method NRM, as exhibition the last line of Tab.1.

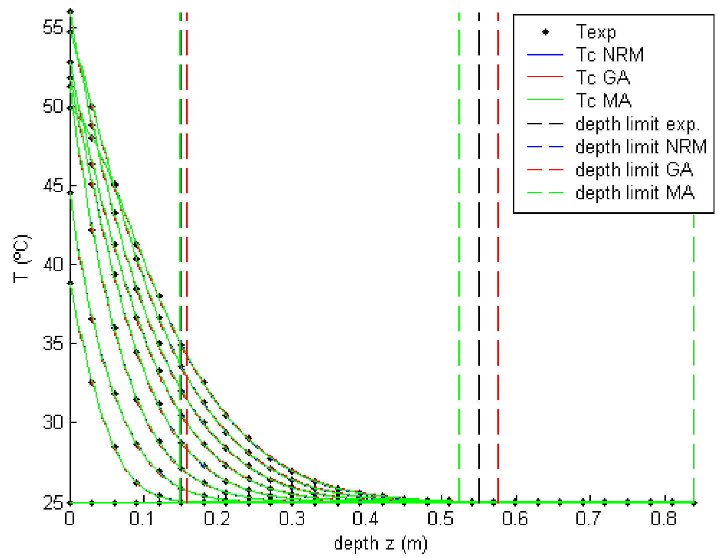

Figure 3. Results obtained with the different methods of inverse problems.

For comparison of the methods of IP it shows the Tab. 1 with the data esteemed by the used methods and the experimental data:

Table 1. Thermal Diffusivity and Thickness: experimental and esteemed by different methods

-	NRM	GA	MA	Exp. Data	σ	$\bar{x}$
$\alpha_1\ (m^2/s)$	$5{,}008\cdot10^{-7}$	$4{,}9374\cdot10^{-7}$	$4{,}9758\cdot10^{-7}$	$5\cdot10^{-7}$	$3{,}5345\cdot10^{-9}$	$4{,}9737\cdot10^{-7}$
$\alpha_2\ (m^2/s)$	$5{,}976\cdot10^{-7}$	$6{,}3286\cdot10^{-7}$	$6{,}0253\cdot10^{-7}$	$6\cdot10^{-7}$	$1{,}9094\cdot10^{-8}$	$6{,}1100\cdot10^{-7}$
$\alpha_3\ (m^2/s)$	$7{,}008\cdot10^{-7}$	$6{,}9568\cdot10^{-7}$	$6{,}9574\cdot10^{-7}$	$7\cdot10^{-7}$	$2{,}9389\cdot10^{-9}$	$6{,}9741\cdot10^{-7}$
$z_1\ (m)$	0,148	0,1581	0,1481	0,15	0,0058	0,1514
$z_2\ (m)$	0,428	0,4173	0,3760	0,40	0,0275	0,4071
$z_3\ (m)$	0,264	0,2645	0,3158	0,29	0,0298	0,2815
R^2	0,9999997439	0,9999894990	0,9999987274	-	-	-
Time	1h 51’ 10s	15’ 46s	5’ 34s	-	-	-

6. CONCLUSIONS

The accomplished numeric tests showed that the methods are effective in the search of the unknowns of the inverse problem, obtaining a very good approach with the experimental data. Finally, that methodology is had as a new alternative in the structural evaluation of pavements for restoration. The installation of sensor of temperature in the pavement turns the less destructive process, with smaller cost economic, smaller interference in the traffic and less people involved in comparison to the trenches.

In a posterior stage of the work the authors intends to test the methods of IP with the real experimental data and to verify the possibility of identification of the pavement materials, using information of a database on the thermal diffusivity of these materials, and modify the model to consider the presence of sources of heat, mainly in the deepest layers, with the objective of producing accentuated temperature gradients than those produced by the influences of the thermal conditions of surface, improving like this the precision of the estimate of the incognito ones for the methods of inverse problem.

7. ACKNOWLEDGEMENTS

To CAPES for the research grant and to UNIJUÍ for the availability of the research spaces.

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