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MODELING TURBULENT NEWTONIAN SHEAR USING NON-NEWTONIAN BACKBONE

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Abstract. *Non-Newtonian fluids are known to display distinct differences from Newtonian fluids in many kinds of geophysical and engineering flow situations. Rivlin (1957) was perhaps the first to investigate the relation between the laminar flows of non-Newtonian fluids and the turbulent flows of Newtonian fluids qualitatively. He noted that there is some intrinsic similarity between their constitutive equations. Speziale (1996) used a CEF (Bird et al., 1987) constitutive equation to propose a closure model for the Reynolds stress tensor as a function of the basis constituted by the rate-of-strain tensor, its square, and its contravariant convected time derivative. For steady-state problems this assumption leads to the three-basis tensor explored in Jongen and Gatski (1998), Schmitt and Hirsh (2000) and Schmitt (2007a,b). This model is used to fit DNS data for the channel flow. The results are given as function of entities that are defined in a non-Newtonian viscoelastic context. The turbulent viscosity exhibiting a shear-thinning behavior, is fitted with a Carreau-type model. First and second normal Reynolds stress differences in shear and a Turbulent Weissenberg number, based on a characteristic turbulent time, the relation between first normal stress difference and apparent kinematic viscosity, are given for different Reynolds numbers. The results give another way of understanding and interpreting turbulence results and new insights are in order.*

Keywords: *Closure turbulent model, turbulent Weissenberg number, DNS channel flow*

1. INTRODUCTION

Non-Newtonian fluids are known to display distinct differences from Newtonian fluids in many kinds of geophysical and engineering flow situations. Rivlin (1957) was perhaps the first to investigate the relation between the laminar flows of non-Newtonian fluids and the turbulent flows of Newtonian fluids qualitatively. He noted that there is some intrinsic similarity between their constitutive equations. Later, Liepmann (1961), Moffatt (1965), Townsend (1966), Crow (1968) have performed some research work on the molecular or structural features, elastic behaviour and entrainment process in the flows of Non-Newtonian fluids, and Lumley (1970), Proudman (1970) Builtjes (1977) proposed some new nonlinear models for the constitutive relation for the Reynolds stress considering viscoelastic features. Recently, Tao et al. (1996), Chen et al. (1999), Huang et al (2003, 2004) have performed some "nonlinear" or "viscoelastic" corrections to the previous models to describe the anisotropy and history effects in the flows of Newtonian and non-Newtonian fluids. Especially, Groisman and Steinberg (2000) observed experimentally that the flow of a sufficiently elastic polymer solution can become irregular even at low Reynolds number, high viscosity and in small length scale state, it shows all the main features of developed turbulence.

Speziale (1996) used a CEF (Bird et al., 1987) constitutive equation to propose a closure model for the Reynolds stress tensor as a function of the basis constituted by the rate-of-strain tensor, $\mathbf{S}$, its square, $\mathbf{S}^2$, and its contravariant convected time derivative. For steady-state, homogeneous, 2-D problems, the dependence on the contravariant convected time derivative reduces to a dependence on tensor $\mathbf{SW} - \mathbf{WS}$, making explicit the contribution of vorticity, $\mathbf{W}$, on the model. This three-basis tensor was explored by Jongen and Gatski (1998), Schmitt and Hirsh (2000) and more recently by Schmitt (2007a,b) in the context of nonlinear eddy viscosity models (NLEVM). This class of models are an intermediate step, concerning complexity, between the linear Boussinesq equation and differential models such as RANS-type models (Reynolds average Navier-Stokes). It must be mentioned, that the general assumption of the dependence on such a basis of tensors, also determines a set of tensor invariants the coefficients associated to this basis must depend on. The aim of the present work is to use DNS data obtained for the channel flow to construct functions for the coefficients. One of the basic assumptions made is that these coefficients are strong functions of second invariant of the rate-of-strain and weak functions of the other invariants involved. The general backbone usual in a non-Newtonian viscoelastic context is used to present turbulent data and to construc such functions. Therefore, besides the concept of turbulent viscosity introduced by Boussinesq, we have a turbulent first and second normal Reynolds stress coefficients in shear. A Turbulent Weissenberg, Wi_T, based on a characteristic turbulent time, the relation between first normal stress difference and apparent kinematic viscosity, is given for different Reynolds numbers. It provides a measure of the importance of the normal turbulent stress with respect to the shear turbulent stress.

From Boussinesq hypothesis,we have the tensor form of turbulent Reynolds stress

$$\mathbf{R} = \nu_T \mathbf{S} \tag{1}$$

where $\mathbf{R}_{ij} = \frac{2}{3}\delta_{ij}K - \overline{u_i u_j}$.

In the 2-D nonlinear framework, a new projection initiated for algebraic stress models (Jongen and Gatski 1998) was adapted to nonlinear eddy viscosity models (Schmitt and Hirsch 2000;Schmitt2007a,b) was proposed as follows,

$$\mathbf{R} = \nu_T \mathbf{S} - \beta(\mathbf{SW} - \mathbf{WS}) - \gamma(\mathbf{S}^2 - \frac{1}{3}\{\mathbf{S}^2\}\mathbf{I}) \tag{2}$$

where the coefficients ν_T, β and γ are given as follows,

$$\nu_T = \frac{\{\mathbf{RS}\}}{\{\mathbf{S}^2\}} = \frac{-2\overline{uv}}{a} \tag{3}$$

$$\beta = \frac{\{\mathbf{RSW}\}}{\{\mathbf{S}^2\}\{\mathbf{W}^2\}} = \frac{\overline{uu} - \overline{vv}}{a^2} \tag{4}$$

$$\gamma = \frac{-6\{\mathbf{RS}^2\}}{\{\mathbf{S}^2\}^2} = \frac{6}{a^2}(\frac{2}{3}K - \overline{ww}) \tag{5}$$

2. VISCOMETRIC MATERIAL FUNCTIONS

The mechanical properties of non-Newtonian flows are fully determined when 3 functions are known, either experimentally or theoretically. These functions are called viscometric functions or material functions, and are expressed against the strain variable a (see e.g. Schowalter, 1978; Barnes, Hutton and Walters, 1989; Irvine and Capobianchi, 1998) as follows,

$$-\overline{uv} = \tau(a) = -\eta(a)a \tag{6}$$

$$\overline{uu} - \overline{vv} = N_1(a) = a^2\Psi_1(a) \tag{7}$$

$$\overline{vv} - \overline{ww} = -N_2(a) = -a^2\Psi_2(a) \tag{8}$$

so we have using the approach in (3-5):

$$\nu_T(a) = \frac{-2\overline{uv}}{a} = \frac{2\tau(a)}{a} = -2\eta(a) \tag{9}$$

$$\beta(a) = \frac{\overline{uu} - \overline{vv}}{a^2} = \frac{N_1(a)}{a^2} = \Psi_1(a) \tag{10}$$

$$\begin{aligned} \gamma(a) &= \frac{6}{a^2}(\frac{2}{3}K - \overline{ww}) = \frac{2}{a^2}[(\overline{uu} - \overline{vv}) + 2(\overline{vv} - \overline{ww})] \\ &= \frac{2}{a^2}[N_1(a) - 2N_2(a)] = 2[\Psi_1(a) - 2\Psi_2(a)] \end{aligned} \tag{11}$$

(τ,N_1,N_2) or (τ,Ψ_1,Ψ_2) are viscometric functions or material functions. N_1 and N_2 (or Ψ_1,Ψ_2) are called the first and second normal stress differences. For an unknown viscometric flow, these functions are experimentally estimated and some general properties of the flow are inferred. For Newtonian flows, $\tau(a)/a$ is a constant independent of a, so if this ratio depends on a, the flow possesses non-Newtonian characteristics.

To explore the analogy, we assume here that the total stress (turbulent stress + viscous stress) of a 2-D shear flow corresponds to a viscometric flow, and see what can be said on the different viscometric functions.

First, the viscous stress for a Newtonian flow can be written as $\mathbf{R}_v = \nu\mathbf{S}$, so that the total stress is given by $\mathbf{R}_{total} = \mathbf{R}_v + \mathbf{R} = \nu\mathbf{S} + \mathbf{R}$, so

$$\mathbf{R}_{total} = (\nu + \nu_T)\mathbf{S} - \beta(\mathbf{SW} - \mathbf{WS}) - \gamma(\mathbf{S}^2 - \frac{1}{3}\{\mathbf{S}^2\}\mathbf{I}) \tag{12}$$

So finally, the quadratic constitutive equation for the total stress could be denoted as

$$\mathbf{R}_{total} = (\nu + \nu_T)\mathbf{S} - \Psi_1(a)\mathbf{T}_2 - 2[\Psi_1(a) - 2\Psi_2(a)]\mathbf{T}_3 \tag{13}$$

where

$$\mathbf{T}_2 = \mathbf{SW} - \mathbf{WS}, \mathbf{T}_3 = \mathbf{S}^2 - \frac{1}{3}\{\mathbf{S}^2\}\mathbf{I} \tag{14}$$

3. DNS RESULTS

For 2-D turbulent channel flows, turbulent quantities are normalized as follows,

$$y^+ = \frac{y}{y_0}, U^+ = \frac{U}{u_\tau}, \tau^+ = \frac{\tau}{u_\tau^2} \tag{15}$$

where the $u_\tau = \sqrt{\tau_w/\rho}$ is the characteristic velocity, and $y_0 = \nu/u_\tau$ is the characteristic length scale, τ_w is the shear stress at the wall and ρ is the density of the fluid. Here we want to build a universal model for 2-D turbulent channel flows, so we consider the viscometric functions and coefficients using non-dimensionalized quantities. So

$$\nu_T(a^+) = -2\eta(a^+) = \frac{2\tau^+}{a^+} \tag{16}$$

$$\Psi_1(a^+) = \beta(a^+) = \frac{\overline{uu}^+ - \overline{vv}^+}{(a^+)^2}, \Psi_2(a^+) = \frac{\overline{ww}^+ - \overline{vv}^+}{(a^+)^2} \tag{17}$$

$$\lambda = \frac{\Psi_1}{2\nu_T} = -\frac{\overline{uu}^+ - \overline{vv}^+}{4a^+\overline{uv}^+}, Wi = \lambda a^+ = -\frac{\overline{uu}^+ - \overline{vv}^+}{4\overline{uv}^+} \tag{18}$$

Here Wi is the Weissenberg number, and λ is relaxation time.

In 2-D turbulent flows, total shear stress, denoted as τ_{total}, takes the form

$$\tau_{total} = -\overline{uv} + \nu\frac{dU}{dy} \quad = \quad (-\overline{uv}^+ + a^+\frac{1}{Re_\tau})u_\tau^2 \tag{19}$$

so,

$$\tau_{total}^+ = \frac{\tau_{total}}{u_\tau^2} = -\overline{uv}^+ + a^+\frac{1}{Re_\tau} \tag{20}$$

Then we get the apparent viscosity

$$\nu_{apparent} = \frac{\tau_{total}^+}{a^+} = -\frac{\overline{uv}^+}{a^+} + \frac{1}{Re_\tau} \tag{21}$$

also we can define the parameters of Weissenberg number and relaxation time using $\nu_{apparent}$,

$$\lambda' = \frac{\Psi_1}{2\nu_{apparent}}, Wi' = \lambda' a^+ \tag{22}$$

We consider here different publicly available DNS databases characterized by a range of Reynolds numbers from Re_τ=180 to Re_τ=2000. The variation of velocity gradient versus y^+ is shown in fig. 1 to verify the databases for the cases of Re_τ=180, 395, 590, 640, 950, 1020 and 2000. As expected, a^+ is very large in the near wall region and very small in the center region of the plane channel. There is also some similarity for the variation of a^+ in the modest y^+ region for all the cases.

Figure 2 shows the relationships between turbulent and apparent viscosities and shear rate. This result shows that apparent viscosity can be used to remove the divergence problem of turbulent viscosity at large a^+, since the curves collapse for higher values of this variable. Additionally, it can be noticed that the viscosity is close to be a constant at fully developed turbulent region of $a^+ < 10$. There is a very good universal slope of about 0.9718 for apparent viscosity at the near wall region of $a^+ > 10$ for all the Re_τ cases. It indicates that the values of apparent viscosity considered here cannot correspond to a laminar Newtonian flow because $\nu_{apparent}$ depends on the shear, but the form of the $\nu_{apparent}$ curve can give more information in the framework of a viscoelastic analogy that it has the same shape, which is first increasing, then decreasing, as a pseudoplastic flow.

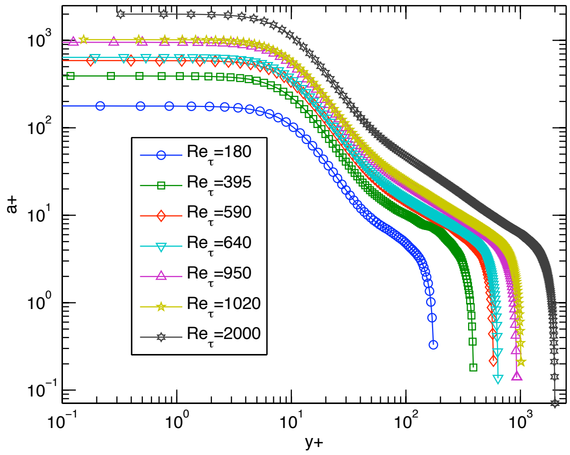

Figure 1. The profiles of velocity gradient a^+ vs. y^+ in log-log plot.

The distributions of Weissenberg number is shown in fig. 3 versus a^+ and y^+, respectively, it exhibits that Weissenberg number is close to be a constant to a^+.

Fig. 4 (a) shows the curve of turbulent Reynolds stress $-\overline{uv}$. We observe a power-law region of small a^+ and also some certain weak Reynolds number effects in this turbulent region. Also there is some same evolution law for the total stress plotted in fig. 4 (b), the slope is a little larger than the value of 1, this indicates that the traditional gradient transport hypothesis is not enough to describe the relationship between turbulent stress and mean shear rate. Also the results of total stress illuminates that there is some benefits for us to model viscous stress and turbulent stress together to eliminate some divergence problem.

Figure 5 plots the curve of the first viscometric function Ψ_1 and second viscometric function Ψ_2, it shows that there is a good slope for us to build the model for $\Psi_1(a)$, and the value of the slope is very close to 5/3. The link with the K41 5/3 exponent is not clear here.

The results of viscosity and viscometric functions lead us to recall one of the Non-Newtonian fluid, the Carreau Model which is expressed by following equation

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = [1 + (\lambda_c a)^2]^{\frac{n-1}{2}} \tag{23}$$

Where η_0 is the zero shear viscosity, η_∞ is the limiting viscosity at high shear rates (suppose to be zero here) and λ_c is a time constant calculated from the reciprocal of the strain rate at which the zero strain rate component and the power-law component of the flow curve intersect.

Here we use this Carreau model to build a new model for viscosity and the viscometric functions, first for $\nu_{apparent}$ we get the parameters (Fig. 2 (a)) as follows

$$\nu^0_{apparent} = 0.0755, n = 0.0282, \lambda_c = 1/12 \tag{24}$$

so

$$\nu_{apparent} = \nu^0_{apparent}[1 + (\lambda_c a^+)^2]^{\frac{n-1}{2}} = 0.0755[1 + (\frac{a^+}{12})^2]^{-0.4859} \tag{25}$$

For the first viscometric function Ψ_1, since the two slopes in fig. 5) are very close, so we can consider a model with the following form

$$\Psi_1 = \left(\psi_1 + \psi_2[1 - exp(-\lambda_c a^+)]\right)(a^+)^{(n-1)} \tag{26}$$

For small a^+ close to zero, Ψ_1 tends to be $\psi_1(a^+)^{(n-1)}$, and for large a^+ goes to infinite, Ψ_1 tends to be $(\psi_1 + \psi_2)(a^+)^{(n-1)}$. Refer fig. 5 (a), we can obtain the values of the parameters

$$\psi_1 = 0.3792, \psi_2 = 0.8372, \lambda_c = 0.3, n - 1 = -1.8275(fit1)or - 1.75(fit2) \tag{27}$$

here in fit 1 the slope $n - 1 = -1.8275$ is evaluated from the small shear rate, however in fit 2 the slope $n - 1 = -1.75$ is evaluated between the values of the two slope -1.8275 and -1.7145, it looks model fit 2 is better than fit 1. so we have the model for Ψ_1

$$\begin{aligned}\Psi_1 &= \left(0.3792 + 0.8372[1 - exp(-0.3a^+)]\right)(a^+)^{-1.75} \\ &= \left(1.2164 - 0.8372e^{-0.3a^+}\right)(a^+)^{-1.75}\end{aligned} \tag{28}$$

For viscometric function Ψ_2, we have (Fig. 5 (b))

$$\Psi_2^0 = 0.006, n = -0.6036, \lambda_c = 1/9 \tag{29}$$

so

$$\Psi_2 = \Psi_2^0[1 + (\lambda_c a^+)^2]^{\frac{n-1}{2}} = 0.006[1 + (\frac{a^+}{9})^2]^{-0.8018} \tag{30}$$

Finally, we have found a way to build this viscoelastic model for turbulent plane channel flow, using the model for apparent viscosity and viscometric functions, the model can be written as follows,

$$\begin{aligned} \mathbf{R}_{total} &= (\nu + \nu_T)\mathbf{S} - \Psi_1(a)\mathbf{T}_2 - 2[\Psi_1(a) - 2\Psi_2(a)]\mathbf{T}_3 \\ &= \{0.0755[1 + (\frac{a}{12})^2]^{-0.4859}\}\mathbf{S} - \left(1.216 - 0.837e^{-0.3a}\right) a^{-1.75}\mathbf{T}_2 \\ &\quad -2\{\left(1.216 - 0.837e^{-0.3a}\right) a^{-1.75} - 0.012[1 + (\frac{a}{9})^2]^{-0.802}\}\mathbf{T}_3 \end{aligned} \tag{31}$$

where

$$\mathbf{T}_2 = \mathbf{SW} - \mathbf{WS}, \mathbf{T}_3 = \mathbf{S}^2 - \frac{1}{3}\{\mathbf{S}^2\}\mathbf{I} \tag{32}$$

For this model for turbulent channel flow, we have introduced a viscoelastic model to find a new way to model the parameters of $\nu_{apparent}$, β and γ, instead of the old way of building the model by introducing some empirical constants and the scale of turbulent kinetic energy and dissipation rate. So obviously, this new viscoelastic model for turbulent flow will do benefits for us to consider the characteristics of memory effects and multiple scale properties of fully developed turbulence.

4. FINAL REMARKS

We have developed a way to construct functions from DNS data to represent the coefficients of a basis of tensors conceived to represent the Reynolds stress tensor. These coefficients are considered functions of the second invariant of the rate-of-strain tensor. The idea is to give a further step on the Boussinesq hypothesis and find, besides the turbulent viscosity, a turbulent first and second normal stress coefficients. This leads to the creation of a dimensionless number, the Turbulent Weissenberg number (analogous to the classical Weissenberg number in non-Newtonian fluids). The results have shown that, to work with the apparent functions, instead of the turbulent ones, generally give more reasonable behavior, easier to reproduce with a master curve.

5. ACKNOWLEDGEMENTS

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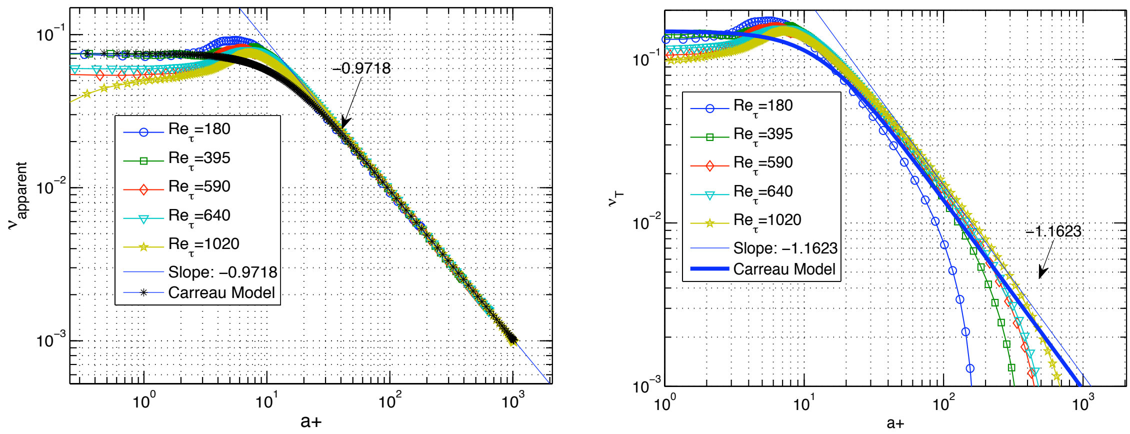

Figure 2. The profile of apparent viscosity $\nu_{apparent}$ (a) and turbulent viscosity ν_T (b) vs. shear rate a^+ in log-log plot and Carreau model fitting.

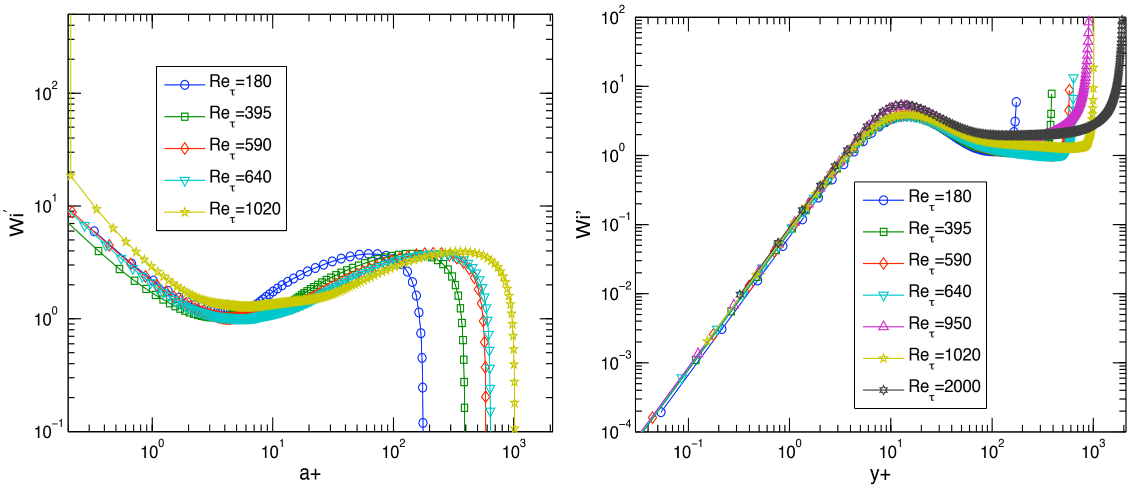

Figure 3. The distributions of Weissenberg number $Wi' = \lambda' a^+$ vs. a^+ (a) and vs. y^+ (b) in log-log plot.

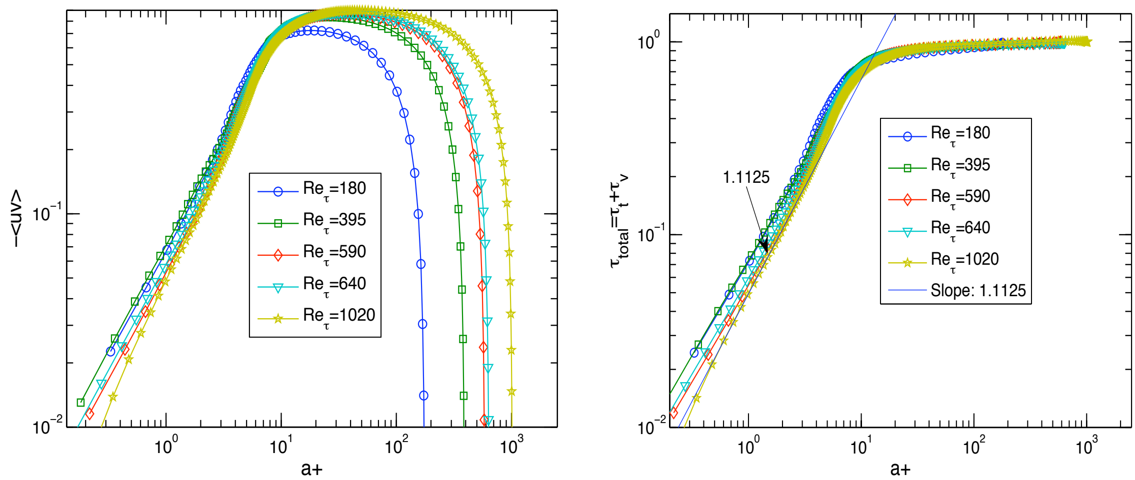

Figure 4. The profile of turbulent Reynolds stress $-\overline{uv}$ (a) and total stress τ_{total} (b) vs. shear rate a^+ in log-log plot.

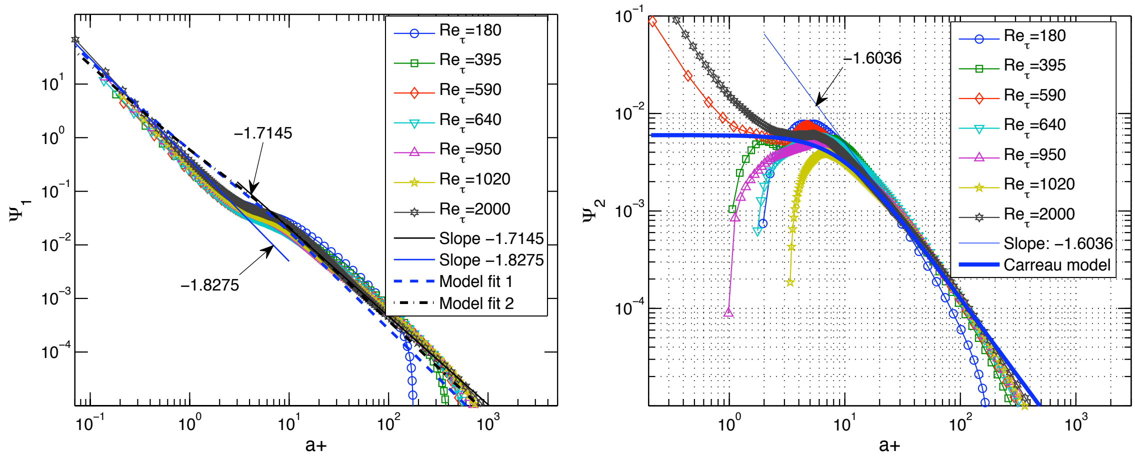

Figure 5. The Carreau model fitting of viscometric functions Ψ_1 (a) and Ψ_2 (b).