

A TOPOLOGY OPTIMIZATION APPROACH APPLIED TO MICROCHANNEL HEAT SINK DESIGN

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Abstract. Fluid flow in small channels are widely present and have many applications for transportation and irrigation systems, biological fluid conduction, and thermal fields, specially in heat sinks since it allows obtaining an efficient device design having better thermal dissipation with small mass and volume, and large convective heat transfer coefficient. This work presents an optimization methodology to design micro-channels applied to heat sink for electronic cooling devices. In this application, maximum dimensions are up to a few millimeters, and the flow velocities are low, retrieving a typical Stokes flow behavior. This methodology is developed by using the Topology Optimization (TO) method, which application in multi-physics problems has been shown a very promising area. This method combines Finite Element Analysis (FEA) and the optimization algorithm called Sequential Linear Programming (SLP) to find, systematically, optimum layout design for the channel heat sinks. Essentially, the optimization problem applied to channel fluid flow consists of determining which points of a given design domain (small heat sink) should be fluid, and which points should be solid to maximize the convective resistance, with minimum pressure drop. The proposed optimization process considers the velocity field distribution and its influence over the temperature distribution, combining both thermal and fluidic fields through a multi-objective function. It is implemented in software, using MATLAB language, and some computational results are shown to illustrate the method.

Keywords: micro-channels, topology optimization, finite element, sequential linear programming.

1. INTRODUCTION

The application of channels in fluid flow has been occurred intensively in many engineering areas, from transportation, in large scale, to biological fluid conduction, in small scale. There are many examples of these applications, such as water distribution systems, air conditioning systems, motor refrigeration, cooling devices for electronic components, oil transportation, and many others.

Although some channel applications take advantage from potential energy gradients for operating, many others must apply external devices for flow pumping, allowing them to operate. Thus, flow pumping systems have improved many of these applications.

Nowadays, most of the engineering processes rely upon power sources, which are vital for their operation. This background demands a detailed analysis of power consumption processes, in order to achieve the most efficient system design, allowing lower power consumption, and consequently lower costs and a lower environment impact, which is also a great concern recently.

In fluid flow systems, one important matter is power dissipation along channels which leads to a pressure drop, compromising their correct operation and their efficiency. Especially in small scale applications, such as micro-channel devices, that operate under low pressure conditions, the fluid conduction can be greatly influenced and compromised by any pressure drop.

As a practical application, we can mention heat sink design for high-power LED's (Light Emitting Diodes). This kind of LED is characterized by a high luminosity, which brings as a consequence, a high heat generation, which it is only removed by an efficient heat sink.

Another application, that is already present in some commercial products, is water cooling systems for high-end microprocessors applied to general computation. These components dissipate a large amount of heat, and need a very efficient dissipation system, to avoid malfunction or even product damage. As long as these microprocessors get more powerful and smaller, the need of efficiency in heat dissipation is even more highlighted.

In both cases cited, the use of water cooling systems has a great potential application, and could provide a more efficient heat transfer. In order to achieve a better heat sink design, it is crucial that these cooling systems have a very efficient fluid flow channel, minimizing pressure drops along its extension, and consequently allowing an efficient full capacity operation. This kind of application is directly affected by the channel performance.

In the past decades, many studies have been conducted, in order to achieve fluid flow channels with better configuration for minimizing power dissipation. These studies take advantage of numerical methods application to analyse the fluid behavior, allowing the study of more complex cases.

The basis of numerical studies on channel flow optimization are given by Pironneau (1973), who conducted a shape optimization analysis in airfoils and other devices, such as diffusers. In its studies he applied the shape optimization process to obtain minimum drag profiles and minimum pressure drop diffusers. Essentially these works contributed for a new application of the optimization methods, which have been mainly applied in the structural field. Other studies in this field are conducted later by Mohammadi and Pironneau (2002 and 2004).

Later in the 90's, there has been a great development of the Topology Optimization (TO) method (Bendsøe and Kikuchi, 1988), which, essentially distributes limited amount of material inside a design domain, analyzing the effects of its distribution over a cost-function. The TO aims to optimize the cost-function, while following some imposed design constraints. The TO had achieved large improvements in the structural analysis field (Bendsøe, 2003), so far.

Lately, these studies disseminated and other field application began to be studied, as shown by Sigmund (2001). Borrvall and Petersson (2003) studied fluid flow channels design by using TO. Thus, instead of controlling only solid material and void regions, the interest is focused on distributing liquid and solid materials, and then, creating optimized fluid flow systems.

One of the great advantages of the TO is the possibility of analyse a much wider range of solutions, due to the “free” material distribution method. By applying this method one can achieve an optimal solution with no need of proposing a “pre-structured” initial guess, which tends to limit the final solution, by directing the optimization process. This issue may be a problem in parametric and shape optimization, where a preliminary solution model must be stated. One can recover a “not-so-intuitive” solution, as shown in Figure 1, which is an example of fluid flow design, extracted from Borrvall and Petersson (2003). At Fig. 1(a), the solution for a design domain of equal length and height (aspect ratio 1:1), with 2 inlets at left side, and 2 outlets at right side. In Fig. 1(b), the same problem is considered for a “stretched” domain (aspect ratio 3:2).

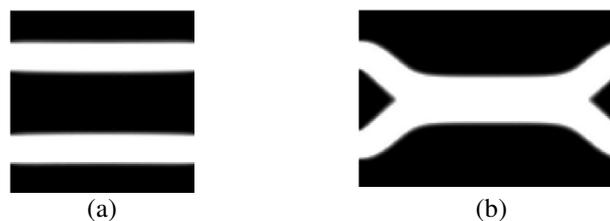


Figure 1- “Non-intuitive” solution for different aspect ratio domains: (a) Aspect Ratio 1:1; (b) Aspect Ratio 3:2.

As one can conclude, for longer domains, the pressure drop on a “single-merged” channel is lower than a “double-way” separated channel, which is not an intuitive solution.

Lately, other studies are conducted, such as Gersborg-Hansen et al. (2005), Evgrafov (2005), Guest and Prévost (2006), Olesen et al. (2006) and Aage et al., (2008).

The main objective of this work is to present the basic characteristics of the TO application for the design of fluid flow channel, mainly focused for heat sink design.

This paper is organized as follows: in section 2 the fundamental theory is presented. In section 3, the Finite Element (FE) model is presented. Section 4 shows an overview on the optimization procedures. Section 5 shows the preliminary results. Finally, in Section 6 the discussion and work conclusion are given.

2. FUNDAMENTAL THEORY

The fundamental theory is given by the constitutive equations for Newtonian fluid flow, based on the well-known fluid flow equations (Navier-Stokes equations), given by:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \Delta \mathbf{u} + \mathbf{f} \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (2)$$

where ρ is the fluid specific mass, μ is the fluid viscosity, \mathbf{u} is the velocity vector, p is the pressure and \mathbf{f} is the body load. Equation (1) refers to the conservation of momentum, and Eq. (2) refers to conservation of mass, or continuity equation.

Like in Borrvall and Petersson (2003), in this study the Navier-Stokes equations are simplified to a linear form, considering a steady-state, incompressible fluid flow (Newtonian fluid) at low Reynolds, where the viscous effects overlap the inertia effects, obtaining the following Stokes flow equations:

$$-\mu\Delta\mathbf{u} + \nabla p = \mathbf{f} \quad (3)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (4)$$

Equation (3) dictates the fluid flow, coupled with Eq. (4), that acts like a constraint in the velocity field to ensure the incompressibility condition.

Besides the fluid flow in free regions of the domain, it is necessary to model the solid region behavior.

Borrvall and Petersson (2003), Gersborg-Hansen et al. (2005) and Guest and Prévost (2006) studied a formulation that combines a fluid-like and a solid-like behavior. This problem is solved by combining the standard Stokes flow equation with a contribution from a porous medium flow, known as Darcy flow, given by:

$$\mathbf{u} = -\frac{\kappa}{\mu}(\nabla p - \rho_f \mathbf{b}) \quad (5)$$

$$\nabla \cdot \mathbf{u} = 0$$

where κ is the porous media permeability.

The main idea is to apply the Stokes equations to model the fluid flow behavior, and to control the velocity field in solid regions through the Darcy equation, by assuming it to be a porous medium with nearly-zero flow permeability.

The combination of Eq. (3) and (5) results in the Brinkman's equation form (Eq. 6).

$$\mu\Delta\mathbf{u} + \alpha\mathbf{u} = \nabla p - \mathbf{f} \quad (6)$$

$$\nabla \cdot \mathbf{u} = 0$$

where α is the fluid inverse permeability.

In Eq. (6), it is included a penalization term that is controlled by α , which is the inverse permeability of the porous medium region, and penalizes the velocity, enforcing a very small flow in solid regions. This approach allows the optimization to work with a continuum variation on the liquid-solid model, which is relevant during the optimization phase, discussed ahead in this manuscript.

Additionally to the fluid movement equations, the energy equation is given by:

$$\rho_f c_p (\mathbf{u} \cdot \nabla T) = k_f \nabla^2 T \quad (7)$$

where ρ_f , c_p , k_f are, respectively, fluid density, specific heat and thermal conductivity and T is temperature.

3. FINITE ELEMENT METHOD

The Finite Element Method (FEM) is applied to solve the equations presented in the previous section (Zienkiewicz e Taylor, 2002). The design domain is divided into rectangular bilinear elements, which have 4 nodes for velocity field and 1 node for pressure field. Although it is recognized as a not full-stable element according to the LBB or div-stability-condition (Hughes et al., 1986), the adopted element has shown a good accuracy for velocity field calculation, with expected spurious oscillation in the pressure field, which in this particular application, do not affect the results decisively (Borrvall and Petersson, 2003). This spurious oscillatory behavior in the pressure field is similar to the *checkerboard* instability present in many structural TO application (Bendsøe, 1989). It's caused by the velocity and pressure fields coupling and it is triggered by the adopted finite element discretization, which may result in an insufficient solution space.

By applying the FEM to Brinkman's equation, and writing it to the discrete matrix form, the following equation system (Bendsøe and Sigmund, 2003) is obtained:

$$\begin{bmatrix} \mathbf{K} & -\mathbf{G}^T \\ -\mathbf{G} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix} \quad (8)$$

where \mathbf{K} is the velocity stiffness matrix and \mathbf{G} represents the divergent operator. \mathbf{u} and \mathbf{p} are the nodal velocity and nodal pressure distribution respectively, and \mathbf{f} is the nodal body load component. By solving the system presented in Eq. 8, the velocity and pressure fields can be determined.

4. TOPOLOGY OPTIMIZATION

The TO method combines an optimization algorithm with an analysis method, such as the Finite Element Method, and, in this work, uses the Sequential Linear Programming (Haftka, 1996) for solving the topology optimization problem. For convenience, the design domain is discretized following the FEM mesh, allowing each of these finite elements to perform as fluid or solid material, according to the design variable, the pseudo-density, ρ .

The material model defines how to describe either a solid and fluid behavior in this design domain. It combines the characteristics of both materials, and allows defining which kind of material is placed on each element. This is controlled by the design variable ρ , in a way that for $\rho=0$, one retains a solid material and for $\rho=1$, one retains a fluid material, characterizing a discrete 0-1 problem.

Although there is not physical application for intermediate values of ρ and it is not desirable to have them at the final design, it's very common to work with a continuous problem, allowing ρ to assume these intermediate values. This approach is due to a known issue, while working with the discrete problem, preventing the problem solution. This question is widely discussed by Bendsøe and Sigmund (2003).

The Brinkman's equation (Eq. (6) and (8)) describes the material model. In this equation, the inverse permeability (α) is a continuous function of the design variable ρ . The material model is the same applied by Borrvall and Petersson (2003) and later by Gersborg-Hansen et al. (2005). This model essentially describes a Stokes flow behavior for fluid elements. For solid elements, the combined porous medium model predominates, with permeability controlled by the design variable ρ , such as for a full solid element ($\rho \rightarrow 1$), the velocity is nearly zero ($u \rightarrow 0$).

The TO method initiates with the initial domain definition, with problem constraints and boundary conditions definition. The design domain is discretized conveniently, so the FEM analysis can be performed and the optimization process follows. The TO algorithm calculates at each iteration the cost function value and performs a sensitivity analysis of this cost function based on gradient calculations over the design variables. The optimization algorithm uses this sensitivity analysis as a guidance for recalculating material distribution along the design domain, and updates the information, until an optimized topology is obtained. Then, a post-processing step over the optimal configuration is necessary for correction of possible issues. A verification process is performed to analyse and validate the results obtained, and finally the design is ready to be manufactured.

The typical methodology adopted in a TO design process is shown as follows, in Fig. 2.

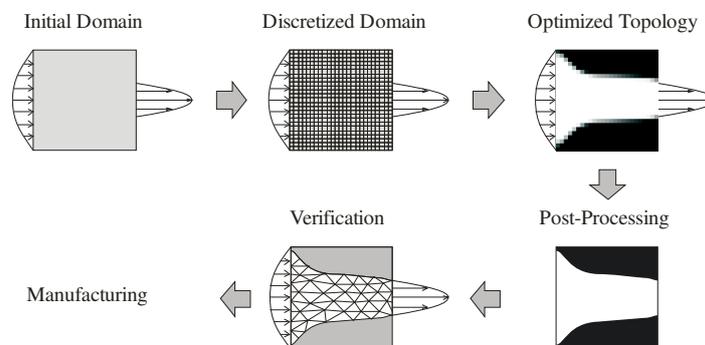


Figure 2 – Design methodology in topology optimization.

4.1. Objective Function, Constraints and Sensitivity Analysis

As cost function chosen for the fluid flow problem, it is adopted the total potential power evaluated at the solution obtained by the FEM analysis. Considering a common case where there is no body forces over the fluid domain, the total potential power represents the power dissipation on the fluid.

Equation (9) shows the cost function evaluated during the optimization process for fluid flow, in its discrete form, where \mathbf{U} represents the nodal velocity field vector and \mathbf{K} is the velocity stiffness matrix.

$$\Phi = \frac{1}{2} \mathbf{U}^T \mathbf{K} \mathbf{U} \quad (9)$$

This equation represents the power dissipation in the design domain, and has the same form of the well-known *mean-compliance* used very often in structural optimization (Bendsøe and Sigmund, 2003). Equation (9) may represent the mean pressure drop over the channel, and it will be used to evaluate performance in the studied channels. The goal here is to minimize power dissipation, and consequently to minimize the pressure drop.

Thus, the topology optimization problem is stated for the fluid flow channel optimization, in a discrete form, as:

$$\text{minimize:} \quad \Phi = \frac{1}{2} \mathbf{U}^T \mathbf{K} \mathbf{U}$$

$$\text{such as:} \quad \begin{bmatrix} \mathbf{K} & -\mathbf{G}^T \\ -\mathbf{G} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix}$$

$$\sum_{i=1}^N \rho_i \leq V$$

$$0 \leq \rho_i \leq 1$$

(10)

For controlling the design heat transfer, another objective function must be stated. The chosen cost function utilizes the temperature distribution, obtained from the energy equation (Eq. 7) to evaluate the system heat transfer performance. It has also the same discrete form as the *mean-compliance* problem, as follows:

$$\Gamma = \frac{1}{2} \mathbf{T}^T \mathbf{K}_t \mathbf{T} \quad (11)$$

where \mathbf{T} is the thermal field distribution and \mathbf{K}_t is the thermal stiffness matrix.

The sensitivity analysis is performed by calculating the objective function gradient in relation to the design variables. The objective function gradients are calculated as follows:

$$\frac{\partial \Phi}{\partial \rho} = \frac{\partial}{\partial \rho} \left(\frac{1}{2} \mathbf{U}^T \mathbf{K}(\alpha(\rho)) \mathbf{U} \right) = \frac{1}{2} \mathbf{U}^T \frac{\partial \mathbf{K}(\alpha(\rho))}{\partial \rho} \mathbf{U} = \frac{1}{2} \mathbf{U}^T \frac{\partial \mathbf{K}}{\partial \alpha} \frac{\partial \alpha}{\partial \rho} \mathbf{U} \quad (12)$$

A volume fraction constraint is adopted in the optimization problem. The volume fraction is the ratio of fluid material volume over the domain total volume.

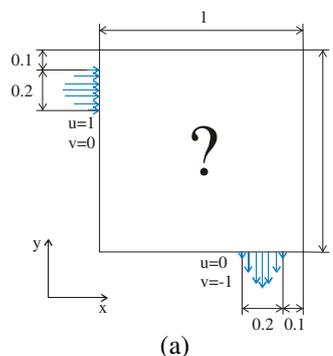
5. RESULTS

This section shows the preliminary results obtained. It will be illustrated initially, minimum pressure drop cases, and then some results combined with heat transfer process.

5.1. Minimum pressure drop

Initially some “benchmark” tests were conducted, based on the results shown on Borrvall and Petersson (2003).

The first model studied was the ‘bend pipe’ design, as shown in Fig. 3. This model has one inlet and one outlet region, both set to a unitary velocity, with direction defined as follows in Fig. 3(a). All other domain boundaries are prescribed as walls, with non-slip and impermeability conditions. At the outlet region, the pressure is set to 0, acting like a sink. This test shows how the optimization problem is stated and illustrates also the volume fraction influence over the solution. The results presented are obtained for a domain composed by 80x80 elements. White regions represent fluid material and black regions represent solid regions.



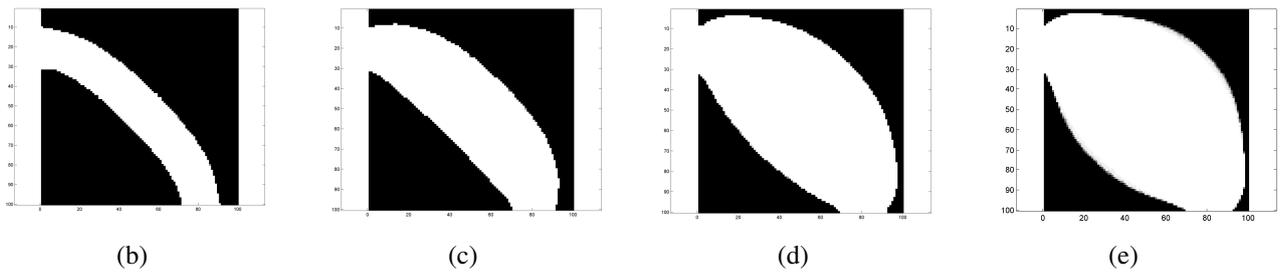


Figure 3 – Volume fraction (V) influence in final solution: (a) stated problem; (b) $V=0.25$; (c) $V=0.4$; (d) $V=0.6$; (e) $V=0.7$;

Figure 4 shows the results for the velocity field.

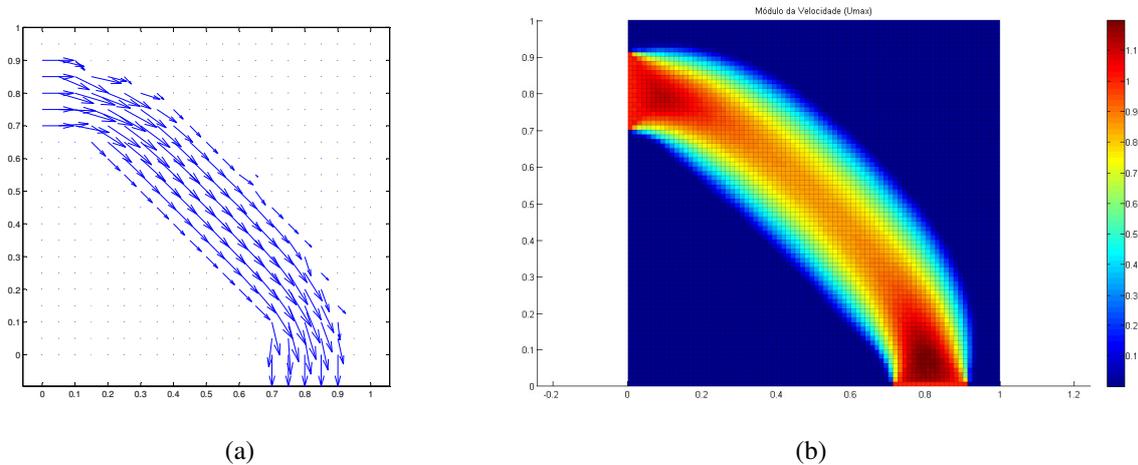


Figure 4 – Velocities for the $V=0.4$: (a) velocity field; (b) velocity magnitude.

Another important result related to TO applications is the mesh-dependency verification. As shown in Fig. 5, the results are not mesh-dependent, which is not a common situation in TO design, without any filtering techniques.

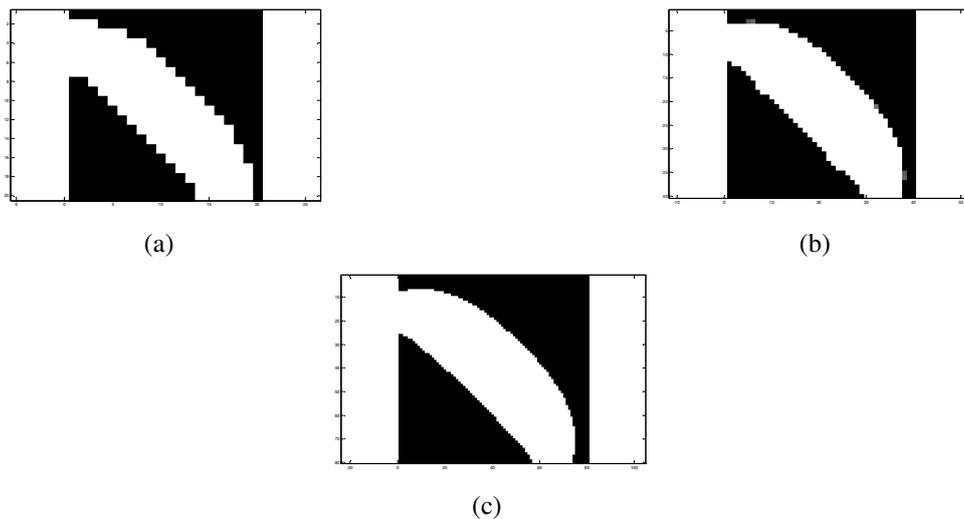


Figure 5 – Mesh-dependency analysis for different meshes: (a) 20×20 ; (b) 40×40 ; (c) 80×80 elements.

5.2. Flow Reverter

Another result is shown here to illustrate a flow reversor. In this example, there is an inlet region at the domain left side, and an outlet region at the right side. It is defined as cost function, the maximization of flow velocity in a specific

region of the domain, more specifically in its center, where it is proposed to be maximized the horizontal velocity in opposition to the inlet region flow direction. Figure 6 shows the results.

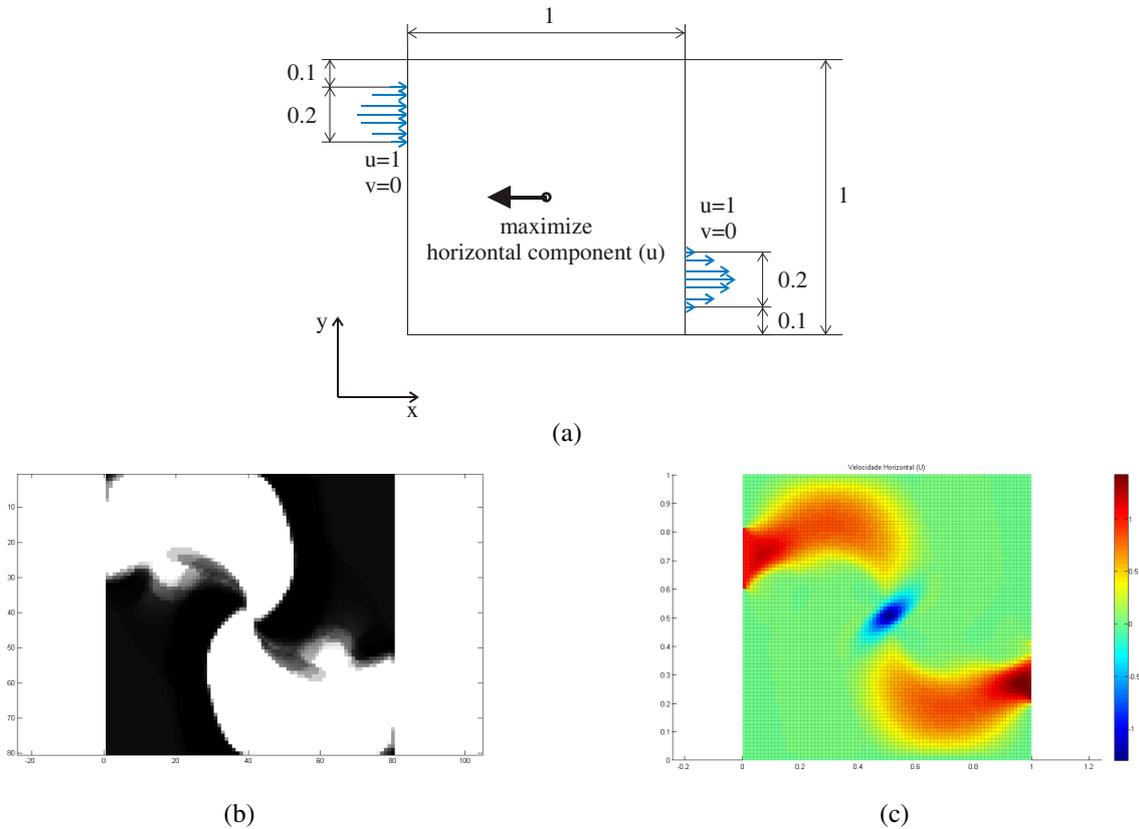


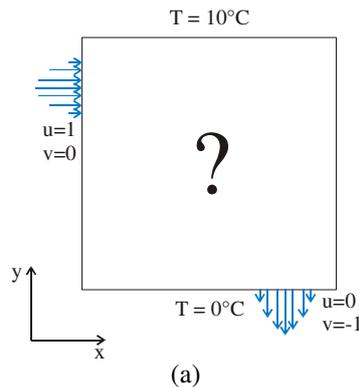
Figure 6 – Flow reversor problem: (a) stated problem; (b) channel configuration; (c) velocity (horizontal component).

Figure 6(c) shows the horizontal component magnitude of the velocity field, where is possible to visualize both, the minimum velocity (dark blue region), and a flow reversion at the center region, as proposed.

5.3. Heat sink design

Finally, a heat sink design is proposed. This problem combines Eq. (10) and the problem stated in Eq. (11), allowing the optimization process to achieve a channel with both low pressure drop and high heat transfer attributes. These two characteristics are evaluated through a multi-objective function which allows the design process to give priority to one of them, or treat both equally.

The result shown in Fig. 7 is based in the ‘pipe-bend’ problem, with additional boundary conditions for the thermal problem. A uniform heat source term is distributed along the entire domain. At the top and bottom of the domain the temperature is fixed to 10°C and 0°C , respectively. For this example, normalized thermodynamic constants are considered. Figure 8 shows the temperature distribution for the example in Fig. 7(d).



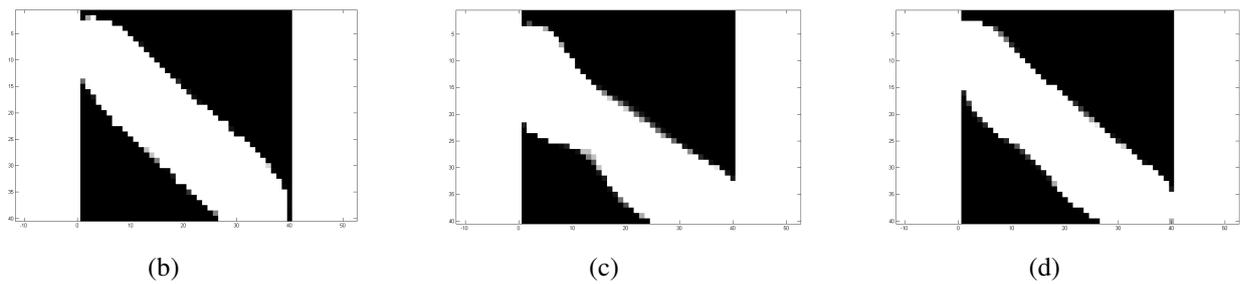


Figure 7 – Heat sink optimization problem: (a) stated problem; (b) fluid flow priority; (c) heat transfer priority; (d) equal priority for both.

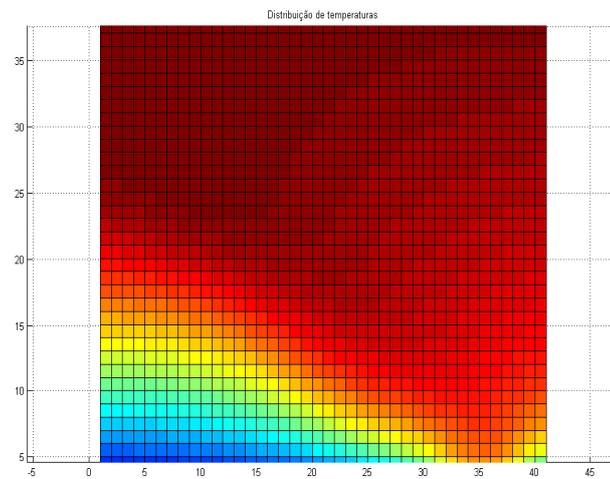


Figure 8 – Temperature distribution for the Fig. 7(d) case.

6. CONCLUSION

By analyzing the results shown in this paper, some points can be highlighted. First of all, the application of the TO in Fluid Mechanics is practical and it allows the systematic design of fluid flow channels. The results obtained here could be compared to other results in the literature, specially those obtained by Borrvall and Petersson (2003), used as a benchmark.

A volume fraction verification has been performed and it is possible to visualize its influence over the results.

Another verification was the mesh-dependency analysis. As it is a common problem in other TO application fields, specially in structural optimization problems, this was an important matter to be verified. The results shown the fluid flow optimization as a mesh-independent problem, even with no applied filtering technique, as it is previously evidenced and discussed by Borrvall and Petersson (2003).

The flow reverter example shows the possibility of TO utilization for designing directional controlled channels, with maximized velocity in a preferential direction.

Finally, an example of application of combined fluid flow and heat transfer channel design is shown. This example illustrates the viability of applying the optimization process to achieve a channel design combining these two distinct characteristics at the same time.

The application of combined Stokes-Darcy flow equations has been shown very efficient within the TO algorithm. Although it has certain limitations, as it is a linear approach of the full Navier-Stokes equations, and suitable only for low Reynolds fluid flow, this model is shown to be applicable for many different problems, from bend-pipes and diffusers to more complex cases.

It is also verified the possibility to work with combined fluid flow and heat transfer attributes in the same channel. This approach is used for more efficient heat sink design, which has a lot of possible applications, as shown previously in this document.

As conclusion, authors consider the application of the TO in fluid flow channel design a very promising field, as shown in this paper. The analysis reported here can be extended to many other study cases for general cooling (or heating) devices design, based on fluid flow channel.

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