

DYNAMIC ANALIZES OF ELASTIC STRUCTURES BY USING MOVING PARTICLE SEMI-IMPLICIT METHOD (MPS)

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Abstract. *This paper presents the theory and hypotheses used in the development of a particle method for dynamic structural analysis. The method was implemented by the authors and the validation of the code was carried out through comparison with analytical results.*

This method is based on the Moving Particle Semi-implicit Method (MPS) that was first developed to simulate the behavior of incompressible fluids. The main strategy of the MPS is to replace the differential operators in the governing equations by a model of interaction between particles. As a meshless method, it is very effective in the simulation of the hydrodynamic problems that involve large deformation and fragmentation of free-surfaces, complex shaped bodies or moving boundaries.

In the recent years, the developments of the method extended its application to the analysis of elastic structures, making possible the analysis of dynamic systems and the coupling of hydrodynamics and structural analysis to investigate hydro-elasticity problems.

For dynamic analysis of elastic structures, the elastic structures are discretized as particles and the theory of small displacements is considered. By using the particles interactions model, the governing equations of the dynamic of the elastic bodies are solved by the gradient and laplacian operators originally developed for fluids, with some adaptations, and a newly defined rotation operator. As a result, the dynamic interaction between two particles can be related to the behavior of normal and tangential springs between the particles, and the rotation of the particles is taken into account.

The qualitative and quantitative validations of the method are carried out herein considering dynamic systems present in numerous engineering problems, such as mass-spring systems, forced or not, and vibrations. Simulation of 2D models were analyzed and compared to analytical models, which showed consistent results.

Keywords: *MPS, elastic structures, particle method*

1. INTRODUCTION

The main objective this article is to present the development of a particle method for simulation of elastic bodies, which, once fully implemented and validated, should be incorporated to hydrodynamic problems allowing analysis of fluid-structure interaction in real time. Particles methods are those that discretize the domain to be studied by particles that interact among themselves and behave according to the governing equations of interest. They are based in Lagrangian description and replace the differential operator by a particle interaction model. Particle methods don't need mesh and allow the analysis of problems with large deformation of interfaces, large displacements and fragmentation easily.

In fluid dynamics, it is often necessary to analyze the interactions between fluid and structures. In such cases, when using the traditional numerical methods of finite differences or elements, the meshes are distorted near the interfaces and the modeling of the phenomena becomes difficult when displacements are very large. The numerical methods that do not use meshes do not present these short comings. Furthermore, the generation of meshes for the analysis of structures with complex geometries may require a considerable processing, which is not required in meshless methods.

The Moving Particle Semi-Implicit Method (MPS) is one of these methods, which was originally developed for incompressible flows with free surface by Koshikuzo and Oka (1996). This method uses a semi-implicit algorithm to perform the simulation.

Another particle based method for analysis of solids without using meshes is the Smoothed Particle Hydrodynamics Method (SPH), approached by Monaghan and Gingold (1983). This method is able to simulate elastic-plastic solid and compressible flow. Parshikov and Medin (2002) have presented the results of the SPH used to analyze elastic materials, in the work they show the bi-dimensional impact of two rubber cylinders. Chikazawa *et al.* (1999) have used the MPS method for fluid-structure interaction and simulated sloshing in a elastic tank represented by a structure composed by particles. A new particle method for elastic visco-plastic structure, using the concepts of MPS method developed for fluids, was proposed by Chikazawa *et al.* (2001). In this study, the governing equations of elastic structures are interpreted by interactions between particles. This method is combined with MPS to analyze the fluid-structure

interaction problems involving large deformations at the interfaces. Song et al. (2003) applied the MPS method in dynamic analysis of elastic solids. In this study, the author simulated the impact of elastic bodies and checked the convergence of the method according to the time-step and the material used in simulations. The same method is evaluated by Koshizuka *et al.* (2001), who examined the fracture of elastic bodies and the problem of fluid and structure simultaneously.

The present study shows an implementation of MPS for dynamic analysis of elastic solids in two dimensions and validation of the model by using simple simulation for modal analysis of bending of double-supported beam. Comparison with the results presented in previous studies is also shown.

2. THEORY

2.1. Governing Equations

The method is based on the dynamics motion equations of solid mechanics and on the linear theory of elasticity. The constitutive model is Hooke's law, which describes the behavior of elastic, linear, homogeneous and isotropic solid. The MPS method replaces the differential operators in the governing equations by particles interaction models presented in following equations and calculates the field forces independently and explicitly.

Equation (1) presents the two-dimensional equation of motion used as a basis for the MPS method analysis.

$$\frac{\partial}{\partial x_i} (\lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij}) = \rho \frac{\partial^2 u}{\partial t^2} \quad (1)$$

Where ϵ_{kk} e ϵ_{ij} are the components of strain tensor, δ_{ij} is the Kronecker's delta and u is the particle displacement vector. Lames' coefficients for 2D problem are given by

$$2\mu = \frac{E}{1+\nu} \quad (2)$$

$$\lambda = \frac{\nu E}{1-\nu^2} \quad (3)$$

Where E and ν are respectively Young's modulus and Poisson coefficient of the material.

The numerical model considers the continuous solid as a set of particles that interact according to given laws of motion. The motion of each particle is represented by spatial coordinates, speed, angle and angular velocity. A rotation of particles should be considered to ensure the conservation of angular momentum in the discretized formulation. Equation (4) shows the dynamic equation of angular momentum conservation.

$$M = I \frac{\partial^2 \theta}{\partial t^2} \quad (4)$$

Where I is the moment of inertia and θ is the rotation angle, that should be twice the angle in the rotation tensor.

2.2. MPS Differential Operators

In MPS method, the differential operators are represented by the interaction between a particle i and its neighbors j (Figure 1). For a given particle i , the influence of a neighbor particle is a function of the distance between them (r_{ij}) and is given by the weigh function w_{ij} and by the effective radius that limits the range of influence, r_e .

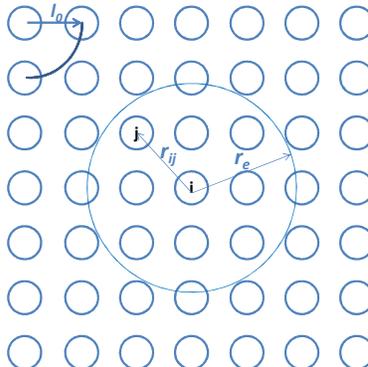


Figure 1: Position of particles and influence radius

Equation (5) shows the calculation of w_{ij} and Figure 2 shows its behavior in the bi-dimensional problem.

$$w_{ij} = \begin{cases} \frac{r_e}{r_{ij}} - 1, (r_{ij} \leq r_e) \\ 0, (r_{ij} > r_e) \end{cases} \quad (5)$$

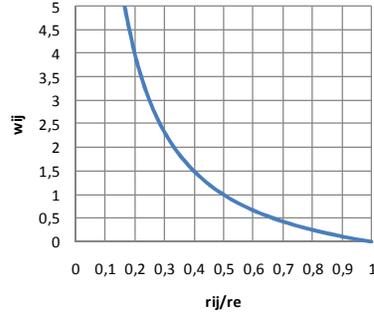


Figure 2: Weight function [source: Koshikuza and Oka (1996)]

Now, the differential operators of interest are modeled by the weighting of influence of neighbor particles. The divergent operator is shown in equation (6) and equation (7) shows the rotational operator.

$$\nabla \cdot \vec{f}_i = \sum_{j \neq i} \frac{(\vec{f}_j - \vec{f}_i) \cdot \vec{t}_{ij}}{r_{ij}} \frac{d}{n_i} w(r_{ij}) \quad (6)$$

$$\nabla \times \vec{f}_i = \sum_{j \neq i} \frac{(\vec{f}_j - \vec{f}_i) \cdot \vec{s}_{ij}}{r_{ij}} \frac{d}{n_i} w(r_{ij}) \quad (7)$$

Where \vec{t}_{ij} e \vec{s}_{ij} are the normal and tangential vectors between particles, respectively, d is the dimension of the problems and n_i is the particle density defined in equation (8).

$$\langle n_0 \rangle_i = \sum_{j \neq i} w(r_{ij}) \quad (8)$$

2.3. Relative Displacements between Particles

In the present study, an explicit time integration scheme has been adopted. This approach considers the relative displacement between particles, caused by external disturbance excitation. Thus, the strain tensor \underline{E} of each particle is calculated explicitly using the relative motion between particles in each time-step. Therefore, the connection between each pair of particles can be interpreted as a normal spring and a tangential vector to the position between them, as is shown in Figure 3.



Figure 3: Representative interaction between two particles

The displacements, stresses and strains are calculated in the plane defined by the rotated vector $\mathbf{r}_{\theta ij}$, the current position of particle i and the current position vector \mathbf{r}_{ij} , as it is shown in Figure 4.

The vector $\mathbf{r}_{\theta ij}$ is obtained by a rotation of angle θ of the position vector \mathbf{r}_{ij} . This angle comes from the angular acceleration calculated in the last time-step in consequence of angular momentum conservation. Equations (9) shows how the rotation of vector is done in 2D from the rotation angle θ_{ij} .

The relative displacement ($\Delta \mathbf{u}$) between particles is defined by the difference between the rotated position vector $\mathbf{r}_{\theta ij}$ and the current position vector \mathbf{r}_{ij} and is decomposed in two directions: normal to the position vector ($\Delta \mathbf{u}_n$) and tangential to the same vector ($\Delta \mathbf{u}_s$) (Figure 4). These components will define the normal and tangential stresses.

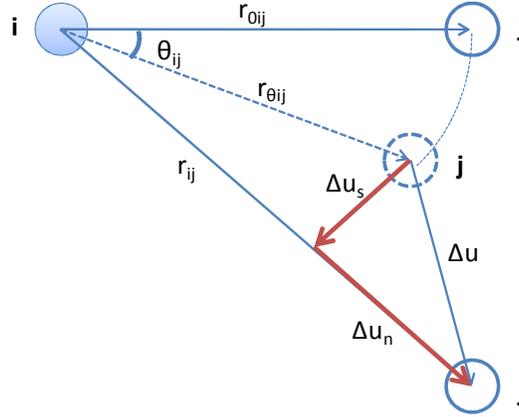


Figure 4: Representation of the displacement between particles

$$\vec{r}_{\theta ij}(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \cdot \vec{r}_{i0} \quad (9)$$

The components of displacements cause a strain of the position vector (r_{ij}) also decomposed in these two directions, being ϵ_n and ϵ_s , calculated by equations (10) and (11) respectively.

$$\epsilon_n = \frac{\Delta u_n}{r_{ij}} \quad (10)$$

$$\epsilon_s = \frac{\Delta u_s}{r_{ij}} \quad (11)$$

Finally, the stress tensor can be calculated, as shown in equations (12) e (13).

$$\sigma_{ij} = 2\mu\epsilon_n = 2\mu \frac{\Delta u_n^\epsilon}{r_{ij}} \quad (12)$$

$$\tau_{ij} = 2\mu\epsilon_s = 2\mu \frac{\Delta u_s^\epsilon}{r_{ij}} \quad (13)$$

The first term of the motion equation (1) represents the volumetric deformation and is described using the divergent of displacement. In this case, the divergent operator of MPS is used to calculate the term, as it is show in equation (14).

$$\epsilon_{kk}|_i = \text{div}(u^\epsilon)_i = -\lambda \frac{d}{n^0} \sum_j \frac{(\Delta u_n^\epsilon)}{r_{ij}} w(r_{ij}) \quad (14)$$

2.4. Algorithm

As mentioned before, the motion is calculated by an explicit algorithm. In other words, in each time-step, the stress tensor is calculated from the relative displacement between two particles and generates the accelerations for next time-step, from what the new displacements will originate.

The figure shows the flow chart of the algorithm for solving the problem. From the initial conditions of each particle, it begins to calculate its motions, which will lead to the strain, stress, translation and rotation. After that, it can be calculate the accelerations that will give the motion corrections for the next time-step. At this point, the existing external forces are considered for the next time-step.

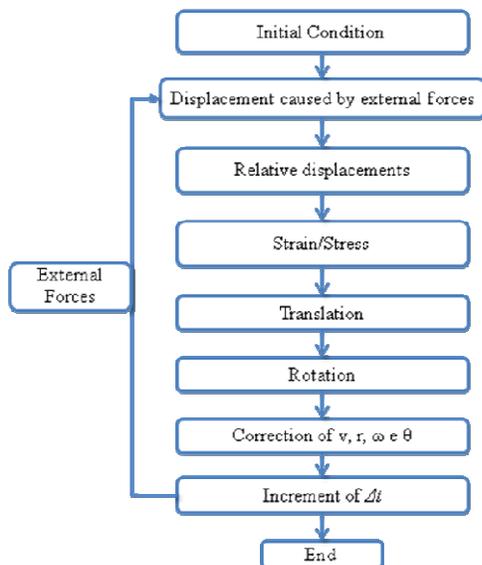


Figure 5: Algorithm for particle method applied to elastic solids

3. VALIDATION

3.1. Block falling in elastic base

The next example is presented to show the behavior of the implemented method. In the simulation, a block is excited with initial velocity of 1 m/s against an elastic base clamped in both extremities. The elastic base has Young's modulus of 1 MPa and Poisson's coefficient of 0.3. Initial distance between particles is 0.01 m.

Figure 6 shows the result of simulation. The restoration of the elastic base pushes the block up and, because the block does not collide in the center of the base, a rotation appears.

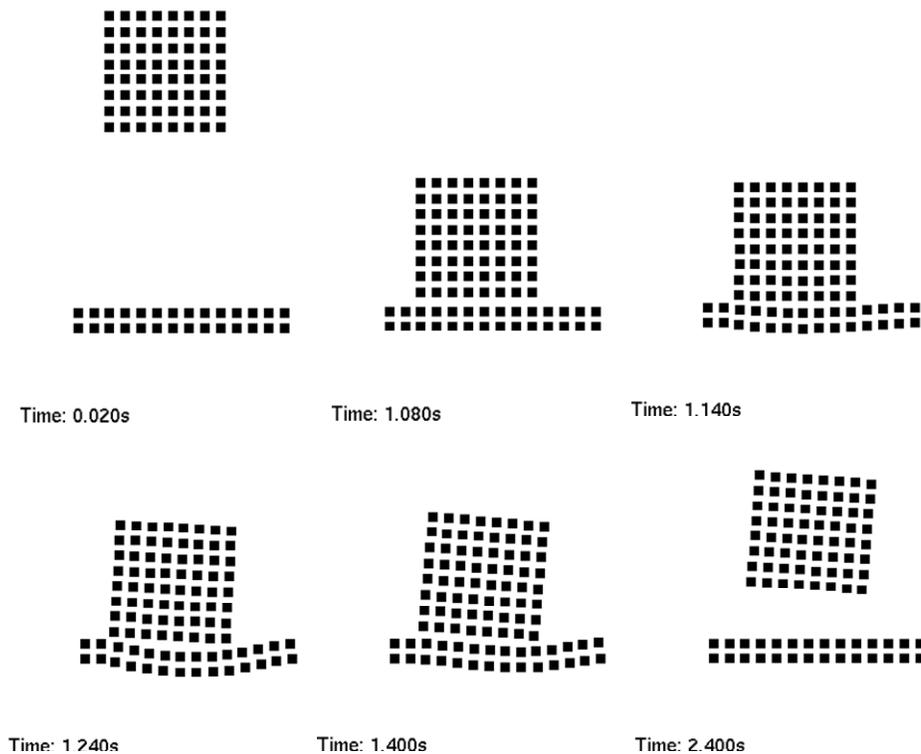


Figure 6: Simulation of a falling block in elastic base

3.2. Collision of elastic rings

As mentioned before, Song et al. (2003) have simulated a dynamic collision of two-dimensional elastic rubber rings. The same simulation was carried out using the algorithm presented in this study. The intention was to compare the results obtained by both models. The simulation is shown in Figure 7 and Table 1 shows its parameters.

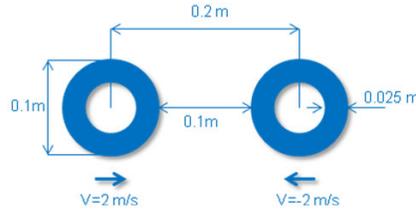


Figure 7: Simulation of collision between two elastic rubber rings

Table 1: Parameters of collision simulation

Simulation	
Number of Particles	128
Distance Between Particles	0.0083 m
Initial Conditions	$V=2.0/-2.0$ m/s
Time-Step	1.0e-6 s
Material	
Young Modulus	2 MPa
Poisson Coefficient	0.48
Density	940 kg/m ³

Both simulations have the same parameters, however the arrangement of the particles of both models are different. Songs et al. utilize a radial arrangement of particles and this study provides a grid position of particles. The comparison shows good agreement of two-dimensional results, as it is shown in Figure 8.

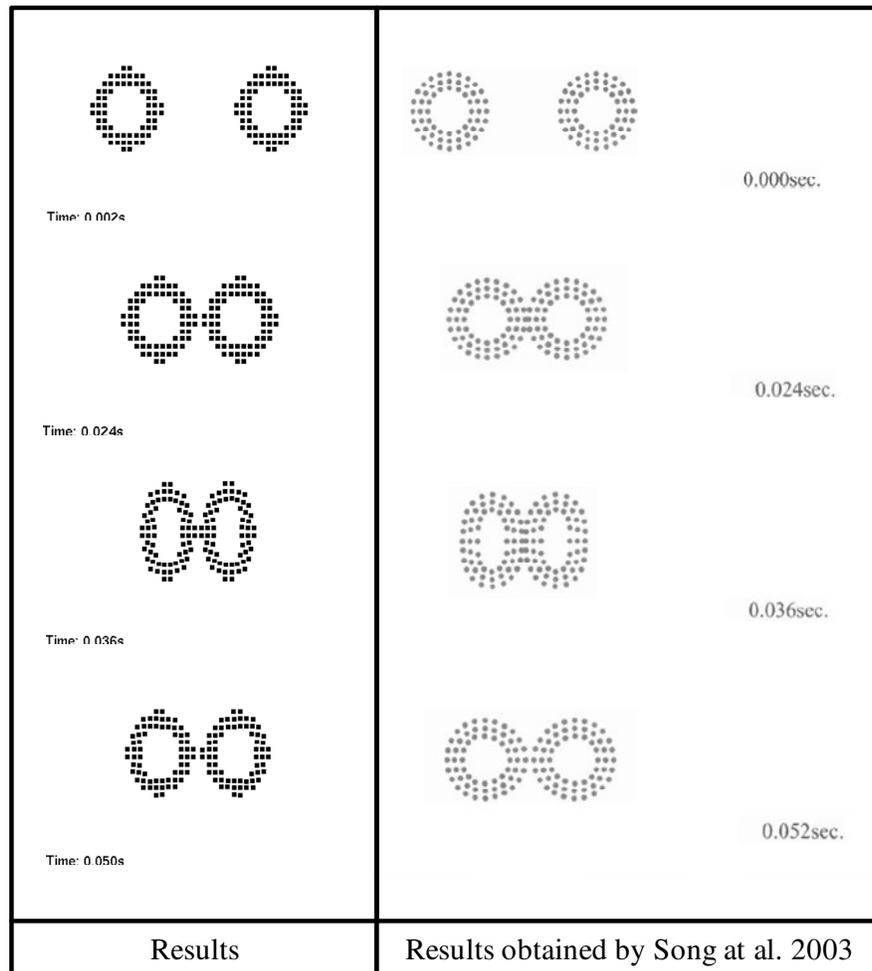


Figure 8: Results of dynamic analysis of two-dimensional impact

3.3. Transverse vibration of flexural beams

The two dimensional case was validated with a simple modal analysis (Figure 9). The first and second mode shape frequency vibration of double-supported beam is analyzed and compared with analytical results. In the simulation, the first vibration mode is excited with gravity, so the amplitudes can also be compared and second mode with a sinoidal velocity field along the beam. Equations (15), (16) and (17) show the formulation to calculate the natural frequency in radians/s and in hertz and period (in seconds) of vibration, respectively.

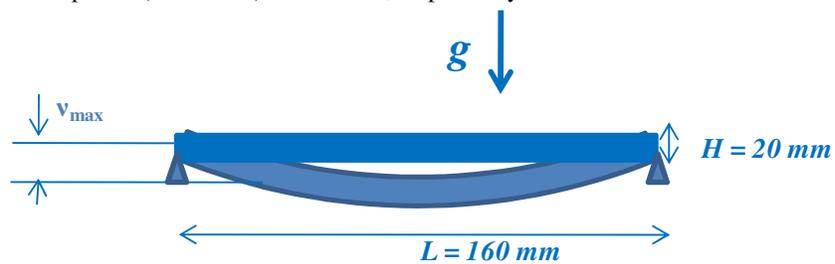


Figure 9: Double-supported beam model

$$w[\text{rad / s}] = \left(\frac{\lambda}{L}\right)^2 \sqrt{\frac{EI}{m}} \quad (15)$$

$$f[Hz] = \frac{w}{2\pi} = \left(\frac{\lambda}{2\pi}\right)^2 \sqrt{\frac{EI}{mL^4}} \quad (16)$$

$$T[s] = \frac{2\pi}{w} = \frac{1}{f} \quad (17)$$

The analytical formula to calculate the maximum vertical deflection (Figure 9) is given in equation (18).

$$v_{max} = \frac{5qL^4}{384EI} \quad (18)$$

The values of λ for double-supported beam problem are π and 2π for first and second vibrating mode, respectively.

The dimensions of the problem are shown in Figure 9. And the material properties are Young's modulus $E = 10$ MPa and Poisson coefficient of 0.3.

Table 2 shows the results of natural frequency and amplitude for the first shape mode. The plotted results can be seen in the graphic presented in Figure 10. The result shows good agreement despite deviations that can be considered as numeric errors and simplifying assumptions.

Table 2: Results of flexural beam simulation in first mode shape

First mode results			
Model	Natural period (s)	Frequency (rad/s)	Maximum displacement (mm)
MPS (16x2 cm)	0.0225	279	-0.318
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MPS (16x2 cm)	0.0225	279	-0.314
Analytical	0.0249	252	-0.253

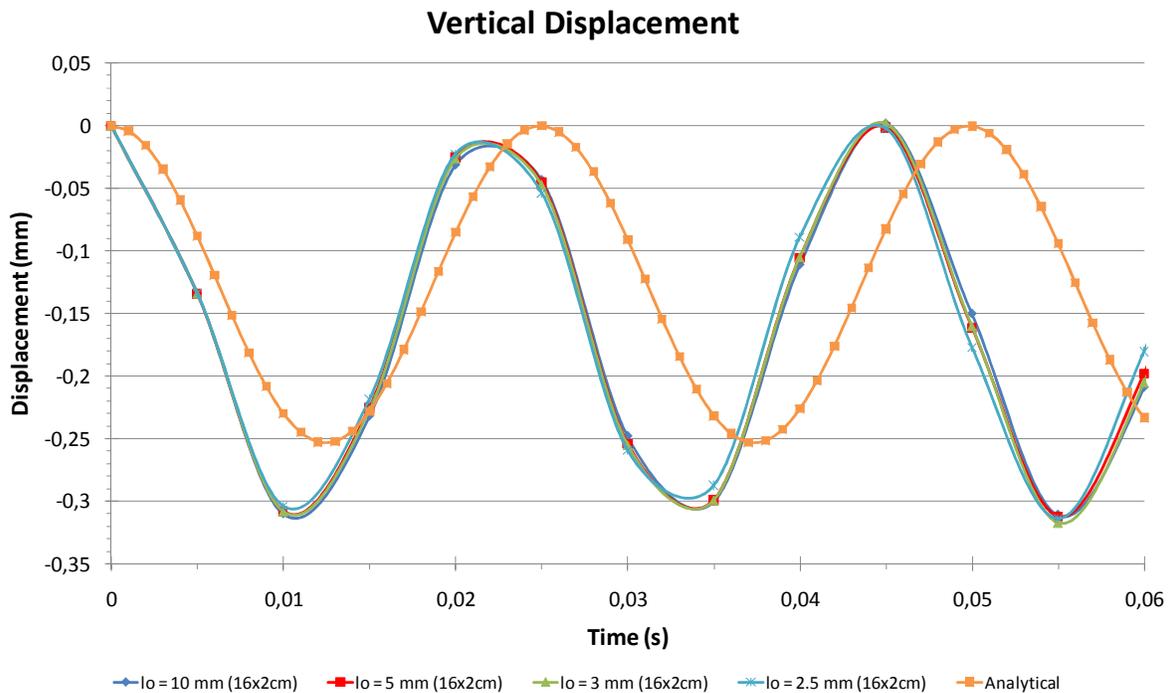


Figure 10: Vibration of 2D flexural beam

For the second mode, Table 3 show the results obtained compared to analytical values for natural frequency and period, it shows an error of 0.001s, given probably by numerical and assumptions deviations.

Table 3: Results of flexural beam simulation in second mode shape

Second mode results		
Modelo	Natural period (s)	Frequency (rad/s)
MPS	0.008	744
Analytical	0.007	891

4. REFERENCES

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