NONLINEAR ANALISYS OF THE RESPONSE OF AN AEROELASTIC SYSTEM USING A MAGNETORHEOLOGICAL DEVICE

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Abstract. *In this research, a theoretical study of the nonlinear response of a two degree of freedom typical airfoil section using a MR device to change the structural damping is presented. The Bingham model of the MR damper, which describes the viscous-plastic behavior of the device, is integrated to the structural model of the aeroelastic nonlinear dynamic system that results from the Aeroelastic Theory. The objective of the present work is to obtain the dynamical response and a qualitative behavior of a two degree of freedom airfoil subjected to a incompressible flow represente by a linear equation, taking into account structural nonlinearities. In the present investigation, the MR damper is connected in the plunge degree of freedom.*
The differential aeroelastic equations of motion for the aeroelastic system are reformulated into a system of four first-order autonomous ordinary differential equations. The term bifurcation is used to describe qualitative changes that occur in the orbit structure of a system as a consequence of parameter changes,this qualitative changes is showed in this work by the evolution of Phase Portraits. Numerical simulations show that the coupled nonlinearities can generate a variety of motions, including chaotic motions.

Keywords*: Aeroelasticity, MR damper, Nonlinear Dynamics, Limit Cycle Oscillations.*

1. INTRODUCTION

Aeroelasticity is the dynamic interaction of structural, inertial, and aerodynamic forces. Conventional methods of examining aeroelastic behavior have relied on a linear approximation of the governing equations which describe both the flow field and the structure. The success of linear flutter analysis is attributed to negligible nonlinear effects, yet aerospace systems inherently contain structural and aerodynamic nonlinearities which are critical for many circumstances (Dowell et al. 2008).

The tendency to increase structural flexibility, operating speed and, mainly, reduction of weight, certainly increases the likelihood of the flutter occurrence within the aircraft operational envelope. Moreover, combat aircraft can experience, during their operational life, dramatic reductions in the flutter speed that can affect their survivability (Marzocca et al., 2001, 2002a, 2002b; Librescu et al., 2002, 2003a, 2003b). Conventional methods of examining aeroelastic behavior have relied on a linear approximation of the governing equations of the flow field and the structure. However, aerospace systems inherently contain structural and aerodynamic nonlinearities and it is well known that with these nonlinearities present, an aeroelastic system may exhibit a variety of responses that are typically associated with nonlinear regimes of response, including Limit Cycle Oscillation (LCO), flutter, and even chaotic vibrations (Dowell et al., 2008). These nonlinearities result from unsteady aerodynamic sources, such as in transonic flow condition or at high angle of attack, large deflections, and partial loss of structural or control integrity. Aeroelastic nonlinearities have been identified in (Lee et al., 1999) and analyses, focusing on LCO behavior and flutter boundaries, have been performed on similar airfoils.

It is well known that the Magnetorheological fluid (MR) consists of a based fluid, with micron magnetic particles in suspension that, in the presence of a magnetic field, these magnetic particles is lined up in parallel to the field, forming a species of "chain". When the structure it is submitted to a vibration, these "chains" break, wasting energy and, the magnetic field cause the resetting of this current. The continuous breaking and reconstitution of chains, allow the fluid to waste energy of the system (Lyu et al. 2000). To describe the behavior of MR damper, the Bingham model it is adopted (Dyke et al, 1996; Tang et al. 2004). A modified form of this model had been used in (Kecik and Warminski, 2007). It is noticed in its work that the increasing of these rates may diminish the amplitude of the motion in the resonance regions. (In Litak et al, 2008) control the chaotic motions of a quarter car model using a nonlinear spring. Controlling the current applied to the MR damper, show the qualitative change of kind of motions trough bifurcations diagrams and Poincare sections and uses Melnikov theory to estimate the critical amplitude of the road surface profile above which the system can vibrate chaotically.

In this paper, the effects of coupled nonlinearities in the pitch and linear plunge degrees of freedom of an airfoil placed in an incompressible airflow are studied using a numerical time-marching scheme.

2. MATEMATICAL MODEL

A classical airfoil section with two degrees-of-freedom, pitch and plunge, will be used to show the effect of the MR damper in the aeroelastic system. The schematic for this aeroelastic system, a typical section, is shown in Fig. 1,

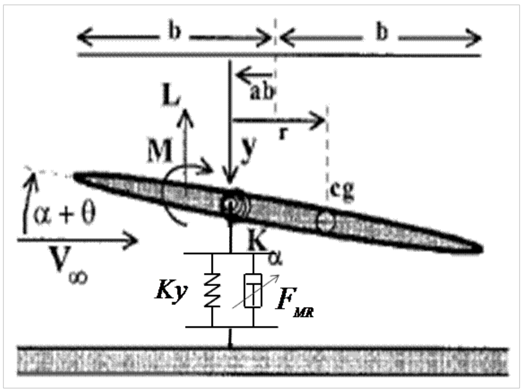

Figure 1 – 2-DOF pitching and plunging airfoil section

where the springs represent the structural bending and torsion stiffness of the airfoil section. This model is known as the typical section, which can represent a 2-D wind tunnel model, or a finite section of a wing. The equations of motion are formulated by the using of the Newton's laws and generalized aerodynamic forces (Fung, 1969; Hodges and Pierce, 2002). The nonlinear aeroelastic governing equations are:

$$m\ddot{h} + S\ddot{\alpha} + c_h\dot{h} + K_h h = -Q_h \quad (1)$$

$$I_\alpha\ddot{\alpha} + S\ddot{h} + c_\alpha\dot{\alpha} + M(\alpha) = Q_\alpha \quad (2)$$

where h (y, in the Fig. 2) is the plunge displacement and α is the pitch angle displacement. In the equations above, is presented the function $M(\alpha)$ that represents the structural nonlinearity considered in the model to obtain nonlinear motions (Marzocca, 2002a, 2002b; Librescu et. al. 2003a, 2003b; Lee et. al. 1999) and have the following expression:

$$M(\alpha) = \alpha + \varepsilon\alpha^3 \quad (3)$$

The terms Q_h and Q_α represents the aerodynamic lift and moment, that are derived by assuming the forces in each degree of freedom in equilibrium. The aerodynamic lift and moment have the follow equations for unsteady aerodynamics (Fung, 1969):

$$Q_h = -C_{L\alpha}\rho bU^2\int_{-\infty}^{\tau}\phi(\tau-\tau_0)\left[\dot{\alpha} + \frac{\ddot{h}}{b} + \left(\frac{1}{2}-a\right)\ddot{\alpha}\right]d\tau_0 - \frac{1}{2}C_{L\alpha}\rho U^2(\ddot{h} - ab\ddot{\alpha}) - C_{L\alpha}\rho bU^2\dot{\alpha} \quad (4)$$

$$\begin{aligned} Q_\alpha = {} & \left(\frac{1}{2}-a\right)C_{L\alpha}\rho b^2U^2\int_{-\infty}^{\tau}\phi(\tau-\tau_0)\left[\dot{\alpha} + \frac{\ddot{h}}{b} + \left(\frac{1}{2}-a\right)\ddot{\alpha}\right]d\tau_0 \\ & + \frac{1}{2}abC_{L\alpha}\rho U^2(\ddot{h} - ab\ddot{\alpha}) - \frac{1}{2}\left(\frac{1}{2}-a\right)C_{L\alpha}\rho b^2U^2\dot{\alpha} - \frac{1}{16}C_{L\alpha}\rho b^2U^2\ddot{\alpha} \end{aligned} \quad (5)$$

where $\phi(\tau)$ is the Wagner function and has approximated by Jones (1940) as

$$\phi(\tau) = 1 - 0.165e^{0.0455\tau} - 0.355e^{0.3\tau} \quad (6)$$

It should be remarked that in Eqs. (4) and (5), the coupling of plunging and pitch motions, referred to as aerodynamic coupling, appears explicitly. The unsteady aerodynamic lift and moment are separated into circulatory and noncirculatory components. The integral terms appearing in Eqs. (4) and (5) correspond to the circulatory effect and are expressed, in the time domain, in terms of Wagner's function. The remaining group of terms belongs to the noncirculatory effects, and is referred to as added mass. These account for the inertia effects in the fluid, and are functions of the motion and the geometry of the airfoil section. In this work, we introduce only the structural non-linearity, which was presented in Eq. (3). It is known that these expressions of the lift and of the moment have aerodynamics non-linearities, but we decided to use the model of steady and linear aerodynamics, which will be presented below.

Magnetorheological fluid (MR) consists of a based fluid, with micron magnetic particles in suspension that, in the presence of a magnetic field, these magnetic particles is lined up in parallel to the field, forming a species of chain. When the structure it is submitted to a vibration, these chains break, dissipating the vibrations energy of the system and the magnetic field cause the resetting of this chain. The continuous breaking and reconstitution of this chains, allow the fluid, in the magnetorheological device, to dissipate energy of the system (Lyu et al. 2000).

A Magnetorheological device is a linear damper completely filled with MR fluid, where is possible to change the current applied to the piston, producing the magnetic field necessary for performance of MR damper. The behavior of the MR fluid and the mechanical schema of the MR damper are presented to follow.

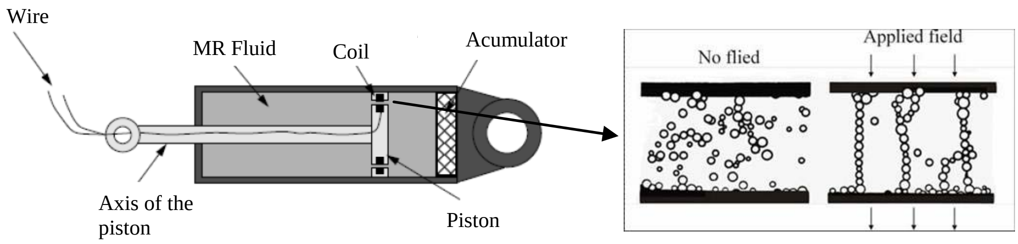

Figure 2 – Mechanical schema of MR damper and rheological behavior of MR fluid in the damper orifice.

To show the effects of the addition of a Magnetorheological damper (MR) in an airfoil to decrease the effect of the structural nonlinearity, we consider the application of the damper on the plunge degree of freedom, that is, in the vertical motion in the airfoil. The mathematical model of the MR damper is defined as:

$$\zeta = \zeta_{y(field)} + \varsigma\dot{\varphi} \tag{7}$$

where $\zeta_{y(field)}$ is the yield stress induced of the magnetic field and ς is the fluid viscosity. This model consists in two elements: an element taken as the same type of Coulomb friction, placed in parallel with a linear viscous damping, as is shown in the Fig. 3

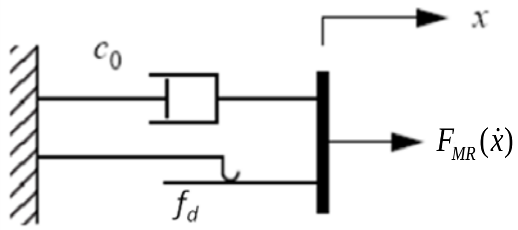

Figure 3 – Idealized layout of the MR damper

A lot of authors have considered this mathematical model of MR damping in your works, known as the Bingham model, (Dyke et al, 1996; Tang et al, 2004; Kecik and Warminski, 2007; Stanway et al, 1985; Stanway et al, 1987). For velocities different of zero, the force generated by the device is given by:

$$F_{MR}(\dot{x}) = f_d \operatorname{sgn}(\dot{x}) + c_0\dot{x}, \tag{8}$$

where, c_0 is the coefficient of viscous damping and f_d is the force related to the rheological behavior, linked to the strain that is produced by the fluid and both depend of the current (magnetic field) applied to the damper. This model does not capture some phenomena resulting from this kind of damping at speeds very close to zero, but at speeds equal to zero, the mathematical model captures the signal change of force generated from the accelerations and displacements, when they change the signal. Then we can say that the model responds well to analysis of global responses to damper, but do not characterize adequately the device for applications of control.

A more complete model, which more accurately captures the nonlinear phenomena caused by this type of device is the Bouc-Wen model (Wen, 1976) adapted for MR dampers. The Bouc-Wen model is extremely versatile, can exhibit a wide variety of hysteretical behaviors and have easy numerical handling.

The equations of this model are:

$$F = c_0\dot{x} + k_0(x - x_0) + \lambda z \tag{9}$$

where the evolutionary variable z is governed by

$$\dot{z} = -\gamma|\dot{x}|z|z|^{n-1} - \beta\dot{x}|z|^n + A\dot{x} \tag{10}$$

By adjusting the parameters of the model γ, β and A, we can control the linearity in the unloading and the smoothness of the transition from the pre-yield to the post-yield region (Spencer Jr. et al 1996). The Bouc-Wen model predicts the force-displacement behavior of the damper well, and it possesses force-velocity behavior that more closely resembles the experimental data. However, similar to the Bingham model, the nonlinear force-velocity response of the Bouc-Wen model does not roll-off in the region where the acceleration and velocity have opposite signs and the magnitude of the velocities are small.

To start the studies in this kind of problem, we use the simplest model of two that were presented, the Bingham model, by presenting equations of easy handling and ease of integration with the mathematical model of airfoil section, so the equations of the airfoil section with the model of the MR damper in the plunge degree of freedom are shown below

$$m\ddot{h} + S\ddot{\alpha} + K_h h + F_{MR}(\dot{h}) = -Q_h \tag{11}$$

$$I_\alpha\ddot{\alpha} + S\ddot{h} + M(\alpha) + c_\alpha\dot{\alpha} = Q_\alpha \tag{12}$$

where the term $F_{MR}(\dot{h})$ in the Eq. (11) represents the Bingham model which is given by the Eq. (8), c_α is the damping coefficient in pitch motion, m is the mass of the airfoil, I_α is inertia moment of the airfoil, S is the static moment about elastic axis, K_h is the stiffness in plunge motion.

To facilitate the analysis and understanding of the problem, through some mathematical manipulations, the equations of motion become dimensionless, this is, we work with displacement and the time without physical dimensions. Then, the dimensionless equations are the following

$$\mu\ddot{h} + \mu x_\alpha\ddot{\alpha} + \bar{F}_{MR}(\dot{h}) + \mu\left(\frac{\omega_h}{\omega_\alpha}\right)^2 h = -\frac{Q_h}{\pi\rho b^3\omega_\alpha^2} \tag{13}$$

$$\mu r_\alpha^2\ddot{\alpha} + \mu x_\alpha\ddot{h} + \zeta_\alpha\dot{\alpha} + \mu r_\alpha^2\alpha + \frac{\varepsilon}{\pi\rho b^4\omega_\alpha^2}\alpha^3 = \frac{Q_\alpha}{\pi\rho b^4\omega_\alpha^2} \tag{14}$$

where ω_h is the natural frequency in plunge motion, ω_α is the natural frequency in pitch motion, μ is the airfoil-mass ratio (apparent mass) and $\bar{F}_{MR}(\dot{h})$ is the dimensionless MR damping. Using the values for the parameters given in (Shahrzad and Mahzoon 2002, Rubillo et al, 2006), as are shown in Table 1, we obtain the following equations

$$12.8h'' + 1.92\alpha'' + (0.2\dot{h} + \bar{F}_{MR}(h')) + 1.97h = -\frac{1}{\pi}\left(\frac{U}{b\omega_\alpha}\right)^2(5.568\alpha + 0.0942) \tag{15}$$

$$1.92\alpha'' + 3.84h'' + 0.2\alpha' + 3.84\alpha + 4.24\alpha^3 = \frac{1}{\pi}\left(\frac{U}{b\omega_\alpha}\right)^2(1.4845\alpha + 0.0084) \tag{16}$$

where the linear viscous damping in the pitch degree of freedom is considered.

Table 1 (Shahrzad and Mahzoon, 2002)					
μ = 12,8	ε = 30	ω_α = 88	ω_h = 34,6	r_α = sqrt(0,3)	x_α = 0.15

Taking the vector $\begin{bmatrix} \alpha & \dot{\alpha} & h & \dot{h} \end{bmatrix}^T = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$ as vector of state variables, we obtain the following equations for the system:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = 0.0361(0.2x_4 + F_{MR}(x_4)) + 0.0078x_3 - 0.0561x_2 - \left(1 - 0.2018U^2\right)x_1 - 1.1871x_1^3 + 0.002U^2$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = -0.0723(0.2x_4 + F_{MR}(x_4)) - 0.1552x_3 + 0.0079x_2 + \left(0.1511 - 0.1579U^2\right)x_1 + 0.1669x_1^3 - 0.00248U^2 \quad (17)$$

where coefficients of Bingham model of the MR damper are given by: $c_0 = 48i + 14$ e $f_d = 62i + 1.5$ and i is the applied current to the damper. (Maślanka,2007)

With these equations, it is possible to start the numerical simulations, which were performed using the Runge-Kutta routine, found in the software MatLab©.

3. NUMERICAL SIMULATIONS AND RESULTS

In this item, we show the numeric simulations and results obtained for this problem. Through computational routines, we obtain the behavior of the system in time and the phase portraits to show the effect of this kind of damper in our problem. We show the phase portraits to emphasize the semi-active control of the LCO's, phenomenon that appears in the model without this kind of damping.

Below, we show the response of the model when the current applied to the MR damper is zero, for a speed U = 15 m/s, Fig. 4, below the flutter speed for the model without damping and with the structural nonlinearity. Since this nonlinearity does not influence the in flutter speed of the system. (Mahzoon and Shahrzad 2002, and Rubillo et al, 2006)

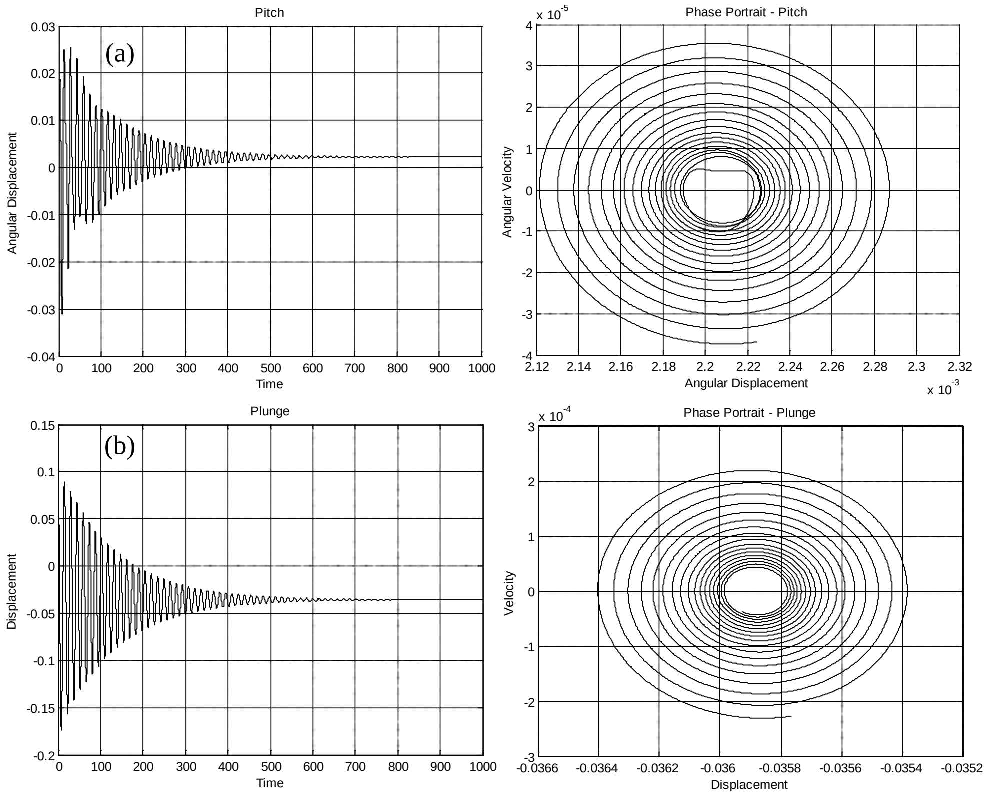

Figure 4. Time History and Phase Portrait, U = 15 m/s, (a) Pitch; (b) Plunge

It was observed that in this case the system seeks to equilibrium without need of the action of the MR damper. After a certain time interval, the system displays LCO with very low amplitudes in pitch and plunge, dispensing the operation of the damper.

When the system exceeds the flutter velocity, which is 17.22 m/s, (Mahzoon and Shahrzad 2002, and Rubillo et al, 2006) where a bifurcation occurs, the amplitudes of the LCO will present a high magnitude, which can be dangerous to the integrity of the aircraft. In this case then becomes necessary to apply a current in the damper to increase the damping to bring these oscillations to the equilibrium. In Figure 5, shows the behavior of the system at a speed above of the flutter speed without the MR damping (U = 20 m/s).

In Fig. 5, we can see the effect of MR damper in the amplitudes of the LCO in Pitch degree of freedom, note that, with increasing current applied to the MR damper, the magnitude of the LCO presents fall in the range of 10^1. For i = 0.1A, we can consider that the system is in equilibrium, showing very small oscillations. For the Plunge degree of freedom, when applies to *i = 0.06A*, the system presents a very small oscillation, as the Fig. 6 show.

When the flow speed reaches 25 m / s, the system presents the LCO amplitude more largest than the previous case, because the aerodynamic forces that act on the system becomes more higher, causing this phenomenon. Thus, need is a bigger current applied to the MR damper. This is done and is shown in Fig. 7.

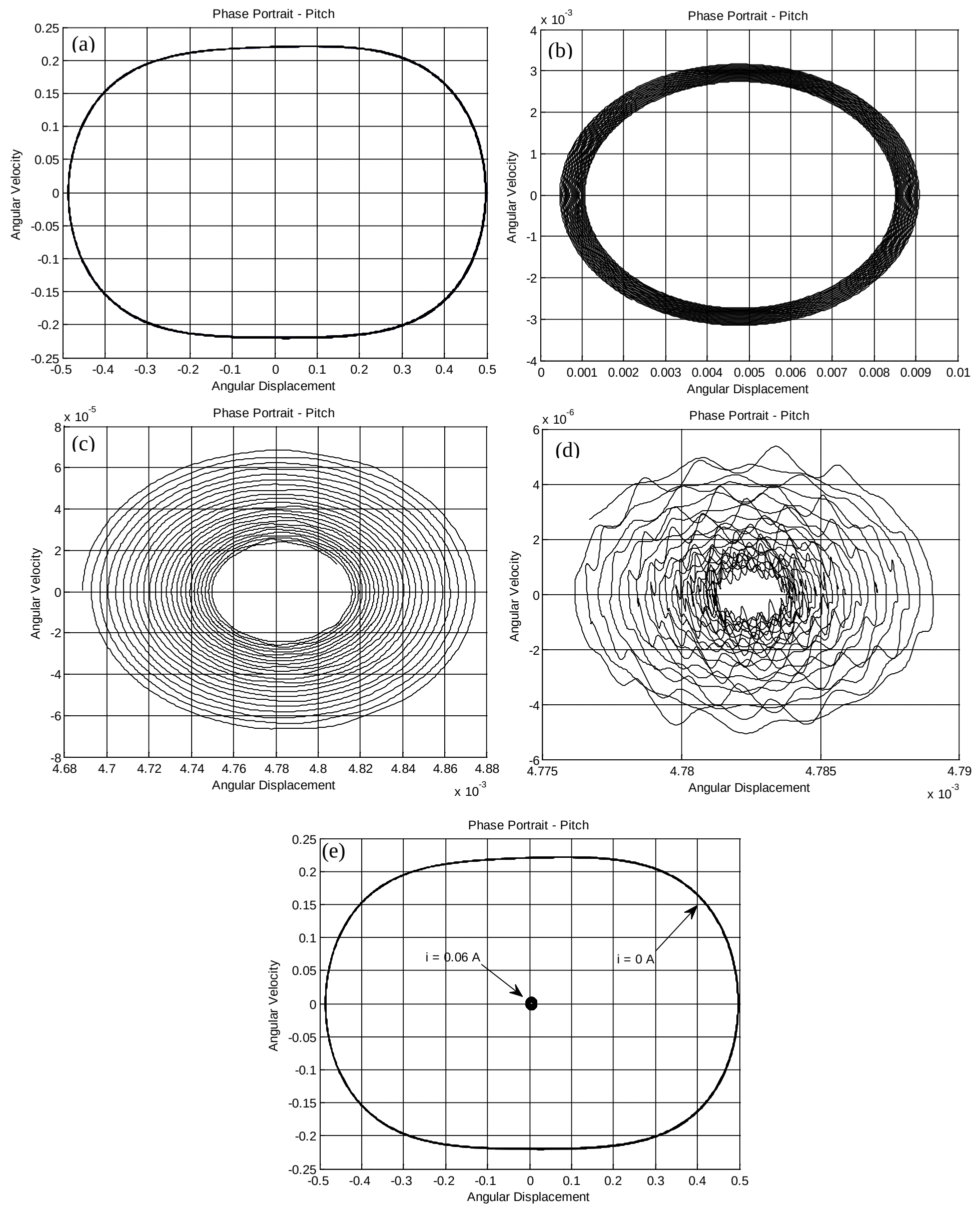

Figure 5. Evolution of Phase Portrait of Pitch degree of freedom, U = 20 m/s, (a) *i = 0A*; (b) *i = 0.06A*; (c) *i = 0.08A*; *(d) i = 0.1A; (e)* LCO's Amplitude comparison.

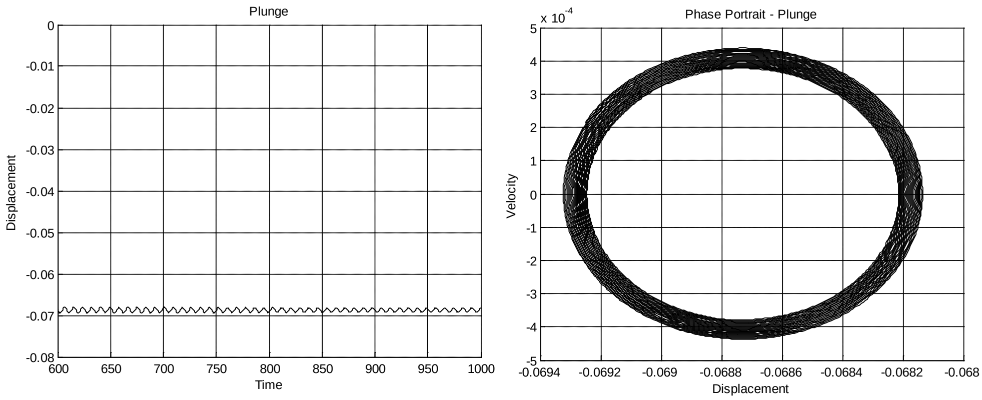

Figure 6. Time History and Phase Portrait, U = 20 m/s, Plunge degree of freedom

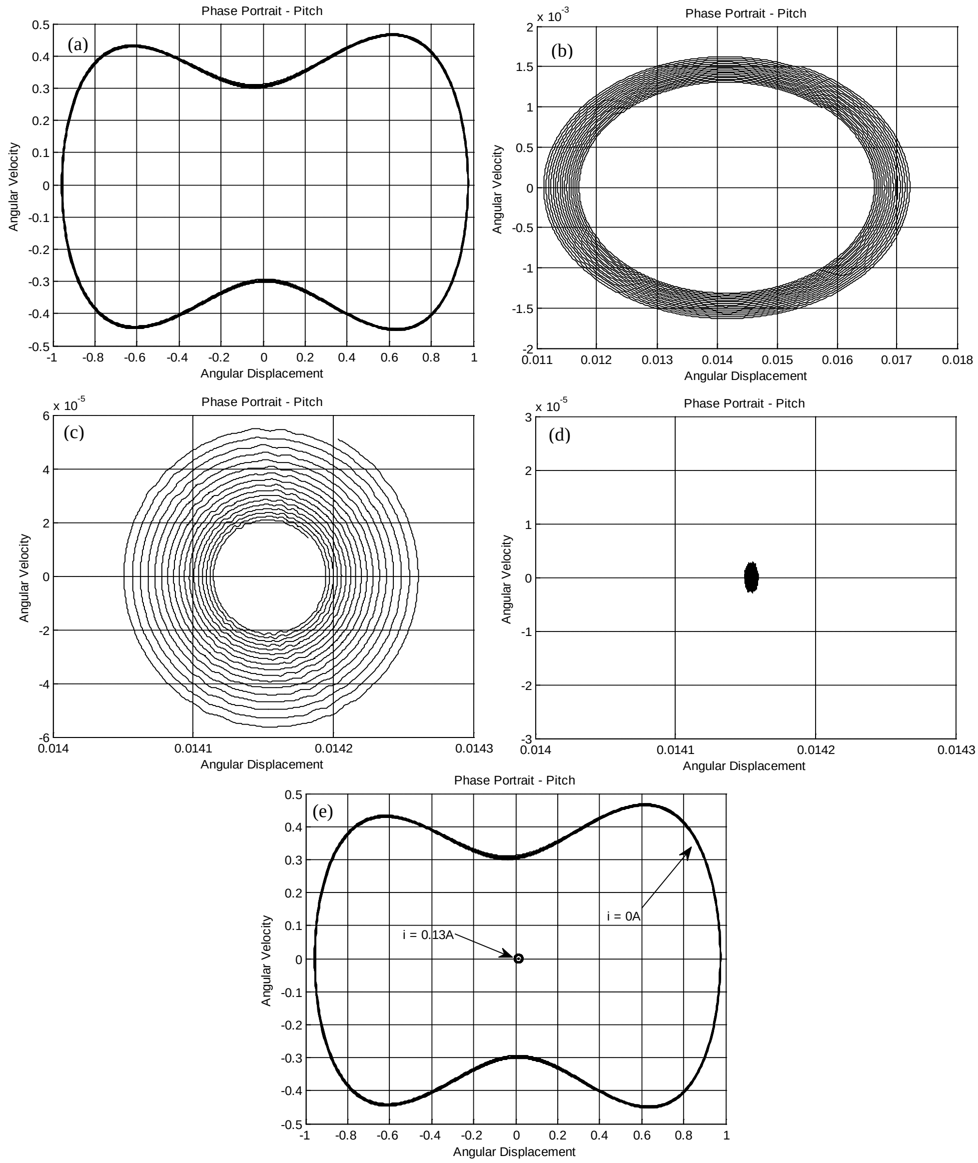

Figure 7. Evolution of Phase Portrait of Pitch degree of freedom, U = 25 m/s, (a*) i = 0A*; (b*) i = 0.13A*; (c) *i = 0.0.16A*; *(d) i = 0.2A; (e)* LCO's Amplitude comparison.

In this case, we can see that we need a higher current for the system to respond, comparing with the previous case, but this is still low. Since the current applied to the damper may range from 0A to 2A. With the increasing of current, is observed a change in the types of LCO found, going of an LCO with high amplitude for a trajectory that can be considered as the equilibrium of the system. This shows that the amount of energy needed to control the LCOs found is very small, how we can perceive by the values of current applied to MR damper.

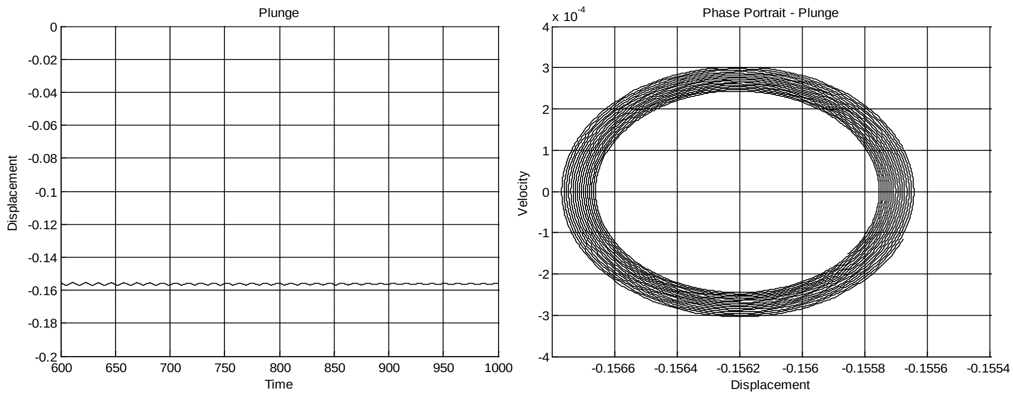

Figure 8. Time History and Phase Portrait, U = 25 m/s, Plunge degree of freedom

For the Plunge degree of freedom, when applies to i = 0.13A, the system presents a very small oscillation, as the Fig. 8 show.

When it increases the value of current applied to the MR damper with the objective of controlling the amplitude of LCOs, it should not be thought that these phenomena do not happen more. We can say then that the LCOs appear at speeds greater than the speed with which the flutter phenomenon occurs, for each value of current applied to the MR damper. Thus, nothing can be said about the speed at which the phenomenon will occur, but the information shown previously, makes possible the "postponement" of this event, so that the studied instability happening in higher speeds, increasing so the flight envelope of each aircraft in which could be used this type of device.

It should be noted that in this paper, is made a feasibility study of the use of this device in aeroelastic systems, for which a real implementation of this study, would need a closed loop control with the objective of finding the "best" current to be applied to the device, to get the best responses of the system in each situation in which it can operate.

4. CONCLUSIONS

In this work, is shown the application of a MR damper in a aeroelastic system that has an structural nonlinearity. This nonlinearity causes the nonlinear behavior analyzed, the appearance of LCOs. Putting the MR damper on the Plunge degree of freedom, it became possible to control the amplitudes of these phenomena, through of the increase of the applied current to the MR device.

From the information shown at work, it can said that the damper is able to control these LCOs and also that the phenomenon will occur at higher speeds where the aerodynamic forces overlap the loads generated by the MR damper.
it also concludes that a closed loop control becomes essential in this problem, since the choice of current to be applied to the damper is not a decision that may be made without information about the behavior of the structure, velocity of flow and the loads that the system is exposed.

3. ACKNOWLEDGEMENTS

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