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LONGITUDINAL CONTROL LAWS FOR AIRCRAFT WITH RELAXED STABILITY

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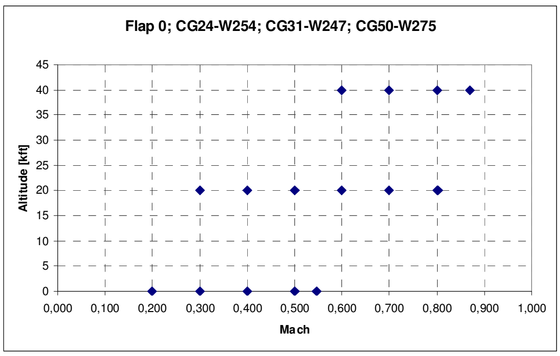
Abstract. *The aim of this work was to develop stability and control laws for a comercial aircraft with relaxed stability. The aircraft stability and response characteristics were evaluated in several cruise phase conditions, including different CG, weight, altitude and speed variations. A critical point of the envelope was chosen and the aircraft control law C*u was applied in order to comply with the C* criterion of flying quality and the speed stability requirement. The control law used consisted of a SAS with angle of attack and pitch rate feedback and a CAS with normal acceleration, pitch rate and airspeed feedback besides a PID controller. The controller gains for the critical point were computed by the LQR (Linear Quadratic Regulator) method, whose matrices were estimated by Bryson and Gangsaas rules. An initial estimation of the gains was performed, before the gains optimization, in order to push away the pole with the greatest real part from the imaginary axis. Due to the large number of design parameters, several responses were simulated. The chosen responses were then applied to the others operating points. These points were divided into four ranges of dynamic pressure, each of them with their respective gains. The gain schedule was then validated by applying the final control law in some operating points with aircraft CG and weight different from the design.*

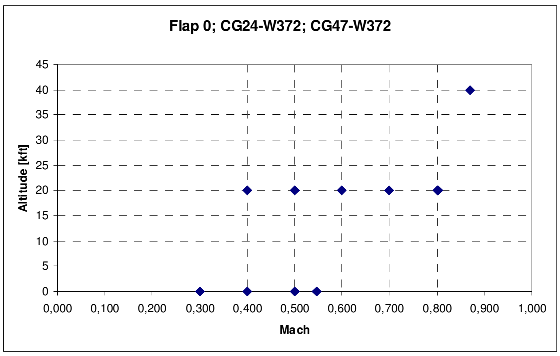
Keywords*: longitudinal control, relaxed stability, C*u control law, linear quadratic regulator, handling quality*

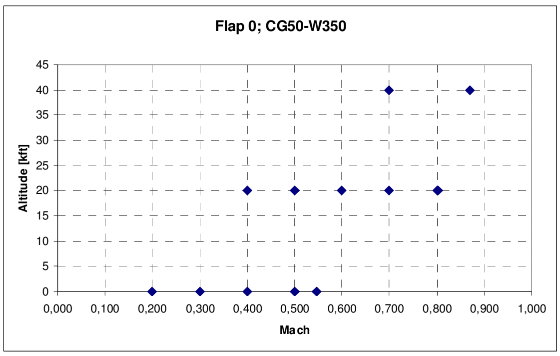
1. INTRODUCTION

Nowadays, commercial aircrafts are designed with a reduced tail volume to minimize drag and reduce their operating cost as much as possible. As a consequence, the stability is decreased or even eliminated and a control system is necessary to stabilize the aircraft. Therefore, fly-by-wire systems are developed to stabilize these aircrafts with relaxed stability.

The purpose of this work was the development of longitudinal control laws based on the C*u criterion for a commercial aircraft in several cruise phase conditions, including different CG, weight, altitude and speed variations as shows Fig. 1 (REISER, 2008). To implement the first C*u controller, the critical point *H=40kft; Mach=0.8; CG=50%* and *W=27,500 kg* was chosen. Where H is altitude, CG the center of gravity and W is the weight.







Where:

Code	CG [% of MAC]	Weight [kg]
CG24-W254	24	25400
CG24-W372	24	37200
CG31-W247	31	24700
CG47-W372	47	37200
CG50-W275	50	27500
CG50-W350	50	35000

Figure 1. Operating Points

2. HANDLING-QUALITIES

2.1. Handling-qualities requirements

Control-law design can only be performed satisfactorily if a set of design requirements or performance criteria is available. In case of control systems for piloted aircraft, generally applicable quantitative design criteria are very difficult to obtain. The reason for this is that the ultimate evaluation of a human-operator control system is necessarily subjective and, with aircraft, the pilot evaluates the aircraft in different ways depending on the type of aircraft and phase of flight.

The Cooper-Harper scale is a systematic approach to handling-qualities evaluation through pilot opinion rating (Tab. 1). Once a rating scale like this has been established it is possible to begin correlating the pilot opinion rating with the properties of the aircraft dynamic model, and hence derive some analytical specifications that will guarantee good handling qualities. Although this may seem simple in principle, it has proven remarkably difficult to achieve in practice, and after many years of handling-qualities research it is still not possible to precisely specify design criteria for control systems intended to modify the aircraft dynamics.

It will be considered first some possible ways in which requirements for dynamic response may be specified. The aircraft model may be linearized in a particular flight condition and the poles and zeros, or frequency response, of a particular transfer function compared with a specification. Alternatively, certain time responses may be derived from the nonlinear model, in a particular flight condition, and be compared with specifications (Stevens and Lewis, 2003).

Table 1. Pilot opinion rating and flying qualities level

Aircraft Characteristics	Demands on Pilot in Selected Task or Required Operation	Pilot Rating	Flying Qualities Level
Excellent; highly desirable	Pilot compensation not a factor for desired performance	1	1
Good; negligible deficiencies		2	
Fair; some mildly unpleasant deficiencies	Minimal pilot compensation required for desired performance	3	
Minor but annoying deficiencies	Desired performance requires moderate pilot compensation	4	2
Moderately objectionable deficiencies	Adequate performance requires considerable pilot compensation	5	
Very objectionable but tolerable deficiencies	Adequate performance requires extensive pilot compensation	6	
Major deficiencies	Adequate performance not attainable with maximum tolerable pilot compensation Controllability not in question	7	3
	Considerable pilot compensation required for control	8	
	Intense pilot compensation required for control	9	
Major deficiencies	Control will be lost during some portion of required operation	10	

2.2. The military flying-qualities specifications

The U.S. Military Specification for the Flying Qualities of Piloted Airplanes (MIL-F-8785C, 1980) does provide some analytical specifications that must be met by U.S. military aircraft. The military specification defines airplane classes, flight phases, and flying qualities levels, so that different modes can be specified for the various combinations (Tab. 2). The flying qualities levels are linked to the Cooper-Harper ratings as shown in Tab. 1 (Stevens and Lewis, 2003).

2.2.1. Phugoid specifications

The military specification dictates that for the different levels of flying qualities, the damping ζ_p and natural frequency ω_{np} of the phugoid mode will satisfy the following requirements:

Level 1: $\zeta_p \geq 0.04$
Level 2: $\zeta_p \geq 0.0$
Level 3: $T_{2p} \geq 55.0$ s

In the level-3 requirement the mode is assumed to be unstable, and T_{2p} denotes the time required for the mode to double in amplitude (Stevens and Lewis, 2003).

Table 2. Definitions – Flying qualities specifications

Airplane Classes	
Class I	Small, light airplanes.
Class II	Medium weight, low-to-medium-maneuverability airplanes.
Class III	Large, heavy, low-to-medium-maneuverability airplanes.
Class IV	High-maneuverability airplanes
Flight Phases	
Category A	Non-terminal flight phases generally requiring rapid maneuvering.
Category B	Non-terminal flight phases normally accomplished using gradual maneuvers without precision tracking, although accurate flight-path control may be required.
Category C	Terminal flight phases normally accomplished using gradual maneuvers and usually requiring accurate flight-path control.
Flying Qualities Levels	
Level 1	Flying qualities adequate for the mission flight phase.
Level 2	Flying qualities adequate to accomplish the mission flight phase, but some increase in pilot workload or degradation in mission effectiveness exists.
Level 3	Flying qualities such that the airplane can be controlled safely, but pilot workload is excessive, or mission effectiveness is inadequate, or both.

2.2.2. Short-period specifications

The short-period requirements are specified in terms of the short period mode natural frequency and damping ζ_{sp} of the equivalent low-order system as shows Tab. 3 (Stevens and Lewis, 2003).

Table 3. Short-period damping ratio limits

Level	Cat. A & C Flight Phases		Cat. B Flight Phases	
	Minimum	Maximum	Minimum	Maximum
1	0.35	1.30	0.30	2.00
2	0.25	2.00	0.20	2.00
3	0.15*	no limit	0.15*	no limit

* May be reduced at altitude > 6,096 m with approval.

The requirements on equivalent undamped natural frequency ($\omega_{n,sp}$) are given in Tab. 4 and specified indirectly, in terms of the quantity $\omega_{nsp}^2/(n/\alpha)$. This term is known as Control Anticipation Parameter, CAP (Field, 1993).

Table 4. Limits on $\omega_{nsp}^2/(n/\alpha)$

Level	Cat. A		Cat. B		Cat. C	
	Min.	Max.	Min.	Max.	Min.	Max.
1	0.28; $\omega_n \geq 0.1$	3.60	0.085	3.60	0.16; $\omega_n \geq 0.7$	3.60
2	0.16; $\omega_n \geq 0.6$	10.00	0.038	10.0	0.096; $\omega_n \geq 0.4$	10.00
3	0.16	no limit	0.038	no limit	0.096	no limit

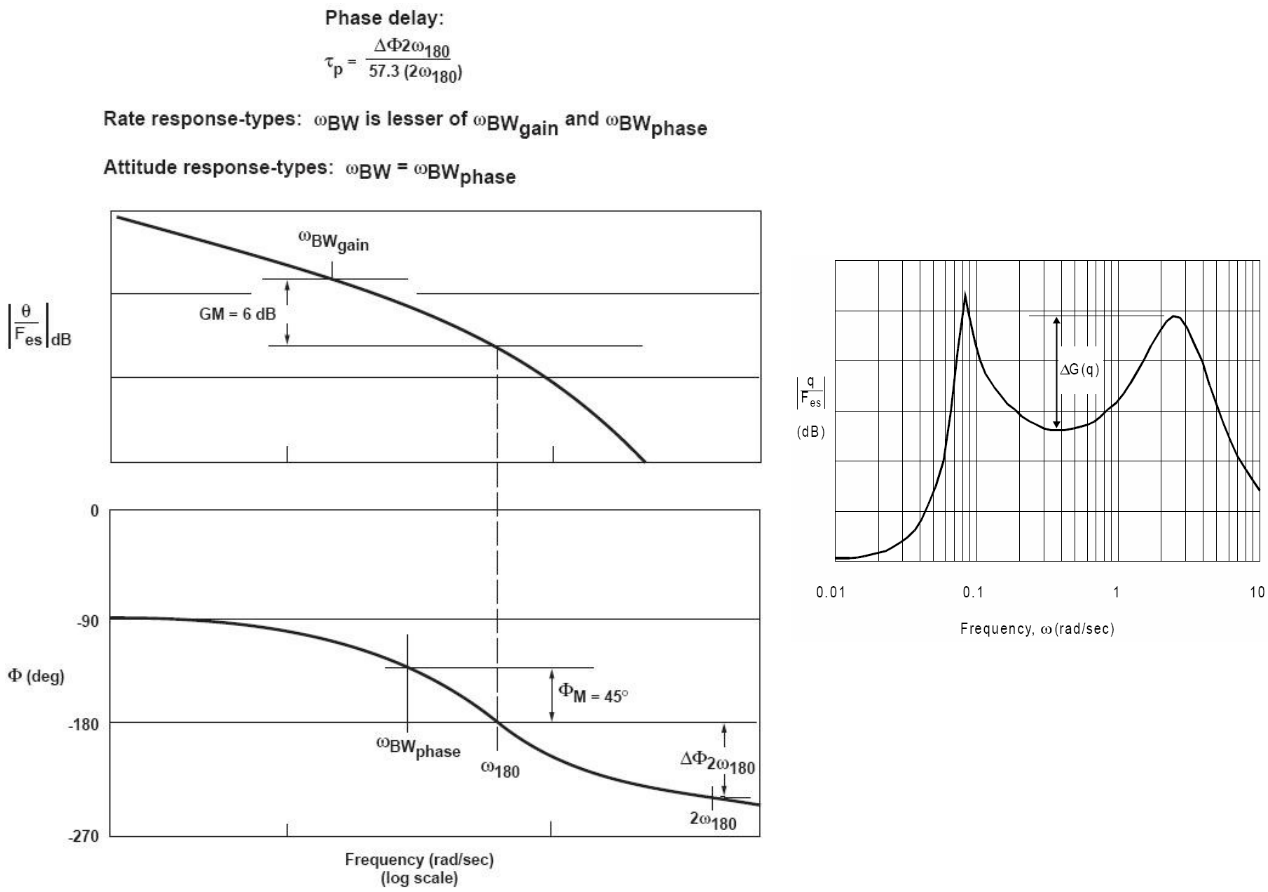
There are some additional limits on the minimum value of *n/α* and the minimum value of ω_n, for different classes of airplane in category C.

2.2.3 Bandwidth criterion

The bandwidth criterion analyses the system for Pilot Induced Oscillations (PIO). They are based on pitch attitude and flight path bandwidth and pitch rate overshoot, using the parameters defined in Fig. 2a. The core of the criterion is a crossplot of angular attitude bandwidth frequency versus phase delay. Bandwidth measures the basic stability of the airplane and determines the frequency range over which piloted control is possible with a minimum of pilot equalization. Phase Delay measures the high-frequency phase loss if the pilot operates at high frequencies (MITCHELL; HOH, 2000).

For the pitch requirements, there are regions where PIO is unlikely on the basis of the attitude bandwidth characteristics alone. In some instances, high pitch rate overshoot is a contributor, and limits are placed on the frequency-domain-based metric, ΔG(q) (Fig. 2b).

Requirements on pitch attitude bandwidth versus phase delay are presented in Fig. 3.



(a) Pitch Attitude Bandwidth and Phase Delay (b) Pitch Rate Overshoot

Figure 2. Bandwidth criterion definitions

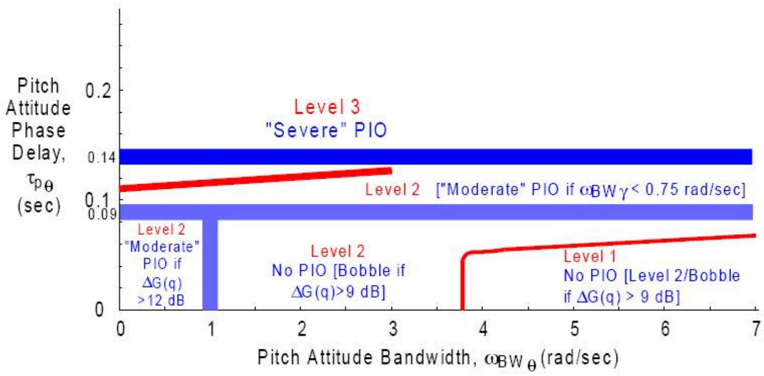


Figure 3. Bandwidth criterion requirements

3. C*U CONTROL LAW

In the 60s, it was discovered that pilots respond to a blend of pitch rate (q) and normal acceleration (n_z), with the ratio varying according to natural variations in the aircraft's response. At low velocities normal acceleration cues are weak; therefore the predominant cue would be pitch rate. At high velocities where slight pitching may produce large normal acceleration changes, n_z cues dominate. This blend of normal acceleration and pitch rate was named C* and is defined in Eq. 1. The ratio of the constants K_{nz} and K_q was determined at the velocity where both cues command equal pilot attention, which was chosen as 122 m/s (Field, 1993).

$$C^* = K_{nz} n_z + K_q q = n_z + 12{,}4 \cdot q \tag{1}$$

To preserve the good characteristics of C* criterion and grant speed stability, the C*u concept was defined as follows.

$$C*u = C* + K_V \cdot (V - Veq) = n_z + 12{,}4 \cdot q - 1{,}4411 \cdot (V - Veq) \quad (2)$$

The gain K_V is used to adjust the force required in the stick for the variation between the current (V) and the steady state (V_{eq}) speeds. According to FAR 25.173, this relation cannot be less than 1 pound/6 kts ≈ 1, 4411N • s/m.

A C*u step input causes a variation in the pitch rate, that returns to its steady state value. The aircraft presents a tendency to climb or descent and changes V accordingly. A positive C*u step implies in a climb with speed reduction, and vice versa. The term $K_V \bullet (V - V_{eq})$ acts as a spring of constant damping and variable steady state value (NAJMABADI et al, 2000).

4. MATHEMATICAL MODELING

4.1 Aircraft modeling

The aircraft longitudinal equations used in this work are presented bellow:

$$\begin{aligned}
\dot{V} &= \frac{F_p \cdot \cos(\alpha + \alpha_F) - D_r - mg \sin\gamma}{m} \\
\dot{\alpha} &= q - \dot{\gamma};\ \ where\ \ \dot{\gamma} = \frac{F_p \cdot \sin(\alpha + \alpha_F) + L - mg\cos\gamma}{m \cdot V} \\
\dot{q} &= \frac{M_a + F_p \cdot z_F}{I_{yy}} \\
\dot{\theta} &= \dot{\gamma} + \dot{\alpha}
\end{aligned} \quad (3)$$

Where V is the aircraft true air speed, F_p is the thrust force, α is the angle of attack, α_F is the angle between the thrust force and the aircraft longitudinal axis, D_r is the drag, m is the aircraft mass, g is the gravitational acceleration, γ is the flight path angle, q is the pitch rate, L is the lift, z_F is the distance between the aircraft longitudinal axis and the thrust force axis, I_{yy} is the inertial moment of the wing axis and θ is attitude angle (REISER, 2008).

These equations were linearized around the operating points showed in Fig. 1. The final space state model has V, α, q and θ as the state variables x, normal accelerations (n_z and n_x) and x as the output variables y and the elevator deflection (δ_p) as the input variable.

4.2 Flight control system

The flight control system is presented in Fig. 4. This system consists of a SAS (Stability Augmentation System) with pith rate and angle of attack feedback ($K_r=[K_\alpha\ K_q]$ gains) closing the loop around the aircraft dynamics and the elevator actuator, a CAS (Control Augmentation System) with a PID (Proportional + Integral + Derivative) controller ($K_c=[K_i\ K_d\ K_p]$ gains) and a C*u feedback ($K_{Cu}=[K_V\ K_q\ K_{nz}]=[-1{,}4411\ 12{,}4\ 1]$ gains). The elevator actuator model has the constant a as the time constant inverse and the angle of attack filter has b as the time constant inverse. On the aircraft dynamics block, **A**, **B, C** and **D** are the aircraft linearized model matrices. **F**, **G, M** and **N** are the matrices of the non-ideal PID model.

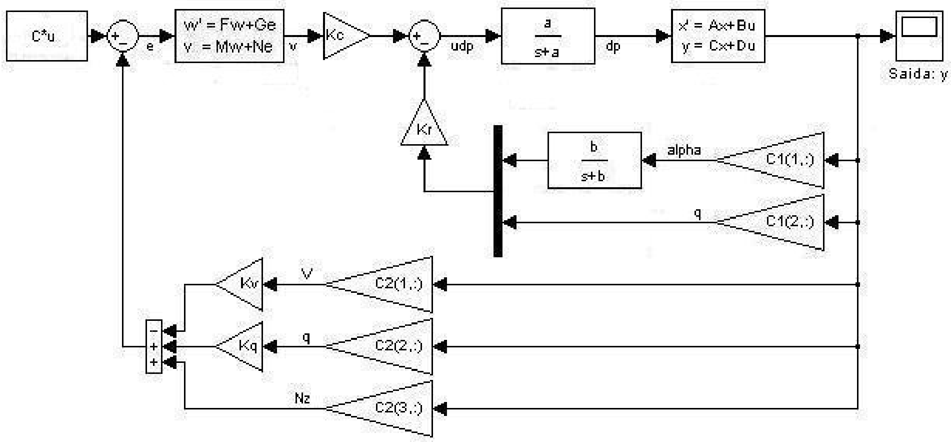


Figure 4. Flight control system

The equations for this system are:

$$\begin{cases} \begin{bmatrix} \dot{x}_a \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} A_a & 0 \\ -G \cdot K_{Cu} \cdot C_2 \cdot C_a & F \end{bmatrix} \cdot \begin{bmatrix} x_a \\ \omega \end{bmatrix} + \begin{bmatrix} B_a \\ 0 \end{bmatrix} \cdot u_{\delta_p} + \begin{bmatrix} 0 \\ G \end{bmatrix} \cdot C^* u_{Cmd} \\ \begin{bmatrix} \overline{\alpha} \\ q \\ v \end{bmatrix} = \begin{bmatrix} C_1 \cdot C_a & 0 \\ -N \cdot K_{Cu} \cdot C_2 \cdot C_a & M \end{bmatrix} \cdot \begin{bmatrix} x_a \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ N \end{bmatrix} \cdot C^* u_{Cmd} \end{cases} \Rightarrow \begin{cases} \dot{x}_f = A_f \cdot x_f + B_f \cdot u_{\delta_p} + G_f \cdot C^* u_{Cmd} \\ y_f = C_f \cdot x_f + D_f \cdot C^* u_{Cmd} \end{cases} \quad (4)$$

Where:

$$A_a = \begin{bmatrix} & A & & 0 & B & \\ 0 & b & 0 & 0 & -b & 0 \\ & 0 & & 0 & -a & \end{bmatrix}; \; B_a = [0 \quad 0 \quad a]^T; \; C_a = \begin{bmatrix} C & 0 & D \\ 0 & 1 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 \\ 0 & -p \end{bmatrix}; \; G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \; M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T; \; N = [0 \quad 0 \quad 1]^T \quad (5)$$

$$C1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}; \; C2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$x_a = [x \quad \overline{\alpha} \quad \delta_p]^T$$

5. C*U CONTROLLER IMPLEMENTATION

The model was simulated on Matlab 6.5/Simulink. The controller gains for the critical point were computed by the LQR method, which is based on the minimization of J (Eq. 6). P is the Lyapunov equation solution, as shows Eq. 7 (OGATA, 2002).

$$J = \frac{1}{2} \cdot trace(P \cdot X); \; X = I \quad (6)$$

$$A_c \cdot P + P \cdot A_c^T + Q + B_c^T \cdot R \cdot B_c = 0 \; where \begin{cases} A_c = A_f - B_f \cdot K_t \cdot C_f \\ B_c = G_f - B_f \cdot K_t \cdot D_f \\ K_t = [K_\alpha \quad K_q \quad K_i \quad K_d \quad K_p] \end{cases} \quad (7)$$

The matrices Q and R are estimated by Bryson and Gangsaas rules. The Bryson rule is defined on Eq. 8, where x_{max} and u_{max} are the maximum variation of each state and input variable, respectively. These variations were based in the several operating point's steady state values. The Gangsaas rule uses the open-loop crossover frequency of each state variable (GANGSAAS et al., 1986).

$$Q = \begin{bmatrix} 1/x_{1_{max}}^2 & 0 & \cdots & 0 \\ 0 & 1/x_{2_{max}}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/x_{n_{max}}^2 \end{bmatrix}; \; R = \begin{bmatrix} 1/u_{1_{max}}^2 & 0 & \cdots & 0 \\ 0 & 1/u_{2_{max}}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/u_{n_{max}}^2 \end{bmatrix} \quad (8)$$

An initial estimation of the gains was performed, before the gains optimization, in order to push away the pole with the greatest real part from the imaginary axis. The initial estimation was computed through the Matlab *fminsearch* function while *fmincon* was applied to the LQR method.

4. RESULTS

The gains computed by the Bryson and Gangsaas rules to the critical point *H=40kft; Mach=0.8; CG=50%* and *W=27,500 kg* are applied to others operating points. Based on the system response to the computed gains, these operating points are divided into three ranges of dynamic pressure (q_c) and a new reference point is chosen (REISER, 2008).

4.1 Dynamic pressure lower than 4,500 Pa

The chosen reference point for this dynamic pressure interval is *CG = 50%; m = 27,500kg; H = 20kft; Mach = 0.3* and q_c *= 2,934Pa*. The best gains were computed by the Gangsaas rule, as shows Fig. 5. As the Bryson rule resulted in a bad elevator temporal response, a weighting increase of the variables associated with the elevator deflection was applied and resulted in the Bryson II approach (REISER, 2008).

According to Fig. 5, the Gangsaas temporal response is:

- Oscillatory, because these operating points are in a region limit of the operational envelope, with high angle of attack;
- Requires too much from the elevator, because its effectiveness is not good in low dynamic pressures.

According to Fig. 6, the Gangsaas rule controller is classified as level 1 for short-period and phugoid mode. The bandwidth criterion evaluates this controller as level 2 for PIO.

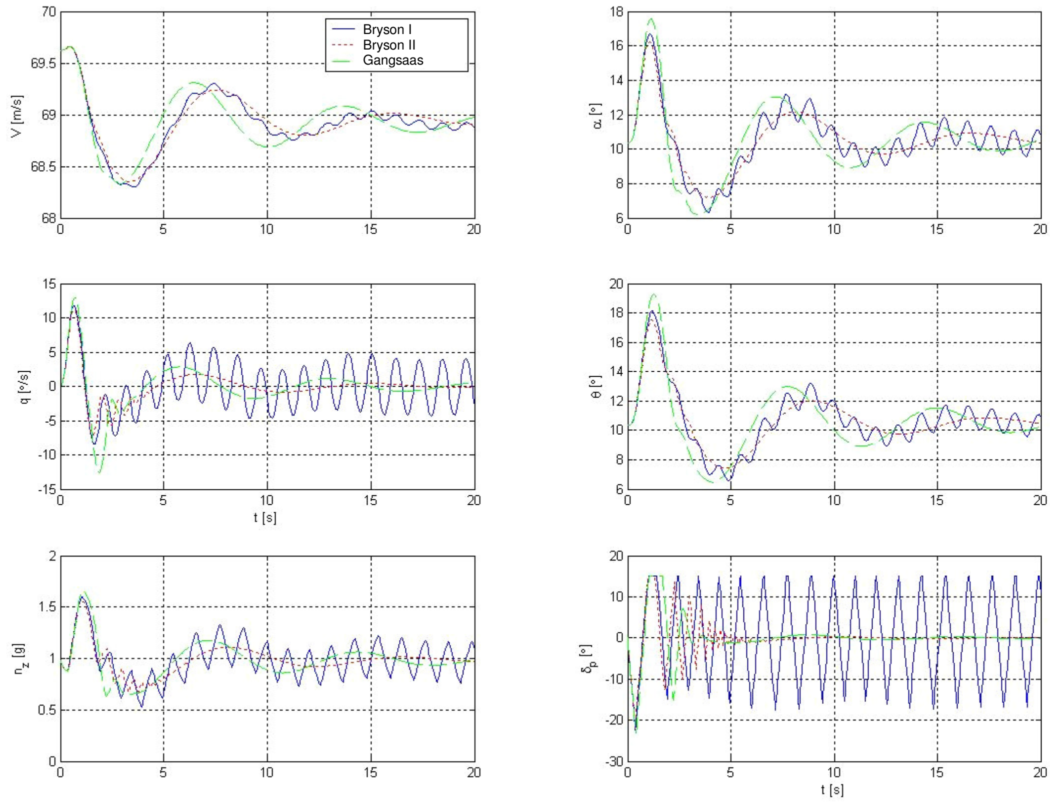
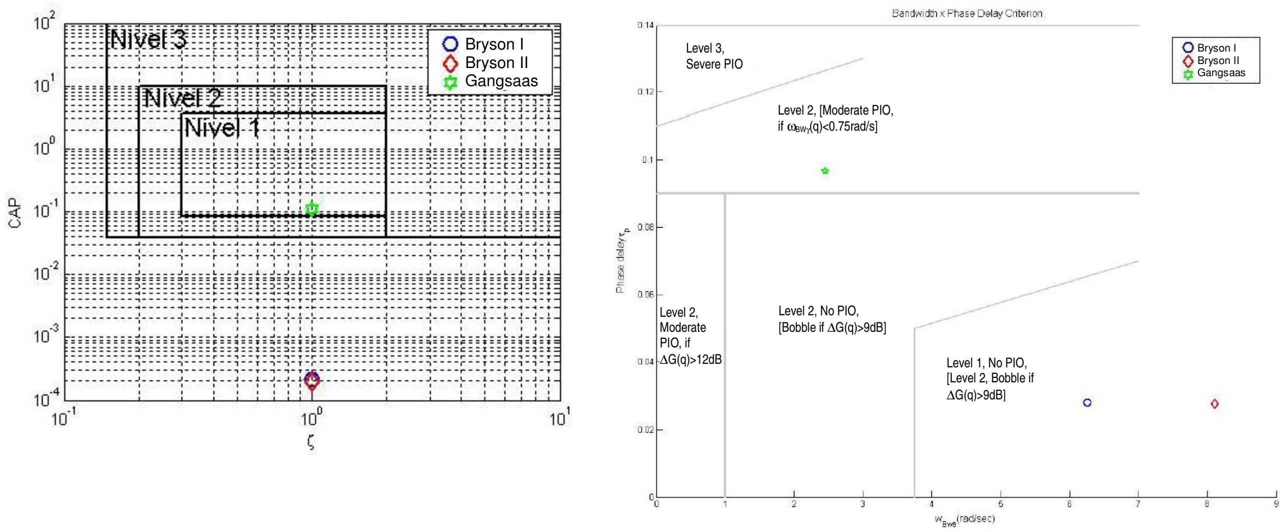


Figure 5. Temporal responses



(a) Short-Period specification (b) Bandwidth criterion

Figure 6. Handling quality levels

4.1 Dynamic pressure between 4,500 Pa and 11,500 Pa

The chosen reference point for this dynamic pressure interval is *CG = 50%; m = 27500kg; H = 40kft; Mach = 0, 8* and q_c = *8,403Pa*. The best gains were computed by the Bryson II rule, as shows Fig. 7 (REISER, 2008).

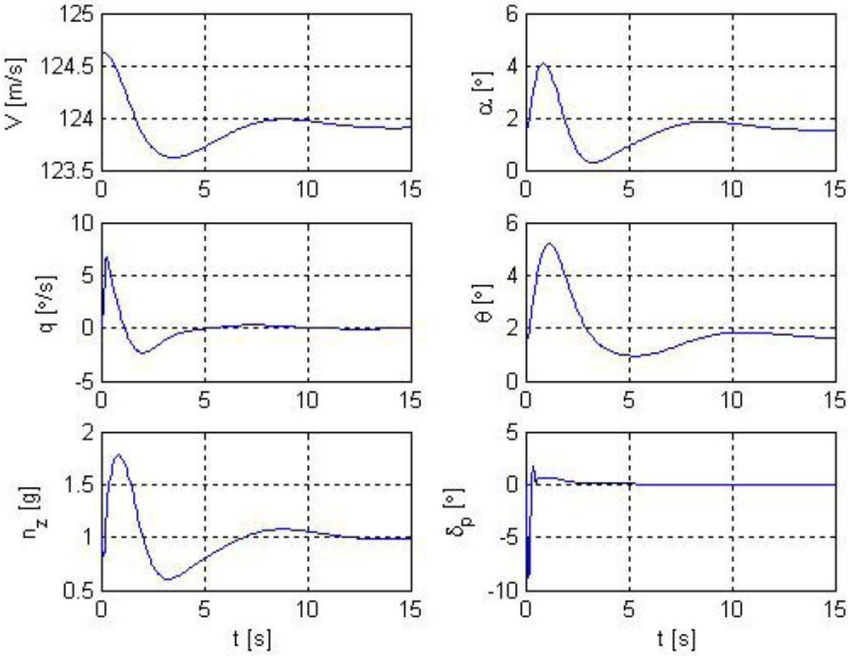
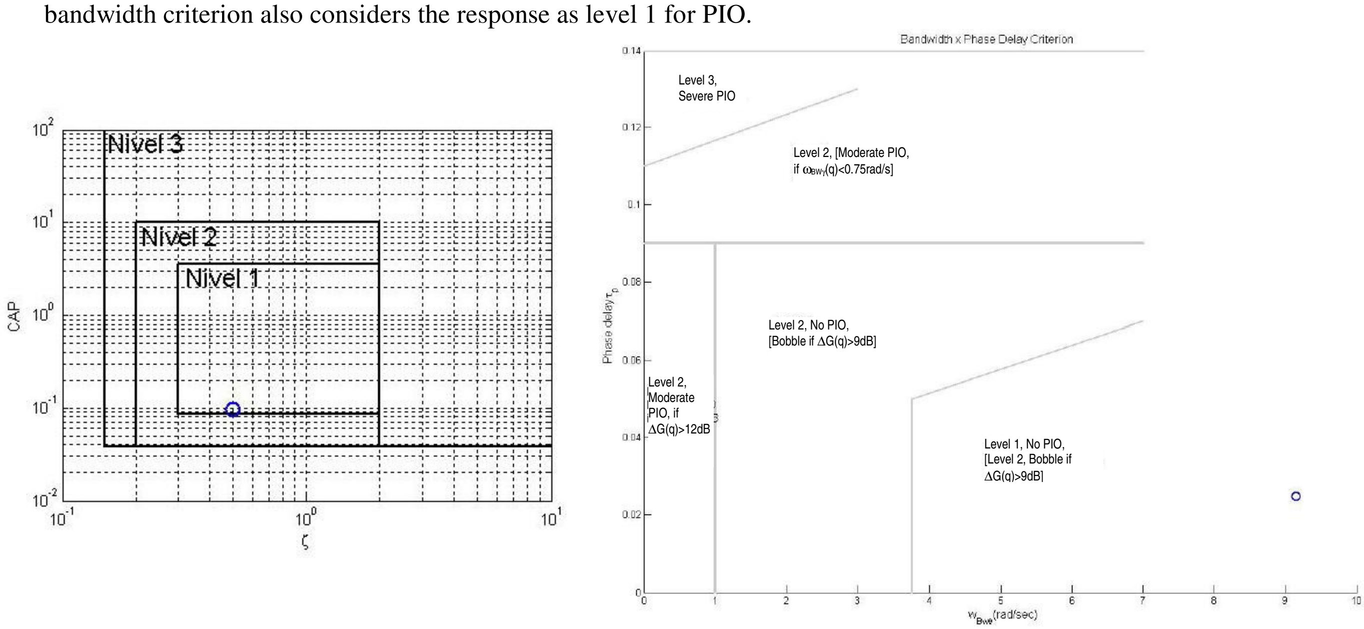


Figure 7. Temporal responses

According to Fig. 8, the Bryson II rule response is classified as level 1 for short-period and phugoid mode. The bandwidth criterion also considers the response as level 1 for PIO.



(a) Short-Period specification (b) Bandwidth criterion

Figure 8. Handling quality levels

4.1 Dynamic pressure higher than 11,500 Pa

The chosen reference point for this dynamic pressure interval is *CG = 50%; m = 27,500kg; H = 0kft; Mach = 0.5* and q_c = *17,732Pa*, as shows Fig. 9. According to Fig. 10, the responses are classified as level 2 for short-period mode and level 1 for phugoid mode. The bandwidth criterion evaluates both Bryson rules as level 1 and the Gangsaas rule as level 2 for PIO (REISER, 2008).

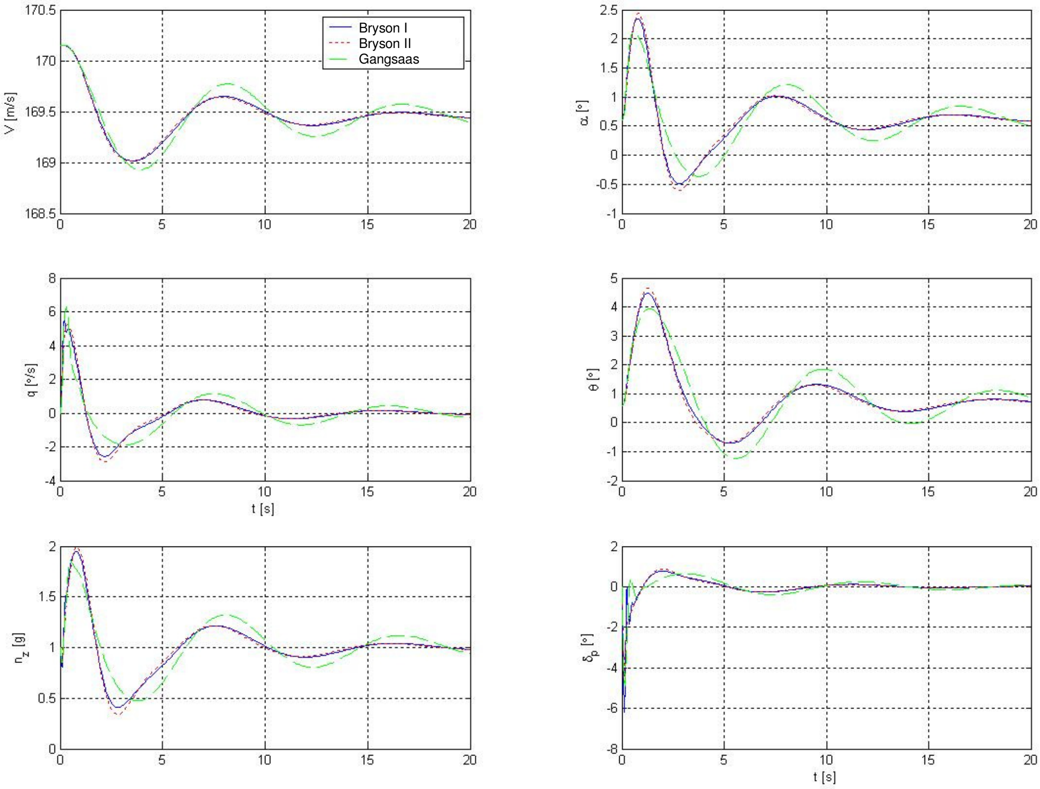
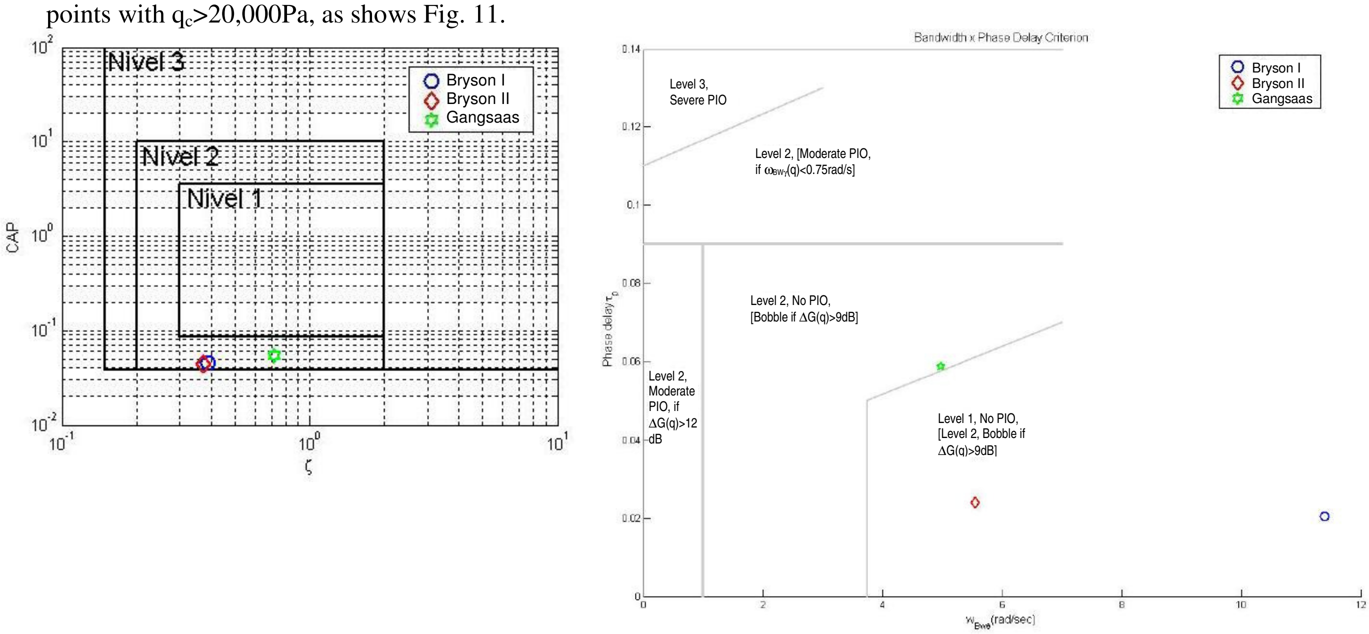


Figure 9. Temporal responses

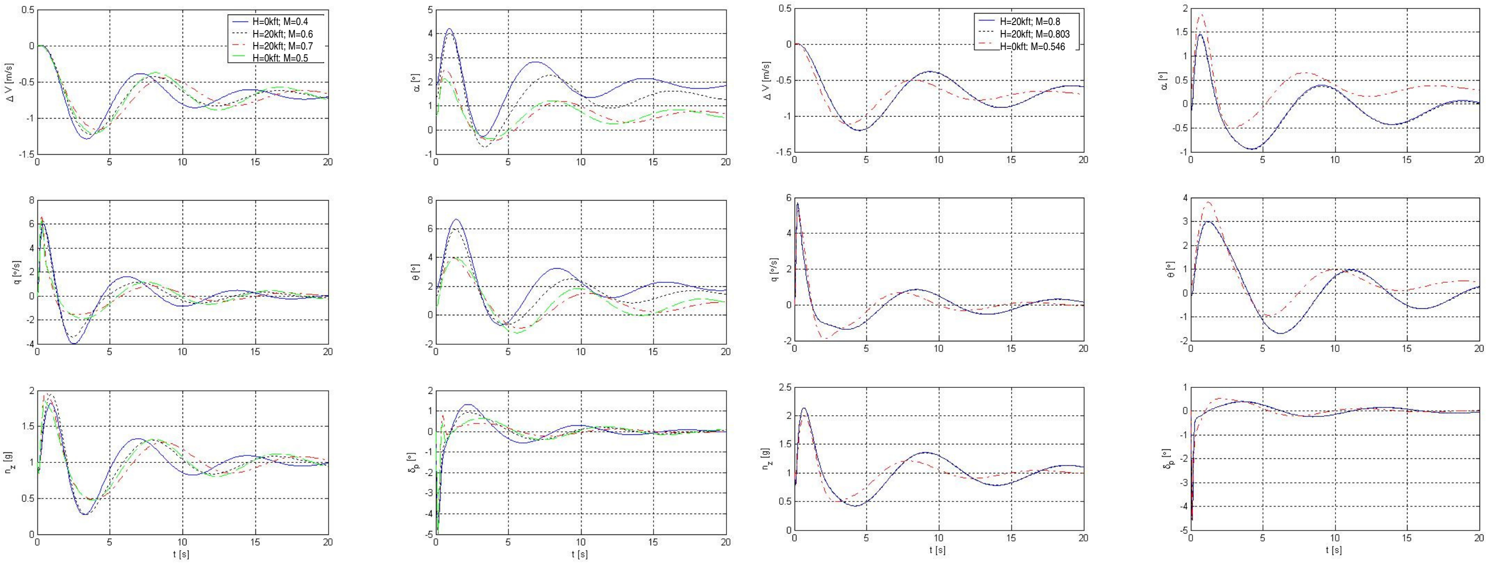
The best responses were computed by the Gangsaas rule for points with q_c<20,000Pa and the Bryson II rule for points with q_c>20,000Pa, as shows Fig. 11.



(a) Short-Period specification

(b) Bandwidth criterion

Figure 10. Handling quality levels



(a) Gangsaas: 11,348Pa < q_c < 17,732Pa (b) Bryson II: 20,861Pa < q_c < 21,145Pa

Figure 11. Temporal responses of others operations points

5. CONCLUSION

This work presented a study of longitudinal aircraft control based on C*u concept. The controller was applied to several operating points, which were classified into four different ranges of dynamic pressure with specific gains. The operating points of each range presented a similar closed-loop response, independent of their CG, weight, altitude and speed. Although the resulting system responses are stable, they do not present adequate handling qualities for all the operating points. As the next steps, each range shall be worked independently for a performance improvement.

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