NONLINEAR DYNAMIC INVERSION APPLIED TO A F-16 AIRCRAFT MODEL

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Abstract. *The main purpose of this work is to introduce and apply the dynamic inversion to a nonlinear aircraft model. The equations of motion are developed and briefly presented. A nonlinear model of a F16 aircraft is developed and simulated. The concept of dynamic inversion is demonstrated and discussions about its application are made. Some desired dynamics are selected and applied to the aircraft model using dynamic inversion. The system behavior is discussed and considerations about robustness and handling qualities are given.*

Keywords*: Dynamic Inversion; Flight Control System*

1. INTRODUCTION

The dynamic inversion aims to replace the system inheriting dynamic by the one selected by the control designer (Georgie and Valasek, 2003). It is a nonlinear control technique that has a long trend of success in aeronautical application, especially in flight control systems (Yeh *et al*, 1995) where has been applied by Honeywell and others (Smit and Craig 1998). Flight control systems are yet one of the most challenging subjects in the automated control area. The complexity of such systems comes from the nonlinearities and uncertainties involved in the aircraft dynamics. The conventional solution of this problem is to establish a certain number of equilibrium points and then design a gain scheduling with respect of the flight condition (Adams and Banda, 1993). This procedure is expensive and uses a considerable processor time, but is well known and has great acceptance in the aeronautical industry.

An alternative method is to use a control law that takes into account the nonlinearities of the system. One of these alternatives is the dynamic inversion. This technique is suitable for a great number of operational conditions, including high angle of attack and supersonic flights (Stevens and Lewis, 2003).

The dynamic inversion is based on the algebraic manipulation of aircraft's equations of motion, resulting in a response within the aircraft's power restriction (Steer, 2001). Although its potentiality and applicability, difficulties arise applying this method due to aerodynamics parameters that can not be totally know (Schumacher, 1999). Further more, when the system operates near a singularity, the control becomes difficult and demands a high fidelity model to be used aboard to make the dynamic inversion practical.

2. AIRCRAFT DETAILS AND MATHEMATHICAL EQUATIONS

The F-16 fighter was developed by Lockheed Martin and more than 4.000 units have been built since 1976. Figure 1 shows the 3 view diagram of the aircraft.

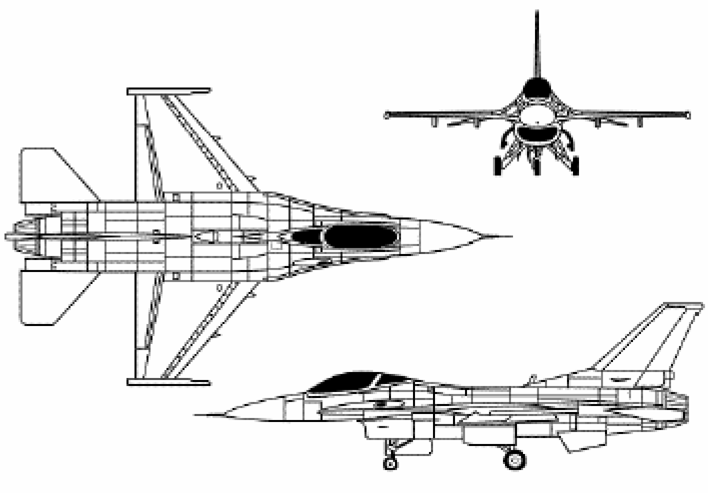

Figure 1. Lockheed Martin F-16 3-view diagram

Data about aircraft parameters are well known and detailed in Garza and Morelli (2003). The main parameters used in the aircraft nonlinear model are showed in the Tab. 1.

Table 1. F-16 main parameters

Mass(kg)	9298.8
Ixx (kg.m2)	12874.8
Iyy(kg.m2)	75673.6
Izz(kg.m2)	85552.1
Ixz(kg.m2)	1331.4
Izy(kg.m2)	0
Ixy(kg.m2)	0
Sref(m2)	27.87
Bref(m)	9.144
Cref(m)	3.45

Where I_{xx},I_{yy},I_{zz} are the moment of inertial, I_{xz}, I_{zy} I_{xy} are the products of inertia.

2.1. Equations of motion

Applying the Newton's second law it is possible to deduce the basic relations that describe the aircraft's equation of motion (Roskam, 2001).

The forces that act in the aircraft can be expressed in the body reference frame as showed in the Eq. (1).

$$\begin{aligned} x_b &: m(\dot{u} - vr + wq) = mg\sin(\theta) + F_{Ax} + F_{Tx} \\ y_b &: m(\dot{u} - vr + wq) = mg\sin(\phi)\cos(\theta) + F_{Ay} + F_{Ty} \\ z_b &: m(\dot{u} - vr + wq) = mg\cos(\theta)\sin(\theta) + F_{Az} + Fz \end{aligned} \tag{1}$$

where m is the aircraft's mass, u,v,w are the aircraft's linear velocities, p,q r are the aircraft's angular velocities, θ and Φ the Euler's angle around inertial Y and X axes, respectively. F_{Ax}, F_{Ay} and F_{Az} are the aerodynamic forces and F_{Tx}, F_{Ty} and F_{Tz} are the propulsive forces.

The moment equations in the body's reference frame are showed in the Eq. (2).

$$\begin{aligned} x_b &: I_{xx}\dot{p} - I_{xz}\dot{r} - I_{xz}pq + (I_{zz} - I_{yy})rq = L_A + L_T \\ y_b &: I_{yy}\dot{q} + (I_{xx} - I_{zz})pr + I_{xz}(p^2 - r^2) = M_A + M_T \\ z_b &: I_{zz}\dot{r} - I_{xz}\dot{p} + (I_{xz} - I_{xz})pq + I_{xz}qr = N_A + N_T \end{aligned} \tag{2}$$

The cinematic equations in the body's reference frame are showed in the Eq. (3).

$$\begin{aligned} x_b &: p = \dot{\phi} - \dot{\psi}\sin(\theta) \\ y_b &: q = \dot{\theta}\cos(\phi) + \dot{\psi}\cos(\theta)\sin(\phi) \\ z_b &: r = \dot{\psi}\cos(\theta)\cos(\phi) - \dot{\theta}\sin(\phi) \end{aligned} \tag{3}$$

where ψ is the Euler's angle around inertial Z axe.

2.2. Modeling a system

A nonlinear system can be modeled by the differential equation in Eq. (4).

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y(t) &= h(x) \end{aligned} \tag{4}$$

Where the state vector $x \in \Re^n$ and the control vector $u \in \Re^m$. *The n* is the number of states and *m* the number of controls.

Isolating the control vector, we have:

$$u = g(x)^{-1}[\dot{x} - f(x)] \tag{5}$$

This equation is valid if *g(x)* is invertible for all values of *x*.

Differently of the conventional control systems, instead of defining the desired state to achieve, the controller demands that the velocity of the controllable variable must be specified (Ito *et al.*, 2002).

Changing $\dot{x}$ by $\dot{x}_{des}$ in the Eq. (5), we have the dynamic inversion controller law.

$$u = g(x)^{-1}[\dot{x}_{des} - f(x)] \tag{6}$$

Equation (6) is valid if *g(x)* was invertible for all *x* values, which sometimes is not true. Another consideration about *g(x)* is that for small values, the nonlinear controller tends to saturate the controller of the system.

2.2. Specifying a desired dynamic

It is very important to choose the dynamic that will satisfy the performance and robustness specifications. The desired dynamic must be achieved considering the surface's deflections constraints. If the dynamics demand more forces and moments that the surfaces can create, the actuators will saturate.

The close loop response can be found according to Fig. (2). The integrator is used to approximate the system dynamics (Ito *et al.*, 2002).

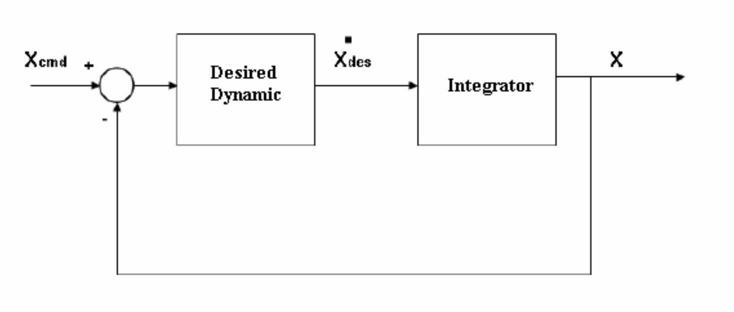

Figure 2. Closed Loop Dynamic

The dynamic inversion requires acceleration terms to make the calculation, like the pitch angular acceleration $\dot{q}$. The desired dynamic block maps the velocity and acceleration terms and in the developed project, it was used a PI controller to get the desired dynamic.

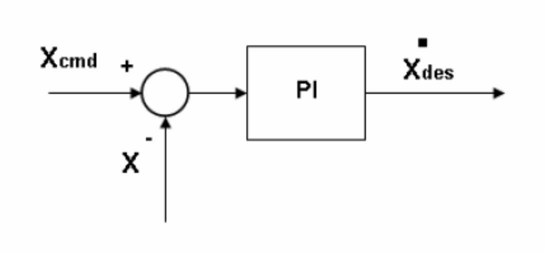

Figure 3. Chosen Controller

3. DEVELOPED PROJECT

In order to verify the dynamic inversion technique it was used a nonlinear F-16 model. First, the aircraft was put in equilibrium point at 7000 meters of altitude with 200 m/s velocity. The input commands are the angular velocities p_{cmd}, q_{cmd} and r_{cmd} commanded by the pilot. These input variables pass a filter designed to achieve some handle quality parameter. After the filter, we have the desired angular velocities p_{des} ,q_{des} and r_{des}.

The filter output signal is subtracted from the aircraft output angular velocities *p*, *q* e *r*.

This error is sent to the PI controller and after to the nonlinear controller. The nonlinear controller is responsible to compute the surface deflections required to obtain the angular velocities desired

Finally, the required deflections are sent to the F-16 model to evaluate the flight dynamics.

Figure 4 shows the developed block diagram.

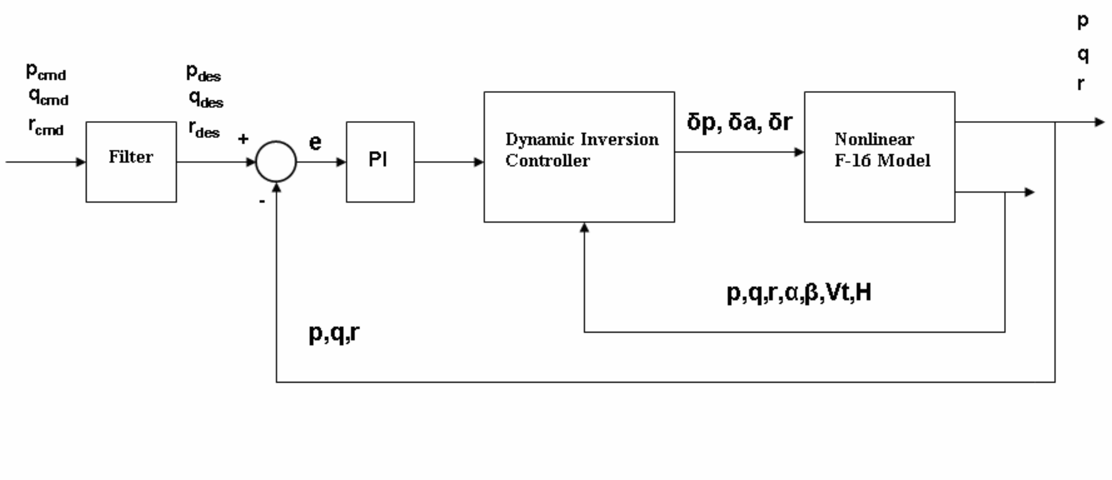

Figure 4. Develop Block Diagram

3.1. The inner loop

The dynamic inversion controller receives feedback from the output variables required to compute the moments coefficient C_l, C_m e C_n. The required variables are the instantaneous angular velocities *p*, *q* e *r*, the angle of attack α and sideslip angleβ, the flight altitude *H* and the total aircraft velocity *Vt*.

Based on that, it is possible to calculate the required moment coefficient and the deflections to perform the maneuver.

The angular accelerations used in the nonlinear controller are showed in the Eq. (8).

$$\begin{aligned} \dot{p} &= (c_1 r + c_2 p)q + c_3 L + c_4 N \\ \dot{q} &= c_5 pr - c_6 (p^2 - r^2) + c_7 M \\ \dot{r} &= (c_8 p + c_2 r)q + c_4 L + c_9 N \end{aligned} \tag{8}$$

Where:

$$c_1 = \frac{(I_y - I_z)I_z - I^2{}_{xz}}{I_y I_y - I^2{}_{xz}} \qquad c_5 = \frac{I_z - I_x}{I_y I_y - I^2{}_{xz}}$$

$$c_2 = \frac{(I_x - I_y + I_z)I_{xz}}{I_y I_y - I^2{}_{xz}} \qquad c_6 = \frac{I_{xz}}{I_y}$$

$$c_3 = \frac{I_z}{I_y I_y - I^2{}_{xz}} \qquad c_7 = \frac{1}{I_y} \tag{9}$$

$$c_4 = \frac{I_{xz}}{I_y I_y - I^2{}_{xz}} \qquad c_8 = \frac{I_x(I_x - I_y) + I^2{}_{xz}}{I_y I_y - I^2{}_{xz}}$$

$$c_9 = \frac{I_x}{I_y I_y - I^2{}_{xz}}$$

The moments are calculated based on the coefficients C_m, C_l and C_n according to Eq. (10).

$$
\begin{aligned}
M &= \overline{q}S.bb.C_n \\
N &= \overline{q}S.cb.C_m \\
L &= \overline{q}S.bb.C_l
\end{aligned}
\tag{10}
$$

where $\overline{q}$ is the dynamic pressure, S the reference area, cb is the mean aerodynamic chord and bb is the wing span of the aircraft.

Using Eq. (10) and Eq. (8). and isolating the moments coefficients:

$$
\begin{aligned}
c_m &= \frac{\dot{q} - c_5 pr + c_6(p^2 - r^2)}{c_7(\overline{q}Scb)} \\
c_l &= \frac{-(rc_4 - qc_8 pc_4 + qc_2 rc_4 - c_9 \dot{p} + c_9 qc_1 r + c_9 qc_2 p)(-c^2{}_4 + c_3 c_9)}{\overline{q}Sbb} \\
c_n &= \frac{(-\dot{r}c_4 - qc_8 pc_4 + qc_2 rc_4 - c_9 \dot{p} + c_9 qc_1 r + c_9 qc_2 p(-c^2{}_4 + c_3 c_9)}{\overline{q}Sbb}
\end{aligned}
\tag{11}
$$

The angular accelerations $\dot{p}$, $\dot{q}$ e $\dot{r}$ are the desired dynamics. The dynamic inversion controller has to calculate the value of C_m, C_l, C_n required to achieve these desired accelerations.

As can be seen in Eq. (11) the L and N values must be calculate simultaneously to obtain $\dot{p}$ and $\dot{r}$, which means that these variables are coupled.

After calculating the required C_{mReq}, C_{lReq} and C_{nReq} it is possible to calculate the surface's deflections. It was used the Matlab's function *solve* in order to manipulate symbolically the equations involved.

3.2. The outer loop

To discovery the PI gains that could achieve the best simulation response, it was used the Matlab's function *fminserch*.

The routine tries to minimize the area between the desired response and the actual response, as showed in Fig. (6).

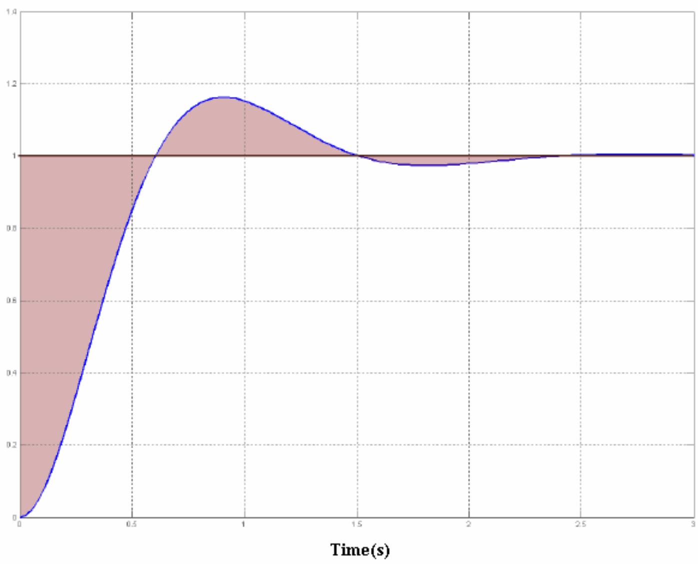

Figure 6. Area that the routine tries to minimize

The optimal values found to the PI controller are indicated in the Tab. 2.

Table 2. PI controller gains

	p	q	r
P	46.5	48.8	51.4
I	6.8	6.46	6.2

3.3. Simulations and results

In this section the results of the nonlinear dynamic inversion controller simulation are showed.

In the following figures, the blue angular velocities represent the pilot commands, the red lines indicate the filtered commands and the black lines display the actual angular velocities obtained from the aircraft simulation.

Figure 7 shows the developed block diagram. The filter used in the simulation was a first order filter with time constant equal to 1 second.

The main idea of the filter is to easily adjust the desired dynamic.

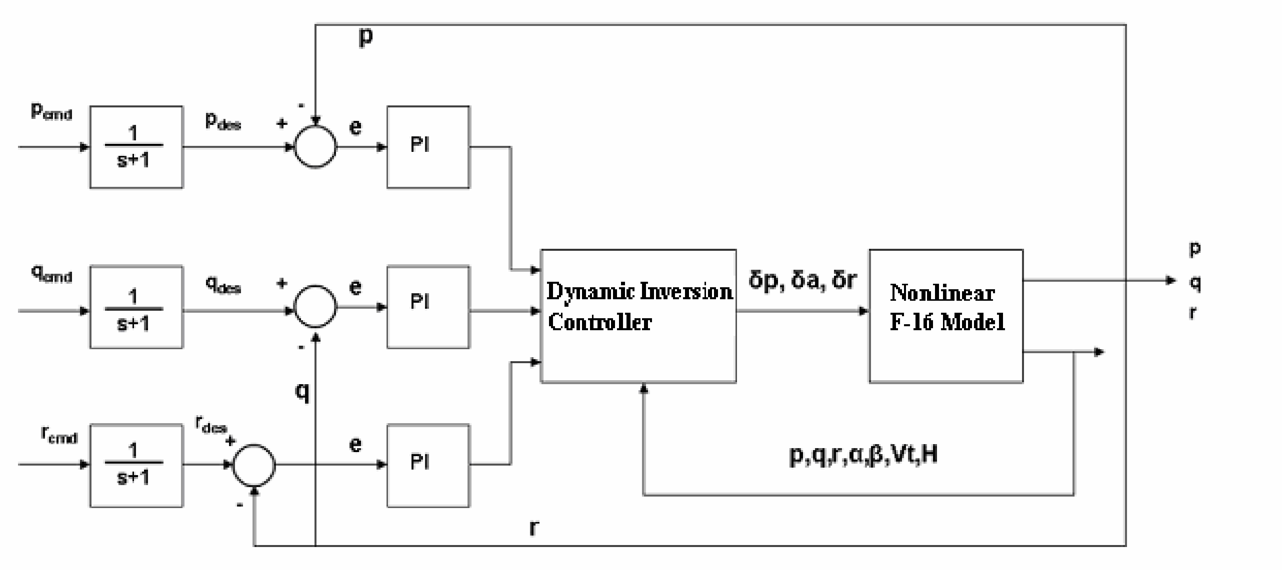

Figure 7. Develop Block Diagram with first order filter selected

Figure 8 shows the commanded angular velocities. At 20 seconds, the pilot commands a 1 degree per second yaw command, inverting this command in 30 seconds and neutralizing it at 40 seconds.

After that, at 50 seconds, .a 1 degree per second roll command is performed. At 60 seconds, the roll command is inverted and neutralized at 70 seconds.

Finally, in 80 seconds a pitch command of 1 degree per second is issued, inverted in 90 seconds and neutralized at 100 seconds

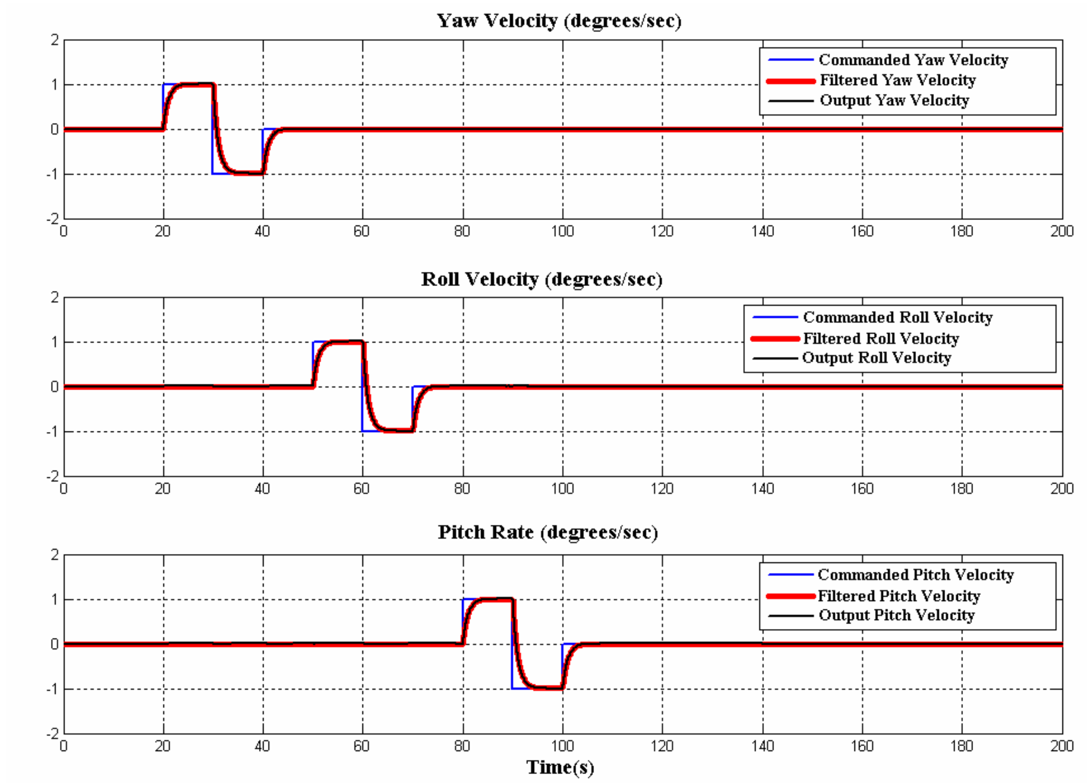

Figure 8: Angular velocities – First order dynamic selected

Figure 9 shows the surface's movements in order to obtain the desired dynamics. As can be seen, the roll and yaw movements require that ailerons and rudder be commanded at the same time.

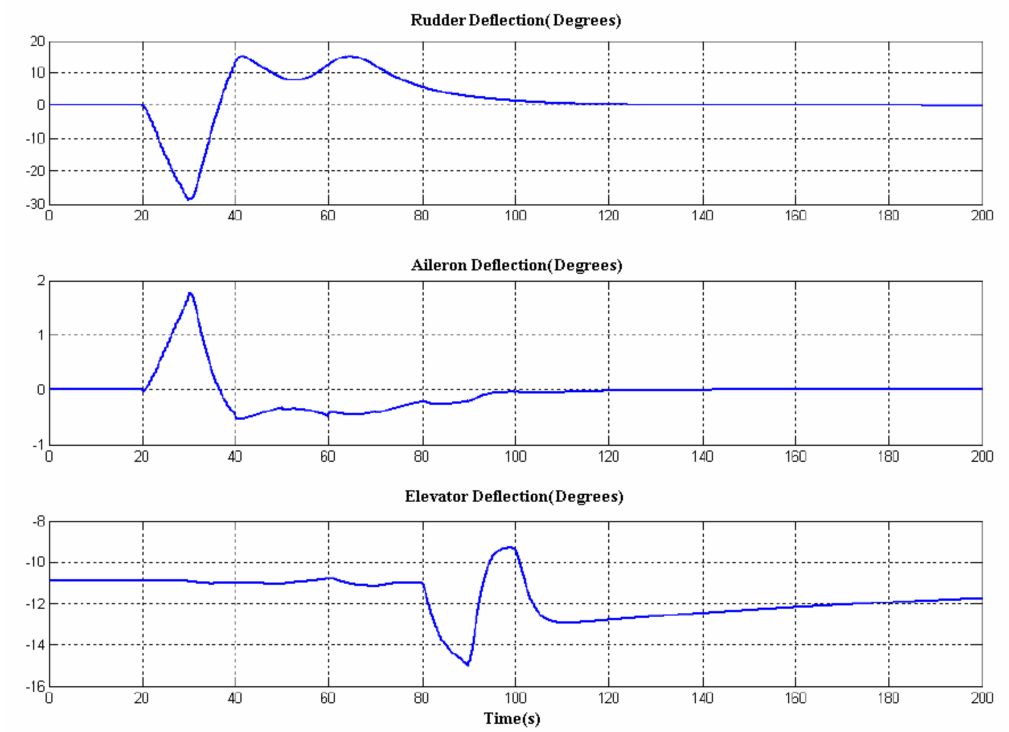

Figure 9. Surface movements to obtain the desired dynamic

Figure 10 shows that filter parameters were changed to perform the next simulation. It was chosen a second order filter to obtain the desired dynamic.

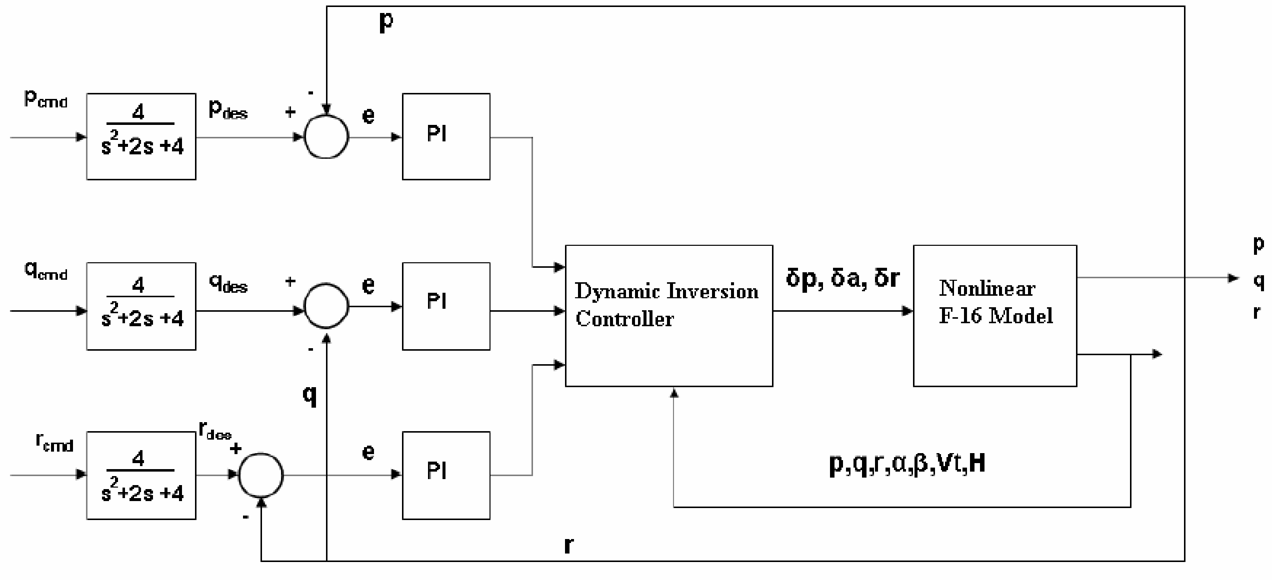

Figure 10. Develop Block Diagram with second order filter selected

In this simulation, the pilot only commands the longitudinal input. First, the pilot commands a 1 degree per second pitch rate at 10 seconds. After that, at 20 seconds, the command is inverted and neutralized at 30 seconds. Now the desired dynamic is a second order dynamic showed in Fig. 11. The dynamic inversion controller is the same used in the previous simulation.

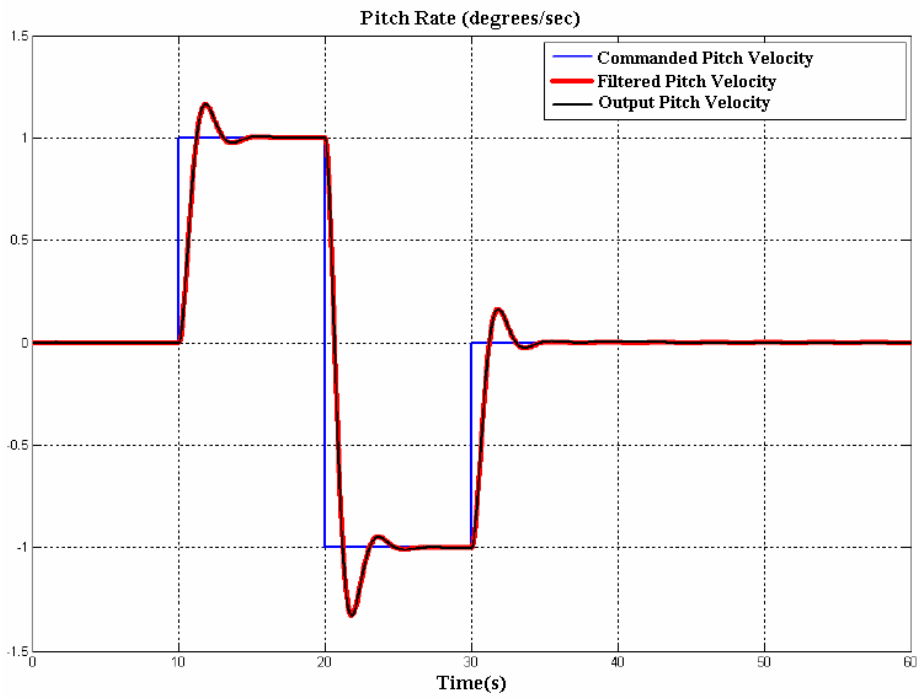

Figure 11. Pitch Rate – Second order dynamic selected

The elevator's movements necessary to obtain the second order dynamic are showed in Fig. 12.

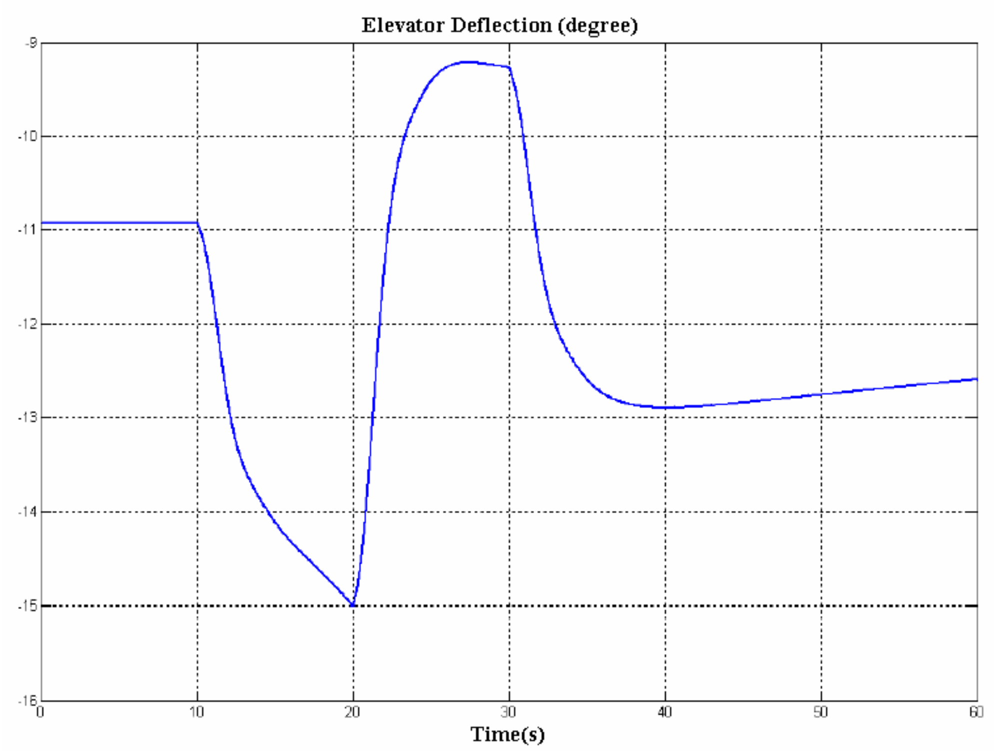

Figure 12. Elevator's movements to achieve second order dynamic

4. CONCLUSIONS

This work presented the powerful dynamic inversion technique. In the simulation figures it was possible to see that the nonlinear controller provided the desired dynamics with great accuracy in both cases, first and second order simulations. It was decided to place a filter between the pilot inputs and PIs controllers. This approach makes the choice of the aircraft's behavior very simple and easy to the designer, that only needs to change the filter parameters to achieve the required dynamics, keeping the inner loop control unchanged.

5. REFERENCES

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