LQG/LTR CONTROLLER FOR COMMERCIAL AIRCRAFT DURING APPROACH AND LANDING

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Abstract. *For a commercial aircraft pilot, approach and landing are the flight phases that demand more workload, restricted to a determined flight and limited freedom for maneuvers. The aircraft must be flown close to the lower boundary of its safe flight envelope and, as consequence, a considerable amount of accidents to civil aircraft occur during approach and landing. The ground effect is one of the factors that may influence these flight phases, which is the aerodynamic influence of the ground as a restraining surface on the flow around the aircraft. It generally increases the lift, reduces the drag and generates a nose down pitching moment. Another factor is the presence of wind shear, which is characterized as a difference in wind speed and direction over a relatively short distance in the atmosphere. The purpose of this paper is to discuss the pros and cons of using a Linear-Quadractic-Gausian with Loop-Transfer-Recovery (LQG/LTR) method as the longitudinal control law for a commercial aircraft during approach and landing. The choice of the LQG/LTR controller resides in the fact that the aircraft model has uncertainties and the chosen control technique shall be robust enough to suppress it. The controller shall also provide good handling qualities to reduce pilot workload. The aircraft handling qualities and the system robustness to atmospheric disturbances are evaluated with a simplified fly-by-wire system in different flight conditions.*

Keywords*:* Approach and landing, flight controls, commercial aircraft, LQG/LTR;

1. INTRODUCTION

Approach and landing are usually the most demanding flight phases for a pilot of commercial aircraft. The pilot must guide the aircraft through a restricted flight path with limited freedom for maneuvers and close to the lower boundary of its safe flight envelope and, as consequence, a considerable amount of accidents with civil aircraft has occured during approach and landing. The ground effect, which is the aerodynamic influence of the ground as a restraining surface on the flow around the aircraft, is one of factors that may influence these flight phases. It generally increases the lift, reduces the drag (for a given lift coefficient) and generates a nose down pitching moment. The aircraft dynamic behavior changes with the altitude, weight, velocity, center of gravity, ground effect, landing gear and flaps configurations. These changes can be seen as modification in the aircraft eigenvalues location. Figures 1 and 2 present the modifications in the eigenvalues due to the aircraft altitude variation and ground effect, where it can be seem that the ground effect has a strong influence near the ground, destabilizing the phugoid eigenvalues (FERREIRA, 2008). The aircraft longitudinal control law must be robust to these changes in such a way that the aircraft must have the same behavior inside the chosen part of the aircraft flight envelope. More than that, the aircraft must have a satisfying handling quality so that the pilot can guide the aircraft easily and precisely during the approach and landing maneuvers.

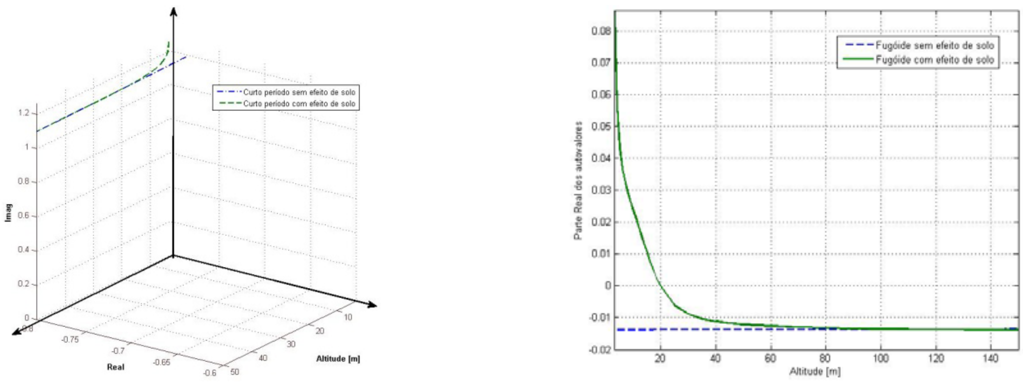

Figure 1 – Influence of ground effect at the phugoid mode

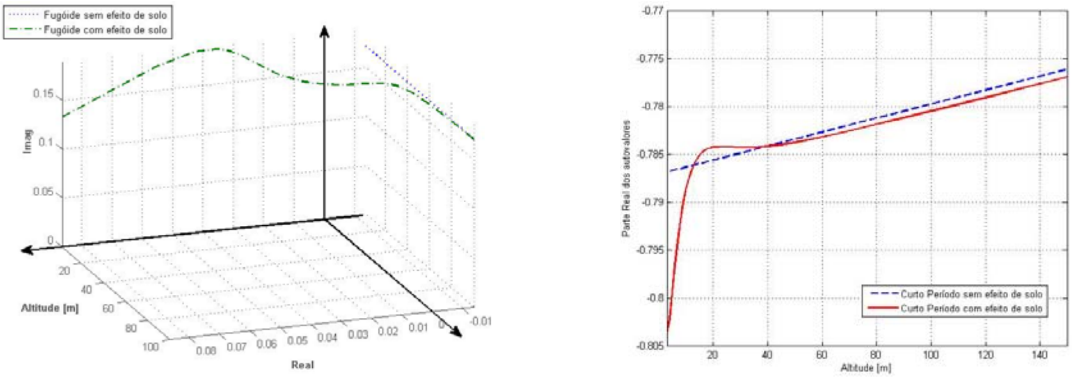

Figure 2 - Influence of ground effect at the short-period mode

These changes in the aircraft model can be interpreted as model uncertainties and the Linear-Quadractic-Gausian with Loop-Transfer-Recovery (LQG/LTR) method was chosen to suppress these uncertainties. More detailed explanation of the LQG/LTR method can be found in (CRUZ, 1996) and (ATHANS, 1986).

The control laws are part of a Fly-By-Wire system, which includes actuators, sensors and the computers with the embedded control. The Fly-by-Wire component implies in additional dynamics and lags in the system, which must be verified to guarantee that the simulated aircraft behavior is close to the real aircraft behavior with the designed control law.

2. DEVELOPMENT

2.1. Objectives

The objective of this work is to "design a longitudinal control augmentation system for the approach and landing phases, which provides precision for tracking and good handling qualities".

To obtain precision for tracking, the controller will be designed to control pitch rate. Pitch rate has shown to be well suited for tracking of a target and also for approach and landing (LEWIS & STEVENS, 2003). The controller must also include an integrator in its direct path, which will make the aircraft respond with a demand of pitch rate instead of pitch attitude. This may reduce pilot workload during approach but may cause a tendency of floating during flare maneuvers.

The handling qualities will be analyzed through the C* criteria. The C* criteria is a temporal envelope which relates the normalized C* response of the aircraft with that of an aircraft with well known handling qualities. The pilots, in general, have a good acceptance for aircrafts whose C* response is contained inside the C* criteria limits (SIQUEIRA et al, 2006). The definition of C* is given in Eq. 1 and more details about the C* criteria and C* flight control law can be found in (FIELD, 1993).

$$C^* = n_z + 12.4q \tag{1}$$

Initially, it was established that the LQG/LTR controller should guarantee the aircraft stability robustness from 100m above ground to the ground including the ground effect but, as it will be shown in the next section, it implied in a small bandwidth for the controlled aircraft resulting in a poor handling quality. The ground effect has a strong influence on the phugoid mode, which is largely unaffected by the feedback of pitch rate (LEWIS & STEVENS, 2003), for this reason it was removed from the set of possible perturbed plant models (Π) and the range of altitude was increased from 100m to 305m. Finally, the aircraft with the LQG/LTR controller response is checked with a simplified Fly-by-wire system.

2.2. Aircraft Model

The aircraft longitudinal model considering the ground effect was presented at (FERREIRA et al, 2008), where the effects on the lift and drag coefficients where modelled as suggested in (ROSKAM, 1997) and the effects on the pitching moment coefficient where modelled as presented in (PINSKER , 1967). It consists of an ultra long range aircraft of which parameters can be found at (FERREIRA et al, 2008). The aircraft linear model is given in Eq. 2 and its state and input vector in Eq. 3. The state variables are respectively the aircraft velocity (V), trajectory angle (γ), angle of attack (α), pitch rate (q) and altitude (H). The input variables are respectively elevator deflection (δp), horizontal stabilizer deflection (δHS) and the thrust lever angle (δπ). Only the elevator deflection can be used by the controller, once the thrust lever angle is set to idle during landing and it is not usual for the pilot to perform landing using the horizontal stabilizer. Moreover, the horizontal stabilizer could be used by an automatic pitch trim system to alleviate the load on the elevator.

$$\begin{aligned} \dot{X} &= AX + Bu \\ Y &= CX \end{aligned} \tag{2}$$

$$\begin{aligned} X &= \begin{bmatrix} V & \gamma & \alpha & q & H \end{bmatrix} \\ u &= \begin{bmatrix} \delta p & \delta HS & \delta\pi \end{bmatrix} \end{aligned} \tag{3}$$

Aircraft are plants with non-linear characteristics but the techniques used to design the LQG/LTR controller use linear models. From the non-linear model presented by (FERREIRA et al, 2008) a set of 67 linear models was extracted, each one with a different trim condition as described on Tab. 1.

Table 1 – Trim conditions of the linear models

Model Nro.	Altitude [m]	Tragectory (º)	Velocity [m/s]	Ground Effect
1-61	5-305	-3	69.77	No
62-64	5-15	0	69.77	No
65-67	5-15	0	69.77	Yes

The linear models parameters differ according to the aircraft speed, altitude, angle trajectory and ground effect. Equation 4 presents the linear model with its parameters variation

$$A = \begin{bmatrix} [-0.0456;-0.0388] & -9.8066 & [-6.7668;-4.6134] & [-0.5584;-0.3819] & [-0.0192;-0.0002] \\ 0.0040 & 0 & [0.7257;0.7932] & [0.0582;0.0641] & [-0.0013;0] \\ -0.0040 & 0 & [-0.7932;-0.7257] & [0.9359;0.9418] & [0;0.0013] \\ 0 & 0 & [-1.6038;-1.4809] & [-0.8262;-0.8187] & [0;0.0028] \\ 0 & 69.77 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} [-0.6282;-0.4296] & [-0.6063;-0.4146] & [4.0773;4.1152] \\ [0.0655;0.0721] & [0.0632;0.0696] & [0.0001;0.0009 \\ [-0.0721;-0.0655] & [-0.0696;-0.0632] & [-0.0009;-0.0001] \\ [-1.3281;-1.3160 & [-1.6561;-1.5292] & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

These parameters variations may be represented as multiplicative uncertainty (SKOGESTAD & POSTLEWAITE, 2005). Figure 3 presents a block diagram of the multiplicative uncertainty where G_p is the uncertainty model, G is the nominal plant, W_I is the weight of the uncertainties and Δ_I is any stable transfer function such that $\| \Delta_I \|_\infty \leq 1$

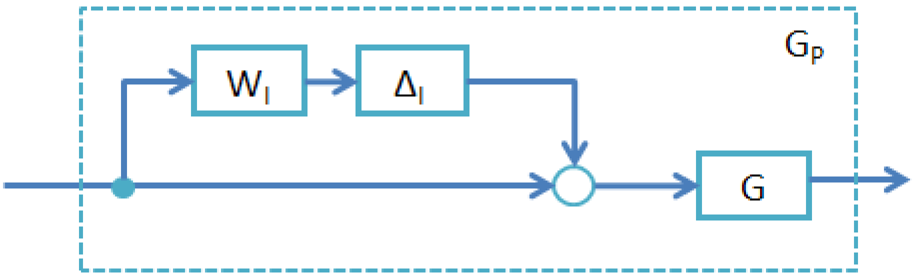

Figure 3 – Plant with multiplicative uncertainty

Table 2 presents the trim conditions of the nominal plant. The choice of these parameters for the nominal plants resides in the fact that the flare maneuvers usually starts at 15m above ground and this is one of the most critical moments of the landing. The longitudinal control law shall provide good handling qualities at this point due to its criticality. The nominal plant is presented in Eq. 5.

Table 2 – Trim Conditions of the nominal plant

Parameter	Value
Altitude	15m (without ground effect)
Tragectory Angle	-3 º
Velocity	69.77 m/s

$$G = \frac{q}{\delta p} = \frac{-1.327s^4 - 0.9296s^3 - 0.06217s^2 - 5.514e-005s + 3.266E-020}{s^5 + 1.6s^4 + 2.093s^3 + 0.1044s^2 + 0.06031s + 6.205E-5} \quad (5)$$

The weight function W_I is obtained through Eq. 6, where G_I belongsto the set of 67 possible perturbed plant models (Π). The singular values diagram of the relative errors is presented in Fig. 4. The three lines with highest singular values belong to linear models which included the ground effect during its linearization. As described in the last section, the ground effect modifies the phugoid eigenvalues into the direction of instability. This work intends to determine a control augmentation system (CAS) which usually improves the short period characteristics but has small effect at the phugoid. The phugoid mode can be improved by the feedback of other states such as velocity and trajectory angle (LEWIS & STEVENS, 2003), which is usually done by the auto-pilot system or by the pilot himself.

Initially, the weight function was designed to involve all the relative errors however this implied in strong limitations of bandwidth for the controlled aircraft. For this reason, the objectives were revised and the LQG/LTR controller should be robust to variations due to velocity, trajectory and altitude, removing the ground effect of the scope. This new set of possible perturbed plant models is composed of the first 64 models of Tab.1.

$$|W_i(jw)| \geq \max \left| \frac{G_i(jw) - G(jw)}{G(jw)} \right| \quad (6)$$

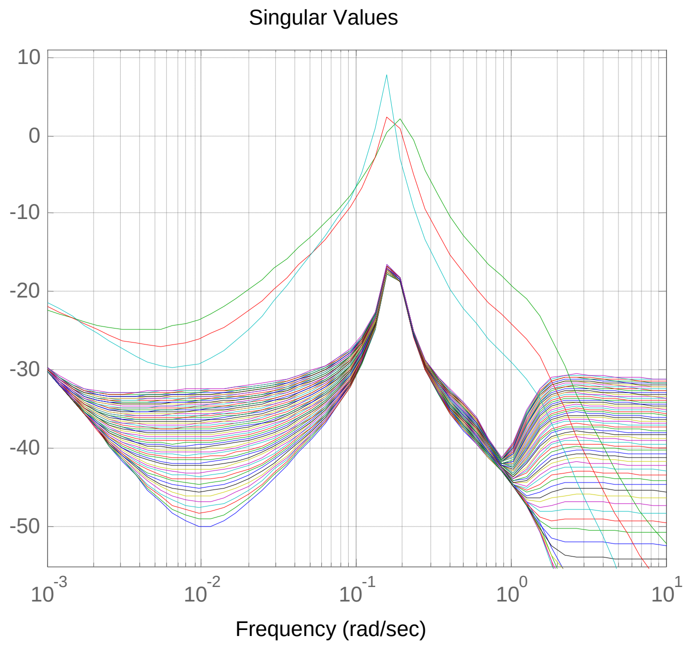

Figure 4 – Diagram of singular values of the set of 67 perturbed plant models

The weight function with the revised requirements is presented in Eq. 7, and its singular values are presented on Fig. 7 together with the revised set of plant models.

$$W_i = \frac{0.05556s^4 + 0.08904s^3 + 0.03926s^2 + 0.002867s + 5.76e-005}{2s^4 + 1.301s^3 + 0.3076s^2 + 0.03123s + 0.001152} \tag{7}$$

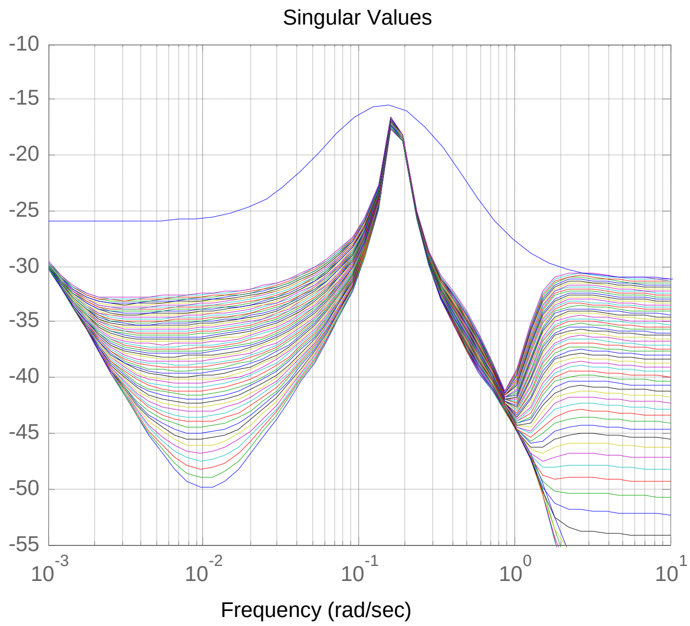

Figure 5 - Weight of the uncertainties function (W_I)

2.3. Project of the LQG/LTR controller

The nominal plant model, with the dynamics G(S), is in series with the controller K(S) as it can be seem on Fig. 6. The K(S) controller is chosen linear and time invariant, with just one input and one output. From now on, the nominal aircraft dynamics G(S) will include and additional integrator due to the requirements described on section 2.1.

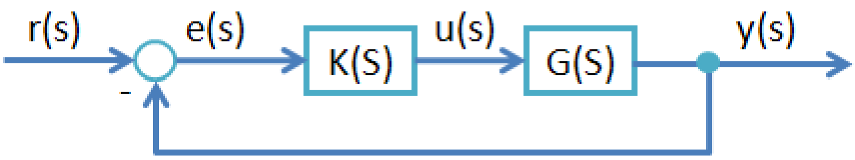

Figure 6 – Control System

The design of the LQG/LTR controller is composed of two steps: first, a target feedback loop where all the specifications are achieved is defined, and in the second step, the controller K(s) is determined which will make the system have the same behavior as the designed target dynamics (ATHANS, 1986).

2.3.1 Target Feedback Loop (TFL)

The dynamic structure is presented in Fig. 7, defined by the parameters $\Phi=(sI-A)^{-1}$ and C of the nominal plant and by the filter gain matrix H.

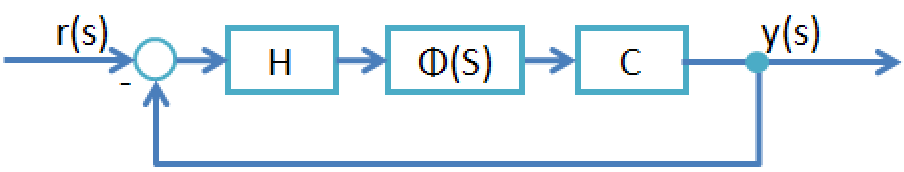

Figure 7 – Desired dynamic structure

As G_{KF} is the open loop transfer function of the desired dynamics (Eq. 8), for stability robustness is it necessary that the condition presented in Eq. 9 is satisfied (ATHANS, 1986). As one of the requirements states that the aircraft shall respond with pitch rate demands and not with pitch angle, the condition presented in Eq. 10 shall also be achieved. It is important to state that those requirements will be achieved only in the interest frequency band.

$$G_{KF}(S) = C\Phi(S)H \tag{8}$$

$$\bar{\sigma}(G_{KF})_i =< \frac{1}{e_m(\omega)} = \frac{1}{W_i} \tag{9}$$

$$\underline{\sigma}(G_{KF}) \geq \frac{1}{j\omega} \tag{10}$$

The parameter H is obtained through the solution (Σ) of the Riccati equation presented on Eq. 11 and 12. The parameters μ e L were obtained empirically, where $\mu=1\times10^{-4}$ and L is given by Eq. 13.

$$H = \frac{1}{\mu}\Sigma C^T \tag{11}$$

$$A\Sigma + \Sigma A^T + LL^T - \frac{1}{\mu}\Sigma C^T C\Sigma = 0 \tag{12}$$

$$L = \mathrm{diag}([0 \quad 0.32 \quad 0.32 \quad 0.352 \quad 0.32 \quad 0])\mathrm{B} \tag{13}$$

Figure 8 presents the singular values diagram of the desired dynamics with the lower boundary for tracking the reference (1/s) and the stability robustness boundary ($1/W_I$). There are conflicts among the requirements, being impossible to achieve all the requirements in all frequencies. The robustness barrier is chosen as a priority and both requirements are achieved in a frequency band of typical pilot inputs (up to 3rad/s).

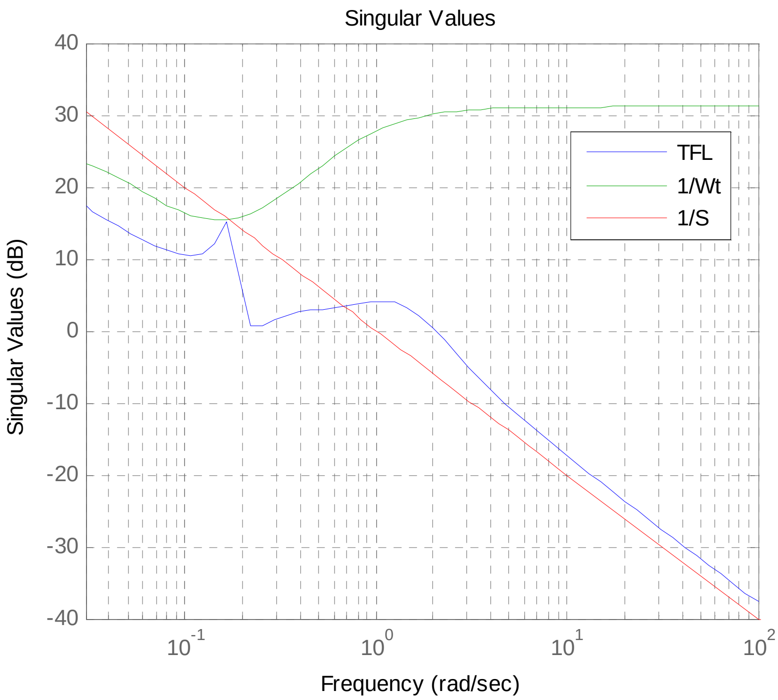

Figure 8 – Project of the Target Feedback Loop (TFL)

2.3.2 Loop Transfer recovery (LTR)

The LQG/LTR controller is based on the desired target feedback dynamics (model-based compensator – MBC). Figure 9 shows the controller structure, where it can be seen that it has a replica of the plant dynamics with two internal feedback loops, one with the filter gain matrix H and another with the gain matrix G, which will be responsible for the recovery of the desired dynamics.

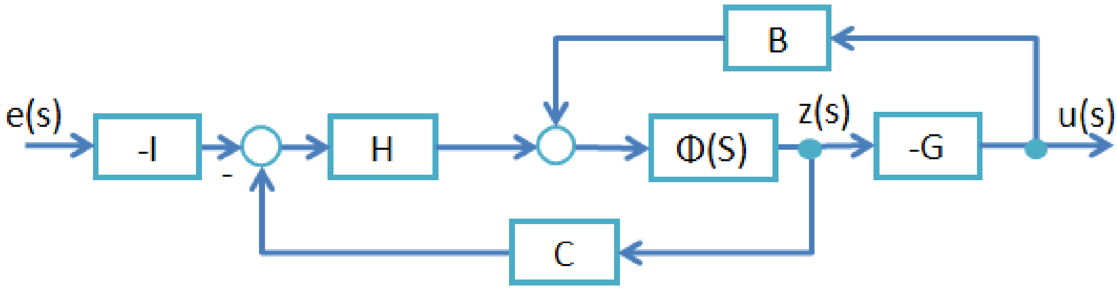

Figure 9 – Structure of the LQG/LTR controller based on the desired dynamics

Equation 14 shows the state space representation of the LQG/LTR controller. The filer gain matrix was already defined and the gain matrix G is obtained through the solution of the Riccati equation of Eq. 16 and Eq. 15. As the parameter ρ tends to zero, the dynamic of the controlled aircraft gets closer to the desired dynamics.

$$\dot{z}(t) = (A - BG - HC)z(t) - He(t)$$
$$u(t) = -Gz(t) \tag{14}$$

$$G_\rho = \frac{1}{\rho} B^T K_\rho \tag{15}$$

$$A^T K_\rho + K_\rho A + C^T C - \frac{1}{\rho} K_\rho B B^T K_\rho = 0 \tag{16}$$

Equation 17 shows the obtained K(s) controller. Figure 10 shows the singular values diagram of the resulted loop transfer recovery (LTR) with the target feedback loop (TFL) and the project boundaries, where it can be seen that up to 11rad/s the aircraft with the LQG/LTR has the desired dynamics.

$$K = \frac{-1338s^5 - 2861s^4 - 518.6s^3 - 138.5s^2 - 12.87s - 0.01311}{s^7 + 53.5s^6 + 1432s^5 + 979.8s^4 + 65.37s^3 + 0.05799s^2 + 1.475E-8s} \tag{17}$$

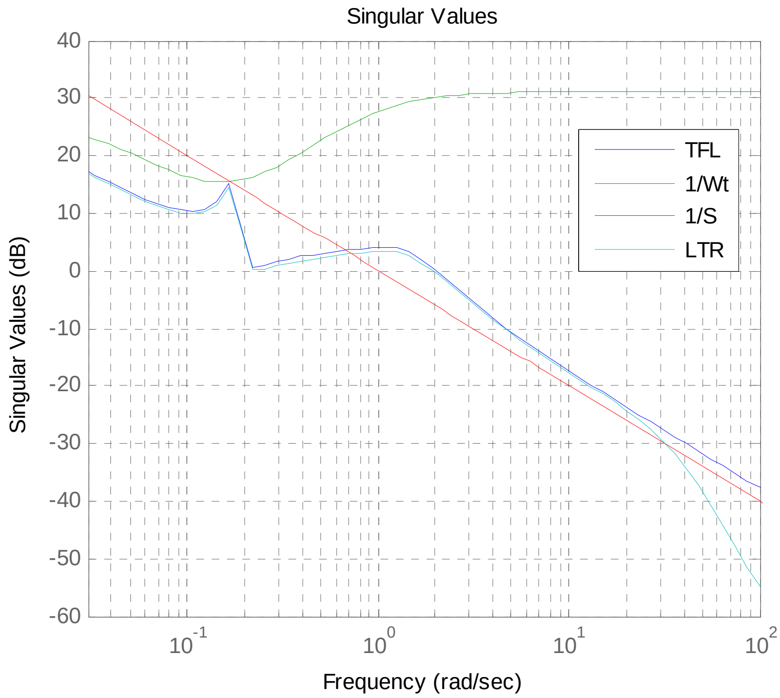

Figure 10 – Loop Transfer Recovery

3.RESULTS

The aircraft with the LQG/LTR controller response was analyzed with a simplified model of FBW system shown on Fig. 11. The elevator actuator transfer function is presented on Eq. 18 and the simplified FBW parameters are presented on Tab. 3.

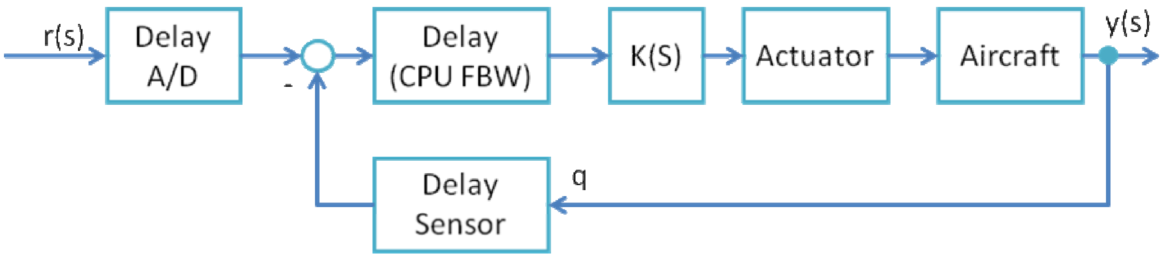

Figure 11 – Simplified FBW system

Table 3 – Simplified FBW system parameters

Component	Delay
A/D conversion	10ms
Processing (CPU GBW)	40ms
Sensor (AHRS)	50ms

$$G_{ACTUATOR} = \frac{1176}{s^2 + 48.02s + 1176} \quad (18)$$

The first analysis is the evaluation of the augmented aircraft flying qualities through the C* criteria. Figure 12 shows the aircraft response for the C* criteria. On the left, it can be seen that the aircraft response without any controller is very poor but with the LQG/LTR controller all the considered 64 flight conditions passed trough this criteria. There is a point that the response touches the non-critical-operations barrier but this is not an issue, due to the fact that it is considering a commercial aircraft and not high performance aircraft such as a fighter.

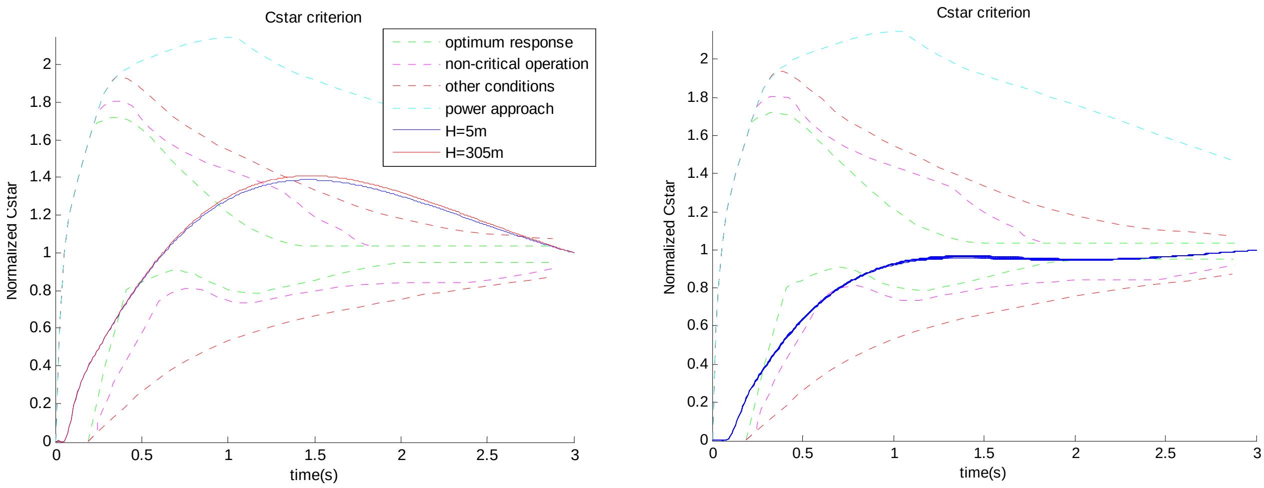

Figure 12 – C* Criteria responce (left – without the LQG/LTR controller / right – with the LQG/LTR controller)

The aircraft response with the LQG/LTR controller is presented on Figures 13 and 14 for two extremities of the considered flight envelope, where it can be seen that the only difference is due to the different trim conditions of the linear models, but in the general it is almost the same response. The pitch rate increases continuously whilst there is an elevator input, which is exactly the desired response as described on section 2.1.

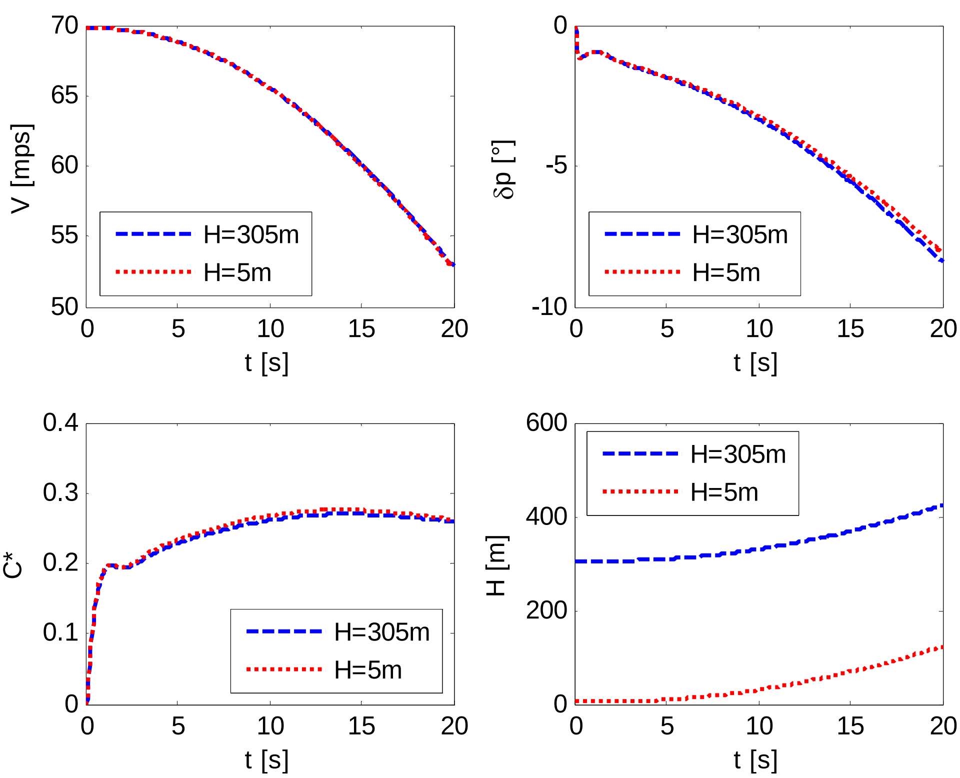

Figure 13 – Aicraft step response with the LQG/LTR controller (a)

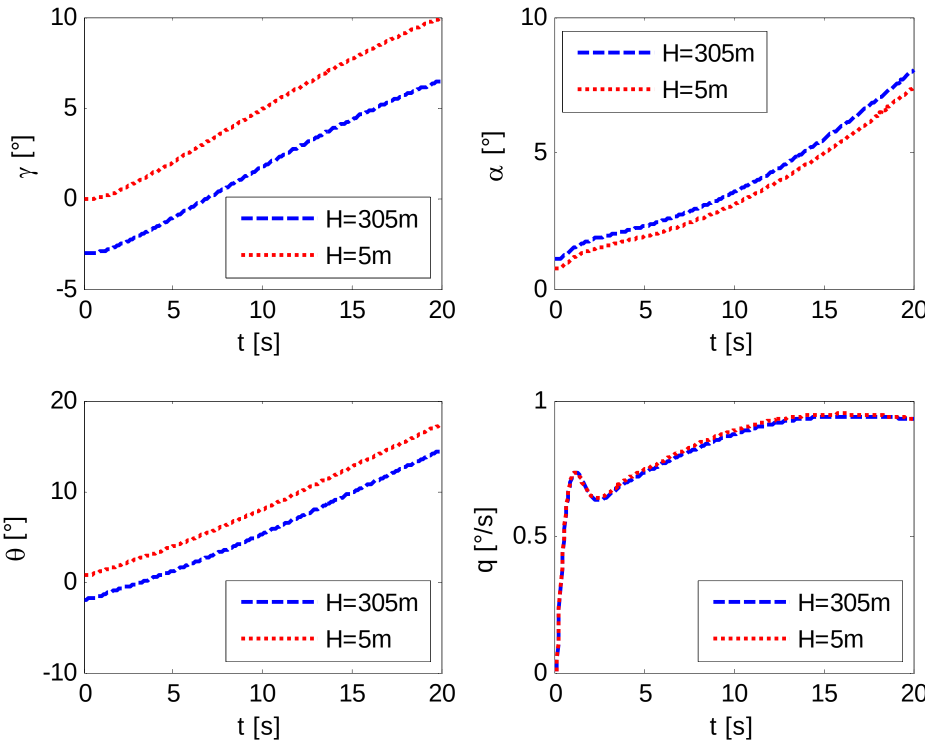

Figure 14 - Aicraft response with the LQG/LTR controller (b)

4.COMMENTS AND RECOMMENDATIONS

The LQG/LTR method offers a simple and straightforward procedure for the controller design. The filter gain matrix (H) demands some iteration to obtain the desired target feedback dynamics, but as the designer acquires more knowledge about the influence of the matrix L parameters and the parameter μ, it is possible to tailor the target feedback dynamics as desired. The aircraft response with the LQG/LTR controller was considered satisfactory and another good point is that it has almost the same behavior along with the chosen flight envelope. Even though the ground effect was removed from the set of perturbed plants, its effects can be mitigated by the auto-pilot system or by the pilot himself, as it normally happens with non-augmented aircraft.

The LQG/LTR controller could also be used with good advantages to provide robustness against components uncertainties such as command asymmetries and friction at flight control system cables. Classical control techniques could be used with a gain schedule to provide robustness against changes in the aircraft dynamics due to different flight conditions but it is not so simple when the designer is dealing with changes in the components.

The use of high order controllers is not common among aircraft manufacturers, but it could bring some benefits, for example, the controller could be robust enough against some failure condition eliminating the need for some sensors.

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