COMPARISON OF EXTENDED KALMAN FILTER AND NON LINEAR SIGMA POINT FILTER FOR ORBIT DETERMINATION USING GPS MEASUREMENTS

Paula Cristiane Pinto Mesquita Pardal, paulacristiane@gmail.com
Helio Koiti Kuga, hkk@dem.inpe.br
INPE (National Institute for Space Research) – DMC

Rodolpho Vilhena de Moraes, rodolpho@feg.unesp.br
UNESP (State of São Paulo University) – FEG – DMA

Abstract. *Herein, the purpose is to present how a non linear extended Kalman filter and a Kalman filter based on the sigma point unscented transformation, aiming at real time satellite orbit determination using GPS measurements, were developed. If the dynamic system and the observation model are linear, the conventional Kalman filter may be used as the estimation algorithm. However, not rarely, the dynamic systems and the measurements equations are of non linear nature. For solving such a problem, convenient extensions of the Kalman filter have been sought. In particular, the unscented transformation was developed as a method to propagate mean and covariance information through non linear transformations. The Sigma Point Kalman filter (SPKF) appears as an emerging estimation algorithm applied to non linear system, without linearization steps. The SPKF will be compared with the extended Kalman filter (EKF) in three queries: accuracy, implementation, and order of complexity. Data from Global Positioning System (GPS) receivers will be used to determine the orbit of an artificial satellite using both the EKF and the SPKF as estimation techniques. The EKF implementation in orbit estimation, under inaccurate initial conditions and scattered measurements, can lead to unstable solutions. The determinist and the stochastic aspects must be taken into account, in order to improve the orbit determination processes and, at the same time, minimize the computational procedure cost. The first aspect encloses the system modeling and the problem formulation in order to adjust the system equations adopted to represent the orbit dynamic. In the second, the statistic estimation theory must be managed to make feasible its utilization. The orbit determination, in particular, presents a big additional inconvenience due to the non linear characteristics of both the dynamics of the motion and the GPS measurements model. In this work, analysis of a different technique to estimate the state vector that characterizes a satellite orbit model, found in non-linear Sigma Point filter, will be shown and compared to the EKF. The formulation of orbital motion differential equations and the GPS measurements equations will be placed in a suitable form. The development of the SPKF, initially in generic form, will be afterwards modified to the specific problem of the orbit determination using GPS measurements.*

Keywords*: Estimation; Sigma Point Kalman Filter; Extended Kalman Filter; GPS Measurements; Orbit Determination.*

1. INTRODUCTION

The problem of orbit determination consists essentially of estimating parameters values that completely specify the body trajectory in the space, processing a set of information (measurements) related to this body. Such observations can be collected through a tracking network on Earth or through sensors, like the GPS receiver onboard Topex/Poseidon (T/P) satellite.

The Global Positioning System (GPS) is a powerful and low cost means to allow computation of orbits for artificial Earth satellites by means of redundant measurements. The T/P is an example of using GPS for space positioning.

On orbit determination of artificial satellites, the dynamic systems and the measurements equations are of non linear nature. It is a nonlinear problem in which the disturbing forces are not easily modeled. Through an onboard GPS receiver it is possible to obtain measurements (pseudo-ranges) that can be used to estimate the state of the orbit.

The Extended Kalman Filter (EKF) implementation in orbit estimation, under inaccurate initial conditions and scattered measurements, can lead to unstable or diverging solutions. For solving the problem of non linear nature, convenient extensions of the Kalman filter have been sought. In particular, the unscented transformation was developed as a method to propagate mean and covariance information through non linear transformations. The Sigma Point Kalman Filter (SPKF) appears as an emerging estimation algorithm applied to non linear system, without needing linearization steps.

2. EXTENDED KALMAN FILTER

The real-time estimators are a class of estimators which fulfill the real time requirements. They are recursive algorithms and produce sequentially the state to be estimated. Amongst them, the Kalman filter and its variations are outstanding.

The Kalman filter is the recursive estimator most used nowadays because it is easy to implement and to use on digital computers. Its recursiveness leads to lesser memory storage, which makes it ideal for real-time applications. The Extended KF (EKF) is a nonlinear version of the Kalman Filter that generates reference trajectories which are updated at each measurement processing, at the corresponding instant (Brown and Hwang, 1985).

Due to the complexity of modeling the artificial satellites orbit dynamics accurately, the EKF is generally used in works of such nature. The EKF algorithm always brings up to date reference trajectory around the most current available estimate.

The KF filter consists of phases of time and measurement updates. In the first, state and covariance are propagated from one previous instant to a later one, meaning that they are propagated between discrete instants of the system dynamics model. In the second one, state and covariance are corrected for the later instant corresponding to the measurement time, through the observations model. This method has, therefore, recursive nature and it does not need to store the measurements previously in large matrices.

Exploiting the assumption that all transformations are quasi-linear, the EKF simply linearizes all nonlinear transformations and substitutes the Jacobian matrices for the linear transformations in the Kalman Filter equations. Although the EKF maintains the elegant and computationally efficient recursive update form of the Kalman Filter, it suffers a number of serious limitations.

The first limitation is that linearized transformations are only reliable if the error propagation can be matched approximated by a linear function. If this condition does not hold, the linearized approximation can be extremely poor. At best, this undermines the performance of the filter. The second is that linearization can be applied only if the Jacobian matrix exists. However, this is not always the case. Some systems contain discontinuities, singularities, and the states themselves are inherently discrete. And the last is that calculating Jacobian matrices can be a very difficult and error-prone process. The Jacobian equations frequently produce many pages of dense algebra that must be converted to code. This introduces numerous opportunities for human coding errors that may degrade the performance of the final system in a manner that cannot be easily identified and debugged. Regardless of whether the obscure code associated with a linearized transformation is or is not correct, it presents a serious problem for subsequent users who must validate it for use in any high integrity system.

Summarizing, the Kalman Filter can be applied to nonlinear systems if a consistent set of predicted quantities can be calculated. These quantities are derived by projecting a prior estimate through a nonlinear transformation. Linearization, as applied in the EKF, is widely recognized to be inadequate, but the alternatives incur substantial costs in terms of derivation and computational complexity. Therefore, there is a strong need for a method that is probably more accurate than linearization but does incur neither the implementation nor computational costs of other higher order filtering schemes. The Unscented Transformation was developed to meet these needs.

3. SIGMA POINT KALMAN FILTERS

3.1. Introduction

If the dynamics system and the observation model are linear, the conventional Kalman filter can be used fearlessly. Although, not rarely, the dynamic systems and the measurement equations are nonlinear, convenient extensions of the Kalman Filter have been sought.

The SPKF is a new estimator that allows similar performance than the Kalman Filter for linear systems and it elegantly extends to nonlinear systems, without the linearization steps. These filters are a new approach to generalize the Kalman Filter for nonlinear process and observation models. A set of weighted samples, called sigma points, is used for normalizing mean and covariance of a probability distribution. This technique is claimed to lead to a filter more accurate and easier to implement than the EKF or a second order Gaussian filter.

The sigma point filter approach is described, as follows (van der Merwe et al., 2004):

1. A set of weighted samples is deterministically calculated, based on mean and covariance decomposition of a random variable. One minimum need is that the first and second order momentums are known.
2. The sigma points are propagated through the real nonlinear function, using only functional estimation, that is, analytical derivatives are not used to generate a posteriori set of sigma points.
3. The later statistics are calculated using propagated sigma points functions and weights. In general, they assume the form of a simple weighted average of the mean and the covariance.

3.2. Basic Idea: The Unscented Transformation

The Unscented Transform (UT) is a recent method to calculate the statistics of a random variable that passes through a nonlinear transformation. The UT builds on the principle that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function (Julier, 1997). The approach: a set of points (sigma points) are chosen so that their mean and covariance are $\bar{\mathrm{x}}$ and $\mathbf{P}_{xx}$ (Julier and Uhlmann, 2004). The nonlinear function is applied to each point, in turn, to yield a cloud of transformed points. The statistics of the transformed points, mean $\bar{\mathbf{y}}$ and covariance $\mathbf{P}_{yy}$ predicted, can then be calculated to form an estimate of the nonlinearly transformed mean and covariance.

Although this method bears a superficial resemblance to particle filters, there are several fundamental differences. First, the sigma points are deterministically chosen so that they exhibit certain specific properties (like a given mean and covariance), and are not drawn at random. The second difference is that sigma points can be weighted in ways that are inconsistent with the distribution interpretation of sample points in a particle filter.

The n-dimensional random variable $\mathbf{x}$ with $\bar{\mathbf{x}}$ mean and $\mathbf{P}_{xx}$ covariance is approximated by $2n + 1$ weighted points, given by

$$\begin{aligned}
\boldsymbol{\chi}_0 &= \bar{\mathbf{x}} & W_0 &= \kappa/(n+\kappa) \\
\boldsymbol{\chi}_i &= \bar{\mathbf{x}} + \left(\sqrt{(n+\kappa)\mathbf{P}_{\mathbf{xx}}}\right)_i & W_i &= 1/\,2(n+\kappa),\ i = 1,\ldots,n \\
\boldsymbol{\chi}_{i+n} &= \bar{\mathbf{x}} - \left(\sqrt{(n+\kappa)\mathbf{P}_{\mathbf{xx}}}\right)_i & W_{i+n} &= 1/\,2\,(n+\kappa),\ i = 1,\ldots,n
\end{aligned} \quad (1)$$

in which $\kappa \in \Re$, $\left(\sqrt{(n+\kappa)\mathbf{P}_{\mathbf{xx}}}\right)_i$ is the i-th row or column of the root square matrix of $(n+\kappa)\mathbf{P}_{\mathbf{xx}}$, and W_i is the weight associated to the i-th point. The transformation occurs as follow:

1. Transform each point through the function to yield the set of transformed sigma points

$$y_i = \mathbf{f}\left[\boldsymbol{\chi}_i\right] \quad (2)$$

2. The observations mean is given by the weighted average of the transformed points

$$\bar{\mathbf{y}} = \sum_{i=0}^{2n} W_i\, y_i \quad (3)$$

3. The covariance is the weighted outer product of the transformed points

$$\mathbf{P}_{yy} = \sum_{i=0}^{2n} W_i \left[\, y_i - \bar{\mathbf{y}}\,\right]\left[\, y_i - \bar{\mathbf{y}}\,\right]^{\mathrm{T}} \quad (4)$$

Despite its apparent simplicity, the UT has a number of important properties.

1. Because the algorithm works with a finite number of sigma points, it naturally lends itself to being used in a “black box” filtering library. Given a model (with appropriately defined inputs and outputs), a standard routine can be used to calculate the predicted quantities as necessary for any given transformation.
2. The computational cost of the algorithm is the same order of magnitude as the EKF. The most expensive operations are to calculate the matrix square root and the outer products which are required to compute the covariance of the projected sigma points.
3. Any set of sigma points that encodes the mean and covariance correctly calculates the projected mean and covariance correctly to the second order. Therefore, the estimate implicitly includes the second-order “bias correction” term of the truncated second-order filter, but without the need to calculate any derivatives.
4. The algorithm can be used with discontinuous transformations. Sigma points can pass over a discontinuity and, thus, can approximate the effect of a discontinuity on the transformed estimate.

3.2. The Unscented Kalman Filter

Using UT, what is processed in the Kalman Filter is summarizes in the following steps, shown first through Fig. 1, and explained after.

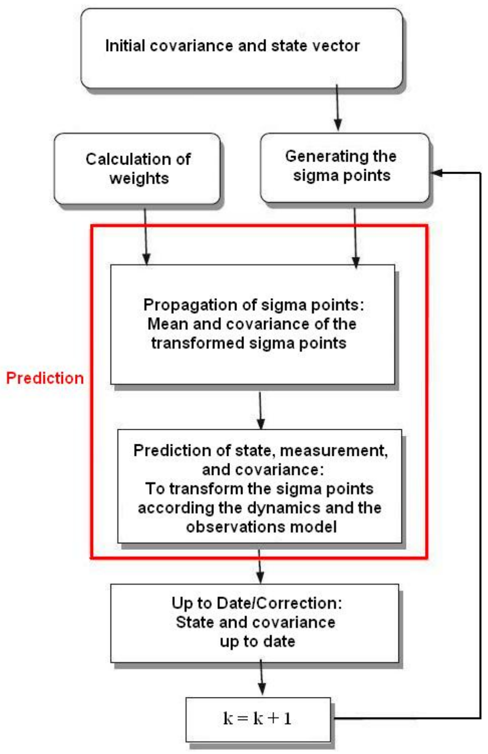

Figure 1. Modified Kalman Filter, leading to UKF

The steps shown in Fig. 1 are detailed next.

1. To predict the new state system $\hat{\mathbf{x}}(k+1 \mid k)$ and its associated covariance $\mathbf{P}(k+1 \mid k)$, taking into account the effects of the process white gaussian noise.
2. To predict the expected observation $\hat{\mathbf{z}}(k+1 \mid k)$ and its residual covariance (innovation) $\mathbf{P}_{\nu\nu}(k+1 \mid k)$, considering the effects of the observation noise.
3. To predict the cross correlation matrix $\mathbf{P}_{xz}(k+1 \mid k)$.

These steps are put in order in the Kalman Filter with the re-structuring of dynamics, state vector and observations models. First, the state vector is added of the noise vector $\boldsymbol{w}_k$, with dimension $q\times1$, in order to obtain a vector of dimension $n^a = n + q$.

$$\mathbf{x}^a(k) = \begin{bmatrix} \mathbf{x}_k \\ \boldsymbol{w}_k \end{bmatrix} \tag{5}$$

The dynamics model is re-written in function of $\mathbf{x}^a(k)$ as

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}^a(k)] \tag{6}$$

and the UT uses $2n^a + 1$ sigma points, generated by

$$\hat{\mathbf{x}}^a(k\mid k)=\begin{pmatrix}\hat{\mathbf{x}}(k\mid k)\\ \mathbf{0}_{q\times 1}\end{pmatrix} \quad \text{and} \quad \mathbf{P}^a(k\mid k)=\begin{bmatrix}\mathbf{P}(k\mid k) & \mathbf{P}_{xv}(k\mid k)\\ \mathbf{P}_{xv}(k\mid k) & \mathbf{Q}_{xv}(k\mid k)\end{bmatrix} \tag{7}$$

The matrices in the principal diagonal of $\mathbf{P}^a(k\mid k)$ are the variances, and the ones out of it, are the correlations between the state dynamic errors and the Gaussian process noises.

There are several extensions and modifications that can be done in this basic method to consider specific details for one given application. It will be presented a discussion of the orbit determination, in real time, using UKF.

3.3. Comparing EKF and UKF

The conventional nonlinear filters, such as the linearized or the extended Kalman Filter, many times have a poor performance when applied to nonlinear problems, due to two known difficulties:

- The linearization (of the dynamic and the measurements models) can lead to a highly instable performance of the filter if the time discretization is not enough small;
- The derivation of the Jacobian is not simple in most applications, and usually difficults the implementation.

The UKF has more advantages, when compared to the EKF, in the following aspects (Lee and Alfriend, 2004):

1. It allows more stable and accurate estimates of mean and covariance;
2. It can estimate discontinuous functions;
3. No explicit derivation of the Jacobian and/or Hessian matrix is necessary;
4. It is suitable for parallel processing.

4. USING UKF ON ORBIT DETERMINATION

4.1. The Orbit Determination

The orbit determination will be based on GPS technology, whose working principle is based on the geometric method. In such method, the observer knows the set of satellites position in the reference system, obtaining its own position in the same reference frame. Figure 2 presents the basic parameters used by GPS for user position determination.

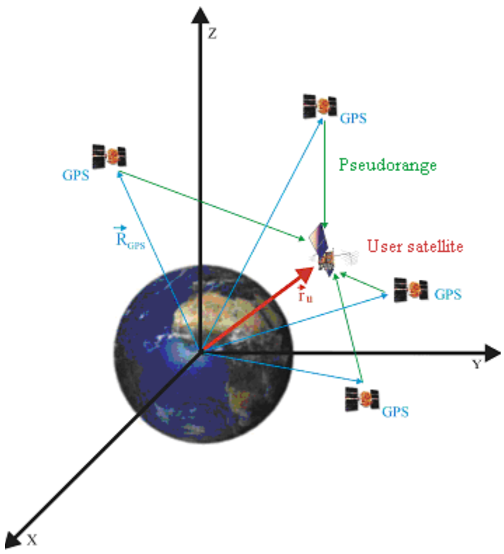

Figure 2. The Geometric Method

Here, $\vec{R}_{\mathrm{GPSi}}$ is the position of the i-th GPS satellite in the reference system; $\vec{\rho}_{\mathrm{i}}$ is the pseudorange, the user satellite position in respect to the i-th GPS satellite; and $\vec{r}_{\mathrm{u}}$ is the user satellite position in the reference system.

4.2. The Dynamic Model

In the case of orbit determination via GPS, the ordinary differential equations that represent the dynamic model are as follows:

$$\begin{aligned} \dot{\vec{\mathrm{r}}} &= \vec{\mathrm{v}} \\ \dot{\vec{\mathrm{v}}} &= -\mu \frac{\vec{\mathrm{r}}}{r^3} + \vec{\mathrm{a}} + \vec{\mathrm{w}} \\ \dot{\mathrm{b}} &= \mathrm{d} \\ \dot{\mathrm{d}} &= 0 + \mathrm{w}_\mathrm{d} \end{aligned} \quad (8)$$

with variables given in the inertial reference frame. In the equations above, $\vec{\mathrm{r}}$ is the vector containing the position components (x, y, z); $\vec{\mathrm{v}}$ is the vector of velocity components; $\vec{\mathrm{a}}$ represents the modeled perturbations; $\vec{\mathrm{w}}$ is the white noise vector with covariance Q; b is the user clock bias; d is the user clock drift; and w_d is the white noise on the drift rate with variance Q_d.

4.3. The Observations Model

The nonlinear equation of the observations model is given by:

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathrm{t}) + \mathbf{v}_k \quad (9)$$

where $\mathbf{z}_k$ is the vector of *m* observations; $\mathbf{h}_k(\mathbf{x}_k)$ is the nonlinear function of state $\mathbf{x}_k$, with dimension *m*; and $\mathbf{v}_k$ is the vector of observation errors with dimension *m*.

4.4. Discussion of the Application

Now that the purpose of the discussion is known, it is possible to present a discussion about the subject.

In order to generate the UKF, it is necessary to re-write the Kalman Filter from UT. First, the state vector is increased, with the measurements noise vector $\boldsymbol{w}_k$, yielding a vector with dimension $n^a = n + q$. The increased version of the state and the covariance are:

$$\mathbf{x}_k^a = \begin{bmatrix} \mathbf{x}_k \\ \boldsymbol{w}_k \end{bmatrix} \quad \text{e} \quad \mathbf{P}_k^a = \begin{bmatrix} \mathbf{P}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_k \end{bmatrix} \quad (10)$$

This increase can still contain $\boldsymbol{\upsilon}_k$, the Gaussian process noise, of dimension $l \times 1$. The new covariance matrix would have $\mathbf{R}_k$ in the principal diagonal, the covariance of such noise, and the new state vector dimension would be $n^a = n + q + l$ (Lee and Alfriend, 2007).

Next, the increased set of sigma points is built:

$$\begin{aligned} \boldsymbol{\chi}_{0,k}^a &= \bar{\mathbf{x}}_k^a \\ \boldsymbol{\chi}_{i,k}^a &= \bar{\mathbf{x}}_k^a + \left(\sqrt{(n_a + \lambda)\mathbf{P}_k^a}\right)_i \quad i = 1, \ldots, n_a \\ \boldsymbol{\chi}_{i,k}^a &= \bar{\mathbf{x}}_k^a - \left(\sqrt{(n_a + \lambda)\mathbf{P}_k^a}\right)_i \quad i = n_a + 1, \ldots, 2n_a \end{aligned} \quad (11)$$

with $\lambda = \alpha^2 (n_a + \kappa) - n_a$, where α control the sigma points scattered about the mean $\bar{\mathbf{x}}_k^a$, and it is usually chosen small, in the interval $10^{-4} \le \alpha \le 1$ (Jwo and Lai, 2008); κ provides an extra degree of freedom; and $\left(\sqrt{(n_a + \lambda)\mathbf{P}_k^a}\right)_i$ is the i-th row or column of the root square matrix of $(n_a + \lambda)\mathbf{P}_k^a$.

In the propagation step, the state vector and covariance predicted are calculated based on the mean and the covariance of the propagated sigma points, transformed from the state vector and dynamical noises.

$$\hat{\mathbf{x}}_{k+1}^{-} = \sum_{i=0}^{2n_a} W_i \, \boldsymbol{\chi}_{i,k+1}^{x}$$
$$\hat{\mathbf{P}}_{k+1}^{-} = \sum_{i=0}^{2n_a} W_i \left[\boldsymbol{\chi}_{i,k+1}^{x} - \hat{\mathbf{x}}_{k+1}^{-}\right]\left[\boldsymbol{\chi}_{i,k+1}^{x} - \hat{\mathbf{x}}_{k+1}^{-}\right]^{\mathrm{T}} \tag{12}$$

The prediction of the observation vector and the innovation matrix, $\mathbf{P}_{k+1}^{\nu\nu}$, is done the same way. That means, the observation and the innovation are predicted from mean and covariance of the transformed sigma points:

$$\hat{\mathbf{y}}_{k+1}^{-} = \sum_{i=0}^{2n_a} W_i^{(\mathrm{m})} \, y_{i,k+1} \tag{13}$$

where $y_{i,k+1}$ represents the sigma vectors propagated through the nonlinear equation of the observation model, yielding the transformed sigma points from the state vector and the dynamics noise, shown before.

In the up to date (correction) step of measurement, the Kalman gain, $\mathcal{K}_{k+1}$, is calculated based on the correlation matrix between the measurement and the observation, $\mathbf{P}_{k+1}^{xy}$, and the innovation matrix, both predicted.

$$\mathcal{K}_{k+1} = \mathbf{P}_{k+1}^{xy}\left(\mathbf{P}_{k+1}^{\upsilon\upsilon}\right)^{-1}, \text{ with } \quad \mathbf{P}_{k+1}^{\nu\nu} = \sum_{i=0}^{2n_a} W_i^{(\mathrm{c})}\left[y_{i,k+1} - \hat{\mathbf{y}}_{k+1}^{-}\right]\left[y_{i,k+1} - \hat{\mathbf{y}}_{k+1}^{-}\right]^{\mathrm{T}}, \quad \mathbf{P}_{k+1}^{xy} = \sum_{i=0}^{2n_a} W_i^{(\mathrm{c})}\left[\boldsymbol{\chi}_{i,k+1}^{x} - \hat{\mathbf{x}}_{k+1}^{-}\right]\left[y_{i,k+1} - \hat{\mathbf{y}}_{k+1}^{-}\right]^{\mathrm{T}} \tag{14}$$

Finally, the up to date state and covariance are

$$\hat{\mathbf{x}}_{k+1}^{+} = \hat{\mathbf{x}}_{k+1}^{-} + \mathcal{K}_{k+1}\left(\mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1}^{-}\right)$$
$$\mathbf{P}_{k+1}^{+} = \mathbf{P}_{k+1}^{-} - \mathcal{K}_{k+1}\,\mathbf{P}_{k+1}^{\nu\nu}\,\mathcal{K}_{k+1}^{\mathrm{T}} \tag{15}$$

where $\mathbf{y}$ is the vector effectively measured in the instant $k + 1$.

The process is repeated for the next instant, and the up to date mean (from state $\hat{\mathbf{x}}_{k+1}^{+}$) and covariance will be used to deterministically generate the sigma points of the next instant.

5. RESULTS

The purpose in this work is to introduce a non linear Kalman filter based on the unscented transformation (UKF) and to compare this filter to the EKF, for the specific problem of real time satellite orbit determination using GPS measurements.

The first line of direction on the UKF algorithm development it was to implement the UKF concept in the propagation step. The algorithm intends to be simple, compact, and a low computational cost recursive estimator, which makes it ideal for real-time applications. The state vector was estimated by the EKF in the update step, and the dynamics model includes geopotential perturbations until order and degree 10. The pseudorange measurements were used as the observations vector.

Figures 3 and 4 present the errors (Δr, Δv) and the pseudorange residuals for the hybrid UKF-EKF while Fig. 5 and 6 show the same analysis to the EKF only implementation, both for 2 short periods: 1 and 2 hours of sampling from 01/05/1994. According to these Fig. 3 to 6 and the Tab. 1, one notices that the residuals present a normal distribution with near zero mean and standard deviation near 1m for 1 hour and near 2m for 2 hours. With regard to the errors in position and in velocity, the hybrid UKF-EKF reaches accuracy around 21m and 25m in position, for 1 and 2 hours of sampling, respectively, and around 0.04m/s and 0.09m/s in velocity, for the same periods. And the EKF reaches accuracy near 22m and 49m in position, for 1 and 2 hours of sampling, respectively, and near 0.05m/s and 0.25m/s in velocity, for the same periods.

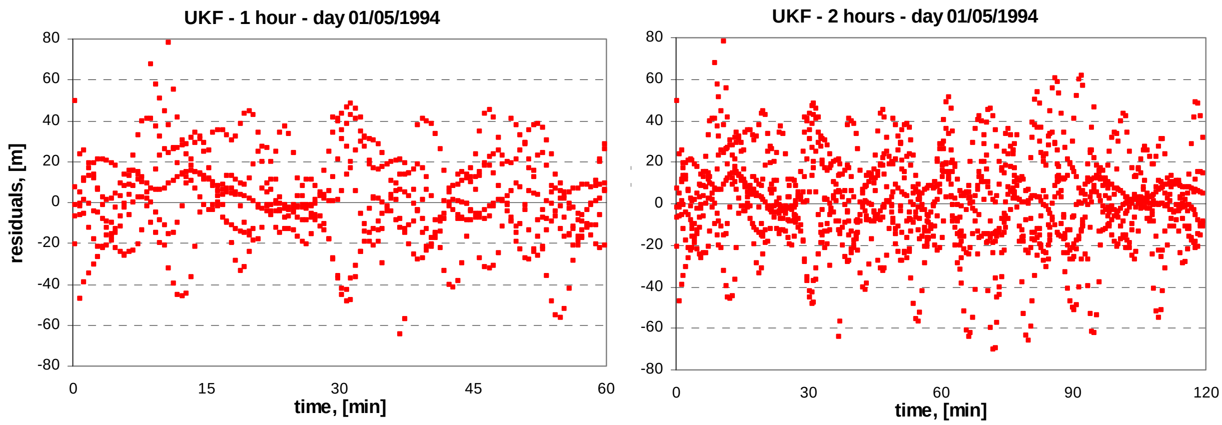

Figure 3. The pseudorange residuals for the 1 and 2 hours, respectively, through UKF-EKF

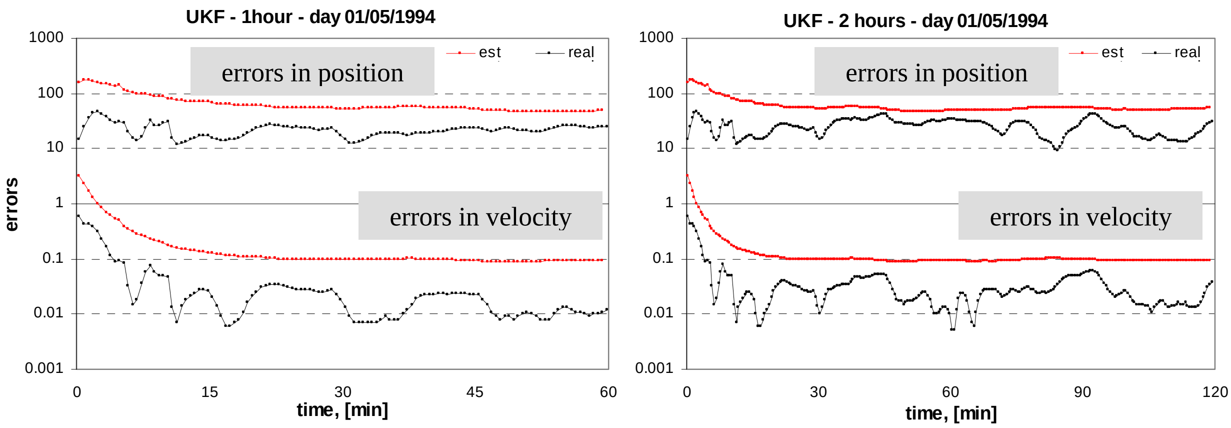

Figure 4. Errors in position and in velocity for the 1 and 2 hours, respectively, through UKF-EKF

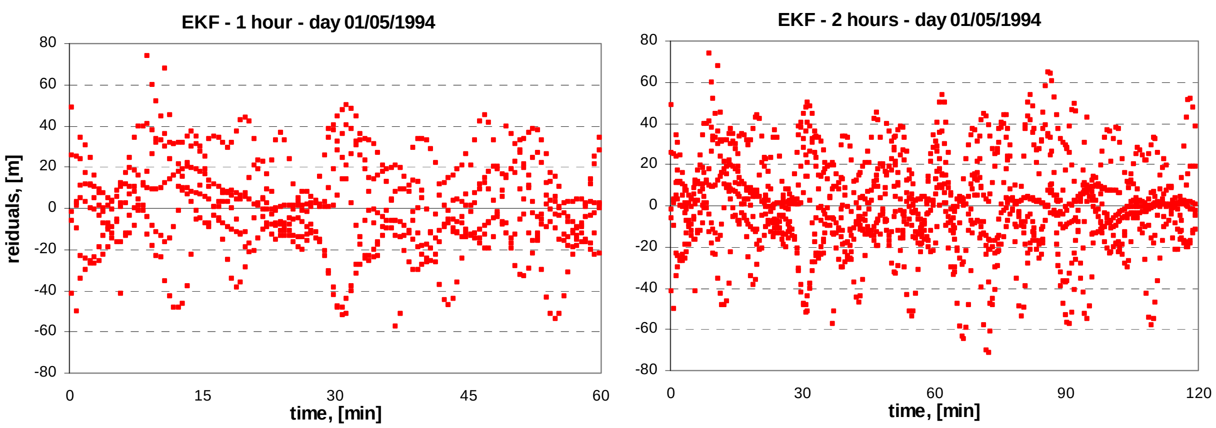

Figure 5. The pseudorange residuals for the 1 and 2 hours, respectively, through EKF

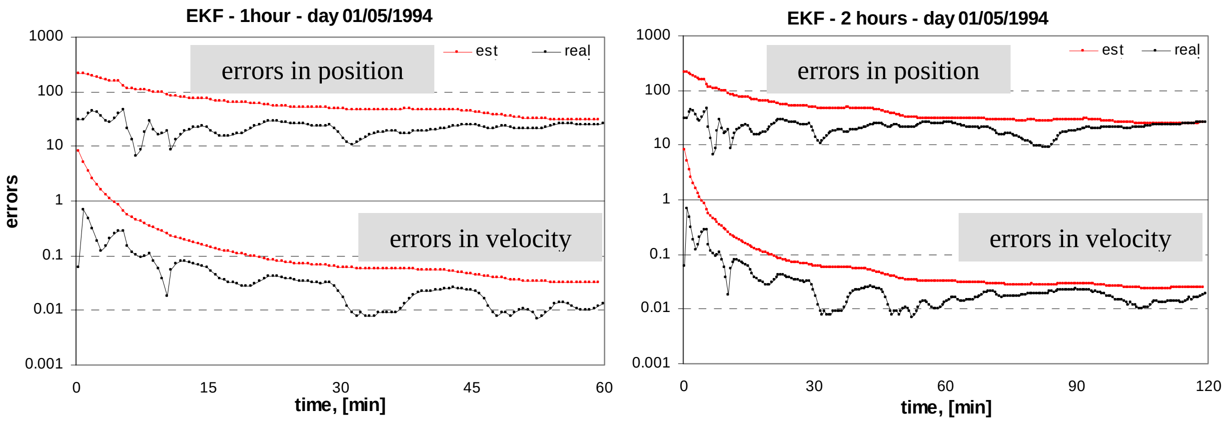

Figure 6. Errors in position and in velocity for the 1 and 2 hours, respectively, through EKF

Table 1. Estimator statistics for UKF-EKF and EKF only

Statistic	Δr (m) UKF	Δv (m/s) UKF	residuals (m) UKF	Δr (m) EKF	Δv (m/s) EKF	residuals (m) EKF
mean 1h	21.97	0.042	2.275	22.36	0.053	1.138
std deviation 1h	6.18	0.087	20.470	6.61	0.091	20.981
mean 2h	25.21	0.174	1.367	47.84	0.249	0.616
std deviation 2h	8.32	0.417	21.290	5.89	1.302	21.557

In both algorithms applied, the real errors (darker curves) in position and in velocity are below the estimated errors (lighter curves), which indicates statistic cohesion. For both algorithms, it seems that the estimators take near 1 hour to reach convergence.

By comparing results from the hybrid UKF-EKF (UKF in propagation step and EKF in update step) with results from EKF (in propagation and update steps), what is summarized in Tab. 1, one can conclude that, for these sampling times and the conditions of this orbit determination problem, both algorithms are competitive and present very similar behavior and statistic values. This indicates that the unscented transformation introduced in the propagation step were able to keep acting like the EKF.

Another test shall be done, for instance: to implement UKF in the update step; to increase sampling time for a long period (24 hours); and to measure computational time wasted. Within such approaches, one can better understand the differences between the algorithms and improve the conclusions about them.

6. ACKNOWLEDGEMENTS

The authors wish to express their appreciation for the support provided by FAPESP (The State of São Paulo Research Foundation), under contract 07/53256-1.

7. REFERENCES

Brown, R. G.; Hwang, P.Y.C.: “Introduction to random signals and applied Kalman filtering”. 3. ed. New York: John Wiley & Sons, 1985. 502p.
Julier, S.J.; Uhlmann, J.K.; Durrant-Whyte, H.F.: “A new approach for filtering nonlinear systems”. In: American Control Conference, June 1995, Seattle, Washington. Proceedings… TA5 - 9:15, p. 1628-1632.
Julier, S.J.; Uhlmann, J.K.: “A new extension of the Kalman filter for nonlinear systems”. International Symposium on Aerospace/Defense Sensing, Simulation and Controls. SPIE, 1997.
J Julier, S.J.; Uhlmann, J.K.; Durrant-Whyte, H.F.: “A new method for the nonlinear transformation of means and covariances in filters and estimators”. In: IEEE Transactions on Automatic Control, 2000. Proceedings… 2000. v. 45, Issue 3, p. 477-482.

Julier, S.J.; Uhlmann, J.K.: “Unscented filtering and nonlinear estimation”. In: IEEE Transactions on Automatic Control, v. 92, n. 3, 2004. Proceedings… Mar 2004.

Jwo, D-J; Lai, C-N.: “Unscented Kalman filter with nonlinear dynamic process modeling for GPS navigation”. GPS Solutions. Springer: Berlin, Sep. 2008, vol. 12, n. 4. ISSN 1521-1886 (on-line).

Lee, D-J.; Alfriend, K.T.: “Precise real-time orbit estimation using the unscented Kalman filter”. Advances in the Astronautical Sciences. v. 114, Suppl.: Spaceflight Mechanics 2003, pp. 20, 2003.

Lee, D-J.; Alfriend, K.T.: “Adaptive Sigma Point Filtering For State and Parameter Estimation”. In: AIAA/AAS Astrodynamics Conference, Providence, 16-19 Aug. 2004. Proceedings… American Institute of Aeronautics and Astronautics: AIAA-2004-5101, 20 p.

Lee, D-J.; Alfriend, K.T.: “Sigma point filtering for sequential orbit estimation and prediction”. Journal of Spacecraft and Rockets. v. 44, n. 2, Mar-Apr 2007.

van der Merwe, R., Wan, E. A., Julier, S. I.: “Sigma-point Kalman filters for nonlinear estimation and sensor-fusion - applications to integrated navigation”. In: AIAA Guidance, Navigation, and Control Conference and Exhibit, 16-19 Aug. 2004, Rhode Island. Proceedings… Providence: American Institute of Aeronautics and Astronautics, 2004.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.