NUMERICAL SIMULATION OF FORCED TURBULENT HEAT CONVECTION IN SQUARE DUCTS

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Abstract. *In the present work, the main parameters associated with the forced turbulent heat convection in square ducts have been determined. Considerations of fully developed and incompressible flow have been confirmed. The turbulence models have been validated based on numerical and experimental results of the literature. The researches show that in turbulent flow in non-circular ducts, appear secondary flows. In this way, these "insignificant" movements can influence the Bulk velocity, with consequences in the thermal energy. Whenever, this energy is transported through of the secondary flow. With these particularities, a good choice of the turbulent model should be done for a good analysis of the involved physical phenomena. To determine the profiles of velocity, the turbulence k-ε no-linear Model (NLEVM) and Reynolds Stress Model (RSM), were adopted and studied. To calculate the temperature field adopted the model: Simple Eddy Diffusivity (SED), it is based in the hypothesis of the Turbulent Prandtl number constant.*

Keywords*: Numerical Simulation, Forced Turbulent Heat Convection, Square Duct, Secondary Flow*

1. INTRODUCTION

Many turbulent flows found in engineering occur in ducts of non-circular cross-section. For instance, in compact heat exchangers, gas turbine cooling systems, cooling channels in combustion chambers, nuclear reactors, such flows are found. A three-dimensional velocity field is found in all these configurations. It has been known since the experimental work of Nikuradse (1926) that a transverse mean flow exists within non-circular ducts even when the flow is fully developed. This transverse mean flow is commonly referred to as Prandtl's secondary flow of the second kind, and it is caused by the anisotropy of the turbulent normal stresses. Although this secondary flow amounts to a few percent of the axial velocity, it exerts a great influence on the flow field. Turbulence-driven secondary motion redistributes the kinetic energy, influences the streamwise velocity, and thereby affects the wall shear stress. Beyond of the turbulent flow, in the present work the turbulent heat transfer is studied. The forced turbulent heat convection in a square duct is one of the fundamental problems in the thermal science and fluid mechanics. Recently, Qin and Plecther (2006) showed that the Prandtl's secondary flow of the second kind has a significant effect on the transport of heat and momentum as revealed by the recent large eddy simulation (LES).

Several experimental and numerical studies have been conducted on turbulent flow though a non-circular duct, namely, (Gessner and Emery, 1976; Gessner and Po, 1976; Melling and Whitelaw, 1976; Nakayama et al., 1983; Myon and Kobayashi, 1991; Lee and Jang, 1997; Assato, 2001) and others. Similarly important works in the turbulent heat convection were developed (Launder and Ying, 1973; Emery et al., 1980; Hirota et al.,1997; Rokni, 1998; Qin and Plecther, 2006) and others.

The experimental work of Melling and Whitelaw (1976) shows detailed characteristics of turbulent flow in a rectangular duct where they used a laser-Doppler anemometer to report the axial development of the mean velocity, streamwise turbulence intensity, contours of transverse turbulence intensity, Reynolds shear stress, turbulent kinetic energy and secondary mean velocity. Nakayama et al. (1983), it shows the analysis the fully developed flow field in ducts of rectangular and trapezoidal cross-sections using a finite-difference method based on the algebraic turbulence stress model of Launder and Ying (1973). On the other hand, Hirota *et al.* (1997) present an experimental work on the turbulent heat transfer in a square duct, shows detailed characteristics of turbulent flow and temperature field, such as Reynolds shear stress, temperature fluctuation intensity and turbulent heat fluxes, etc. Likewise, Rokni (1988), in the doctoral thesis achievement a comparison of four different turbulence models for predicting the turbulent Reynolds Stresses and three turbulent heat fluxes models for ducts of square and trapezoidal cross-sections using a volume finite technique.

It is well known that Linear Eddy Viscosity Models (LEVM) can give rise to inaccurate predictions for the Reynolds normal stresses and so that not have the ability to predict secondary flows of the second kind. In spite of that, they are one of the most popular models in the engineering due to its simplicity (requires less computational effort than complex models, e.g. algebraic or Reynolds stress), good numerical stability and it can be applied to a wide variety of flows. Thus, NLEVM represents a progress of the classical LEVM which permits inequality of the Reynolds normal stresses, a necessary condition for calculating turbulence-driven secondary flow in non-circular ducts within the relative cost of a two-equation formulation.

According to mention previously whenever non-isotropic effects are important we might consider other turbulence models, just as, RSM, also called the second order or second moment closure model, this model it is very accurate in the calculation of mean flow properties and all Reynolds stresses for many simple and more complex flows including wall jets, asymmetric channel and non-circular duct flows and curved flows, also present, disadvantages, just as, very large computing costs (Six extra, equations differential for the Reynolds stress, flow 3D).

The SED model for calculated turbulent heat flux have been adopted and studied. The field of temperature will be determined with the model: SED, it based in the hypotheses of the Turbulent Prandtl number constant, very utilized in commercial codes.

2. GOVERNING EQUATIONS

The governing equations are the continuity, momentum and energy equations (Reynolds Averaged Navier Stokes - RANS). It is considered fully developed turbulent flow and heat transfer and the following hypothesis have been utilized: steady state, condition of non slip on the wall, fluid with constant properties. The turbulent Reynolds stress ($-\rho\overline{u_i^{'}u_j^{'}}$) and the turbulent heat flux ($-\rho\overline{u_j^{'}t^{'}}$) are modeled and solved by solution of algebraic expressions or differential (Kays and Crawford, 1980).

$$\frac{\partial}{\partial x_j}(U_j)=0 \tag{1}$$

$$\frac{\partial}{\partial x_j}(U_iU_j)=-\frac{1}{\rho}\frac{\partial P}{\partial x_i}+\frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\right]+\frac{1}{\rho}\frac{\partial}{\partial x_j}(-\rho\overline{u_i^{'}u_j^{'}}) \tag{2}$$

$$\frac{\partial}{\partial x_j}(U_jT)=\frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\frac{\mu}{\Pr}\frac{\partial T}{\partial x_j}+(-\rho\overline{u_j^{'}t^{'}})\right] \tag{3}$$

Where $t^{'}$, $u_i^{'}$, $u_j^{'}$ are fluctuations of the temperatures and velocities in the direction *i* and *j*, respectively. This statistical approach resolves the problem of the handling of the instantaneous values. This is known as "closed problem". However, for the non-linear convective term, it causes the generation of news incognita (Reynolds stress tensor). Then, we defined the turbulent Reynolds stress tensor, as:

$$\tau_{ij}=-\rho\overline{u_i^{'}u_j^{'}}=-\rho\begin{bmatrix}\overline{u^{'2}} & \overline{v^{'}u^{'}} & \overline{w^{'}u^{'}}\\ \overline{u^{'}v^{'}} & \overline{v^{'2}} & \overline{w^{'}v^{'}}\\ \overline{u^{'}w^{'}} & \overline{v^{'}w^{'}} & \overline{w^{'2}}\end{bmatrix} \tag{4}$$

Where the elements of the diagonal, represent the normal tension components, and the outside elements of the diagonal, the shear tensions. This new set of Equations (1 to 2), containing the Reynolds stress, is known in the literature as equations Reynolds Averaged Navier Stokes (RANS), or simply, equations of Reynolds. Basically, they are two more utilized forms to find the Reynolds stress: the concept of turbulent viscosity and modeling of the equation of transport of the tensor of Reynolds, commonly called of approaches of second order. To first form for the modeling of the tensor of Reynolds was supplied by Joseph Boussinesq (1877). In this work the models of turbulence has been based in the approach of Boussinesq, these have produced satisfactory results for some cases of flows, however in many others (generally associated with effects of curvature, regions of detachment, strong acceleration, etc.) the concept of existence of a linear relation between the tension and the rate of deformation has shown fault. Equation (5) represents such relation:

$$-\overline{u^{'}v^{'}}=\nu_t\frac{\partial U}{\partial y} \tag{5}$$

Where ν_t is the turbulent viscosity. In this way, the scientific community has found alternatives to contour the problem of a linear relation (incapable of represent flows with strong curvature) by means of the enclosure of terms non-linear to the constitutive basic equation. In this way, many approaches have been created between these. We have, for example, the NLEVM of Speziale (1987). It is worth to emphasize that the turbulent viscosity is not a physical phenomenon of the fluid, otherwise a local measure of the level of turbulence, varying of point to point and of flow for flow. On the other hand, the tensor for turbulent heat flux can be represented as:

$$\tau_i = -\rho\overline{u_i't'} = \left(\rho\overline{u't'} \quad \rho\overline{v't'} \quad \rho\overline{w't'}\right) \tag{6}$$

The models for resolve the turbulent Reynolds stress ($-\rho\overline{u_i'u_j'}$) will be commented in the sections 2.1 e 2.2, already the tensor of turbulent heat flux ($-\rho\overline{u_i't'}$), of order zero, or algebraic models, that will be detailed in the sections 2.3 e 2.4.

2.1. Non-Linear Eddy Viscosity Models NVLME

When Eqs. (1) - (2) are written for the geometry of Fig. (1), the forms below are presented. The momentum equations for the calculation of the secondary velocity components *U*, *V* and of the axial velocity *W*, assuming fully developed flow in the axial direction (*z*), can be expressed in the following form:

x-momentum: $$\rho\left(U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}\right) = -\frac{\partial P}{\partial x}+\mu\nabla^2 U+\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y} \tag{7}$$

y-momentum: $$\rho\left(U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}\right) = -\frac{\partial P}{\partial y}+\mu\nabla^2 V+\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y} \tag{8}$$

z-momentum: $$\rho\left(U\frac{\partial W}{\partial x}+V\frac{\partial W}{\partial y}\right) = G+\mu\nabla^2 W+\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y} \tag{9}$$

where $G = -\frac{\partial P}{\partial z}$, in Eq. (9) is the axial pressure gradient constant that drives the flow.

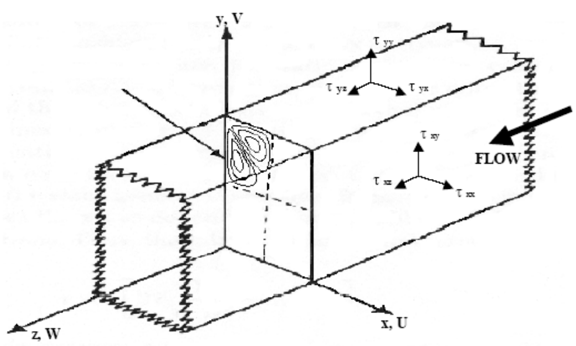

Figure 1. Fully developed turbulent flow in a square duct.

The modeled transport equations for the turbulent kinetic energy *k*, and its dissipation rate *ε*, respectively, are given by:

$$U_i\frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\mu_t}{\rho\sigma_k}\frac{\partial k}{\partial x_i}\right)+P_k-\varepsilon \tag{10}$$

$$U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\mu_t}{\rho \sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) + c_1 \frac{\varepsilon}{k} P_k - c_2 f_2 \frac{\varepsilon^2}{k} \tag{11}$$

The symbols P_k and μ_t, respectively, represent the turbulence kinetic energy production rate and the eddy viscosity, and are defined as:

$$P_k = \tau_{ij}\frac{\partial U_i}{\partial x_j}, \mu_t = c_\mu f_\mu \rho \frac{k^2}{\varepsilon} \tag{12}$$

In the present work for NLEVM, both high and low Reynolds models are compared. Their basic difference lies in the distinct form of the damping functions f_2 and f_μ referred in Eqs. (11) and (12). Expressions for them are shown in Table 1. These functions and a slightly different set of constants have been used in conjunction with the k-ε equations. In the calculating of the shear wall stress with the high Reynolds method (Launder and Spalding, 1974) and in the Table 1 may be varied to simulate the surface roughness and $\kappa = 0.41$ is the von Kármán constant. Subscript P refers to the next node to the wall. Thus u_P and k_P are, respectively, the value of the velocity and turbulent kinetic energy in this point, and y_P is the normal distance to the wall. The symbol n in the low Reynolds model represents the normal distance to the wall. The constants c_μ, c_1, c_2, σ_k and σ_ε for the high Reynolds model are set as 0.09, 1.44, 1.92, 1.0 and 1.33, respectively, and for the low Reynolds model given by 0.09, 1.5, 1.9, 1.4 and 1.3, respectively.
In this work the expression for the Reynolds stress is given as,

$$\tau_{ij} = \left(\mu_t S_{ij}\right)^L + \left(c_{1NL}\mu_t \frac{k}{\varepsilon}\left[S_{ik}S_{kj} - \tfrac{1}{3}S_{kl}S_{kl}\delta_{ij}\right]\right)^{NL} \tag{13}$$

Where the second term on the right hand side of Eq. (13) represents the non-linear term. This quadratic form produces a certain anisotropy degree between the Reynolds normal stresses, which make possible to predict the presence of secondary motion in non-circular ducts. The value of c_{1NL} proposed by Speziale (1987), is equal to 1.68. Here, c_{1NL} will be analyzed and adopted different values for both high and low Reynolds models.

The Reynolds normal and shear stresses present in Eqs. (7), (8) and (9), are expressed as:

Table 1. High and Low Reynolds Model

	High Reynolds model proposed by Launder [12]	Low Reynolds model proposed by Abe[14]
f_μ	1.0	$\left\{1-\exp\left[-\frac{(\nu\varepsilon)^{0.25}n}{14\nu}\right]\right\}^2\left\{1+\frac{5}{(k^2/\nu\varepsilon)^{0.75}}\exp\left[-\left(\frac{(k^2/\nu\varepsilon)}{200}\right)^2\right]\right\}$
f_2	1.0	$\left\{1-\exp\left[-\frac{(\nu\varepsilon)^{0.25}n}{3.1\nu}\right]\right\}^2\left\{1-0.3\exp\left[-\left(\frac{(k^2/\nu\varepsilon)}{6.5}\right)^2\right]\right\}$
τ_w	$\dfrac{u_P \rho c_\mu^{\frac{1}{4}} \kappa k_P^{\frac{1}{2}}}{\ln\left(\dfrac{E\rho c_\mu^{\frac{1}{4}} k_P^{\frac{1}{2}} y_P}{\mu}\right)}$	$\mu \frac{\partial u}{\partial y}$

$$\tau_{xx} = c_{1NL}\mu_t \frac{k}{\varepsilon}\left[\frac{1}{3}\left(\frac{\partial W}{\partial x}\right)^2 - \frac{2}{3}\left(\frac{\partial W}{\partial y}\right)^2\right], \tau_{yy} = c_{1NL}\mu_t \frac{k}{\varepsilon}\left[\frac{1}{3}\left(\frac{\partial W}{\partial y}\right)^2 - \frac{2}{3}\left(\frac{\partial W}{\partial x}\right)^2\right] \tag{14}$$

$$\tau_{xy} = c_{1NL}\mu_t \frac{k}{\varepsilon}\left[\frac{\partial W}{\partial x}\frac{\partial W}{\partial y}\right], \qquad \tau_{xz} = \mu_t \frac{\partial W}{\partial x}, \qquad \tau_{yz} = \mu_t \frac{\partial W}{\partial y} \tag{15}$$

The following difference for the Reynolds normal stresses is:

$$(\tau_{yy} - \tau_{xx}) = c_{1NL}\mu_t \frac{k}{\varepsilon}\left[\left(\frac{\partial W}{\partial y}\right)^2 - \left(\frac{\partial W}{\partial x}\right)^2\right] \quad (16)$$

It has been observed that for this type flow, the derivatives of the velocity components U and V relative to W derivatives are small and for that they are omitted in Eqs. (14) - (16).

The turbulence production term is expressed as:

$$P_k = \tau_{xz}\frac{\partial W}{\partial x} + \tau_{yz}\frac{\partial W}{\partial y} \quad (17)$$

2.2. Reynolds Stresses Equations Models (RSM)

The Reynolds stress model (RSM) (Launder et.al, 1975; Launder, 1989) involves calculation of the individual Reynolds stresses, $\overline{u_i^{'}u_j^{'}}$, using differential transport equations. The individual Reynolds stresses are then used to obtain closure of the Reynolds-averaged momentum equations Eqs. (1-2); The Eq. (18) describes six partial differential equations of the independent Reynolds stresses which are solved along with a equation model for the scalar dissipation rate ε (flow 3D). The strategy modeling originates from work reported in Launder et al (1975).

The exact transport equations for Reynolds stresses, $\rho\ \overline{u_i^{'}u_j^{'}}$, may be written as follows:

$$\frac{\partial}{\partial t}(\rho\overline{u_i^{'}u_j^{'}})(a) + \frac{\partial}{\partial x_k}(\rho u_k\overline{u_i^{'}u_j^{'}})(b) = -\frac{\partial}{\partial x_k}\left[\rho\overline{u_i^{'}u_j^{'}u_k^{'}} + \overline{p\left(\delta_{kj}u_i^{'} + \delta_{ik}u_j^{'}\right)}\right](c) +$$
$$\frac{\partial}{\partial x_k}\left[\mu\frac{\partial}{\partial x_k}\left(\overline{u_i^{'}u_j^{'}}\right)\right](d) - \rho\left(\overline{u_i^{'}u_k^{'}}\frac{\partial u_j}{\partial x_k} + \overline{u_j^{'}u_k^{'}}\frac{\partial u_i}{\partial x_k}\right)(e) - \rho\beta\left(g_i\overline{u_j^{'}\theta} + g_j\overline{u_i^{'}\theta}\right)(f) + \quad (18)$$
$$\overline{p\left(\frac{\partial u_i^{'}}{\partial x_j} + \frac{\partial u_j^{'}}{\partial x_i}\right)}(g) - 2\mu\overline{\frac{\partial u_i^{'}}{\partial x_k}\frac{\partial u_j^{'}}{\partial x_k}}(h) - 2\rho\Omega_k\left(\overline{u_j^{'}u_m^{'}}\varepsilon_{ikm} + \overline{u_i^{'}u_m^{'}}\varepsilon_{jkm}\right)(i) + S(j)$$

The terms in these exact equations (18) for Reynolds stresses:

a) Local time derivate, b) $C_{ij\equiv}$ Convection, c) $D_{T,ij\equiv}$ Turbulent diffusion,

d) $D_{L,ij\equiv}$ Molecular diffusion, e) $P_{ij\equiv}$ Stress production, f) $G_{ij} \equiv$ Buoyancy production , g) $\phi_{ij} \equiv$ Pressure strain, h) $\varepsilon_{ij\equiv}$ Dissipation, i) $F_{ij} \equiv$ Production by System Rotation, j) source term.

Of the various terms in these exact equations, C_{ij}, $D_{L,ij}$, P_{ij} and F_{ij} do not require modeling. However, $D_{T,ij}$, G_{ij}, ϕ_{ij}, e ε_{ij} need to model to close the equations.

2.2.1 Modeling Turbulent Difussive Transport

$D_{T,ij}$ can be modeled by the generalized gradient-diffusion model of Daly e Harlow (1970):

$$D_{T,ij} = C_S\frac{\partial}{\partial x_k}\left(\rho\frac{k\overline{u_k^{'}u_l^{'}}}{\varepsilon}\frac{\partial\overline{u_i^{'}u_j^{'}}}{\partial x_l}\right) \quad (19)$$

But, this equation can result in numerical instabilities, therefore Fluent (2003) uses a scalar turbulent diffusive as follows (Gibson and Launder, 1978):

$$D_{T,ij} = \frac{\partial}{\partial x_k}\left(\frac{\mu_t}{\sigma_k}\frac{\partial\overline{u_i^{'}u_j^{'}}}{\partial x_k}\right) \quad (20)$$

The turbulent viscosity, μ_t, is computed using the Eq. (24).

Lienand e Leschziner (1994) derived a value of $\sigma_k = 0.82$ by applying the generalized gradient-diffusion model, Eq (19).

2.2.2 Modeling of the Pressure Strain Term
Quadratic Pressure Strain Model

The pressure strain term, ϕ_{ij}, in Eq. (18) is modeled according to the proposals by Speziale, Sarkar, and Gatski (1991). This model has been demonstrated to give superior performance in a range of basic shear flows, including rotating plane shear, and axisymmetric expansion/contraction. This improved accuracy should be beneficial for a wider class of complex engineering flows, particularly those with streamline curvature.

2.2.3 Modeling of the Turbulence Kinetic Energy

In general, when the turbulence kinetic energy is necessary for modeling a specific term, it is obtained by taking the trace of the Reynolds stress tensor:

$$k = \frac{1}{2}\overline{u_i' u_i'} \tag{21}$$

An option available to solve a transport equation for the turbulence kinetic energy in order to obtain boundary conditions for the Reynolds stresses is:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + \frac{1}{2}(P_{ii} + G_{ii}) - \rho\varepsilon\left(1 + 2M_t^2\right) + S_k \tag{22}$$

Where $\sigma_k = 0.82$ and S_k is a user defined source term. Equation (22) is obtainable by contracting the modeled equation for the Reynolds stresses, Eq. (18). As one might expect, it is essentially identical to equation used in the standard k-e model.
Although Eq. (22) is solved globally throughout the flow domain, the values of k obtained are used only for boundary conditions. In every other case, k is obtained from Eq. (21).

2.2.4 Modeling of the Dissipation Rate

The scalar dissipation rate, ε, is computed with a model transport equation similar to that used in the standard k-e model:

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{\partial}{\partial x_i}(\rho\varepsilon u_i) = \frac{\partial}{\partial x_j}\left[(\mu + \frac{\mu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial x_j}\right] C_{\varepsilon 1}\frac{1}{2}[P_{ii} + C_{\varepsilon 3}G_{ii}]\frac{\varepsilon}{k} - C_{\varepsilon 2}\rho\frac{\varepsilon^2}{k} + S_\varepsilon \tag{23}$$

Where: $\sigma_\varepsilon = 1.0, C_{\varepsilon 1} = 1.44, C_{\varepsilon 2} = 1.92,$ $C_{\varepsilon 3}$ is evaluated as a function of the local flow direction relative to the gravitational vector, and S_ε is defined source term.

2.2.5 Modeling of the Turbulent Viscosity

The turbulent viscosity μ_t is computed similarly to the k-e models:

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \tag{24}$$

Where $C_\mu = 0.09$

2.3. Simple Eddy Diffusivity Model (SED)

This method it based in the model of viscosity of Boussinesq. To the unknown eddy thermal diffusivity, it can be expressed as:

$\alpha_t = \mu_t / \sigma_t$, where the number of turbulent Prandtl σ_t is prescribed. The zero equation models, SED assumes that the number of turbulent Prandtl is constant in all the region; this is 0.89 for the air, independently of the effect of proximity of the wall

$$\rho \overline{u_j t} = -\frac{\mu_t}{\sigma_T}\frac{\partial T}{\partial x_j} \tag{25}$$

This is one of the simplest models of the literature and they are frequently found in commercial codes. The main disadvantage of this model is that the diffusivities are independent of the direction, which means, they are isotropic.

3. NUMERICAL METHOD

For Reynolds stress tensor predicted, it has been employed the NLEVM and RSM models. This last has been calculated by a CFD commercial package. To obtain the turbulent heat flux in this paper it has been developed a FORTRAN computational program, in which solves the non-dimensional energy equation employing SED model. Similar methodology was developed by Patankar (1970).

4. RESULTS

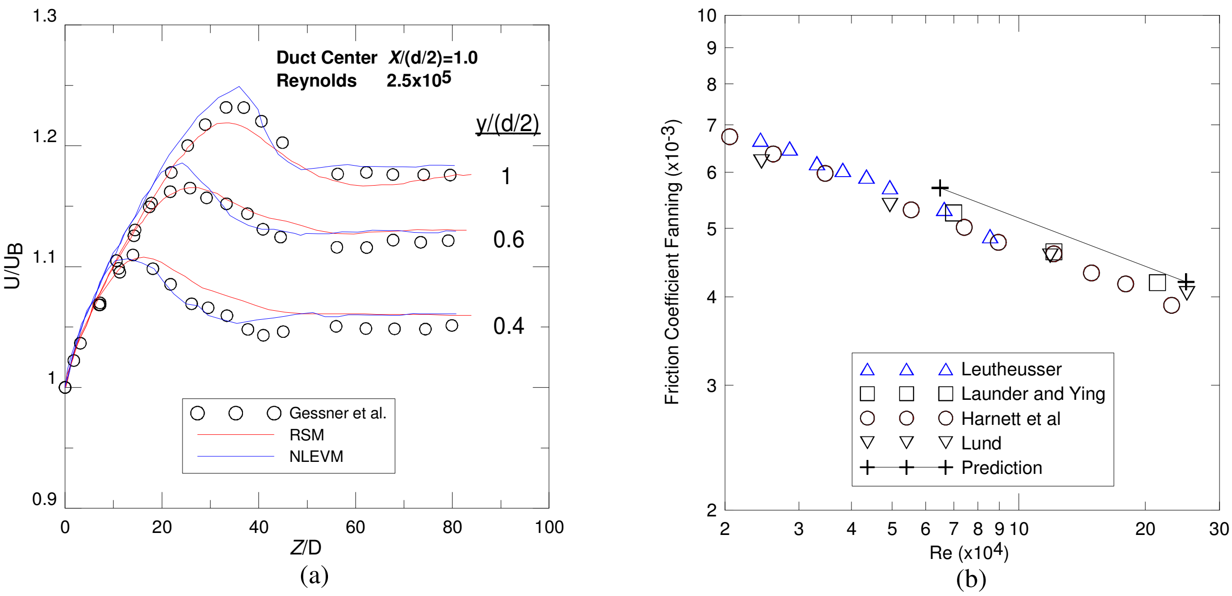

Figure 2. Axial mean velocity distributions at various streamwise locations (a); Friction coefficient for fully developed flows in a square duct (b)

Figure 2a shows comparisons between experimental (Myon et al., 1981) and numerical works (NLEVM and RSM models) for the variation of the axial mean velocity component at $x/(d/2)=1$, with distances from of the wall in the interval $0.4 \le y/(d/2) \le 1$ (where d is the hydraulic diameter). For the position $y/(d/2)=1$ the velocities in the central region increase with streamwise distance, reaching to local peak values after that, the values decrease asymptotically. The maximum center line velocity occurs at downstream of the location where the boundary layers begin to merge, approximately in ($z/d \cong 32$). The comparison among the turbulent models, they agree reasonably with experimental data for a determined calculation domain. Figure 2b shows comparisons among various experimental works. This figure shows also the prediction for the fanning friction coefficient in two numbers Reynolds for fully developed flow (2.5×10^5 and 65×10^3), other authors (Rokni, 1998) reported also this subject. The Fanning friction factor for a square duct follows:

$$f = 0.0971\,\mathrm{Re}^{-0.25} \tag{28}$$

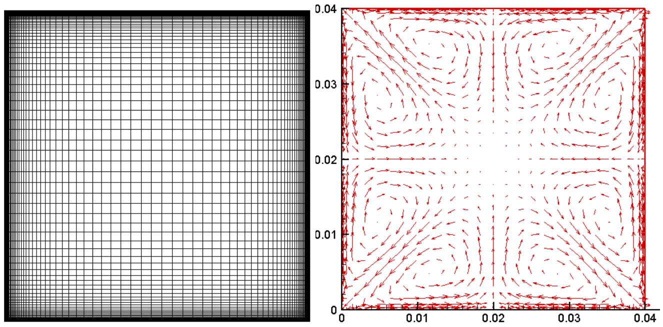

Figure 3. Grids and secondary flow vectors.

Figure 3 represents the grids and also shows the secondary flow vectors for the square duct. The secondary velocity flow is shows in form qualitative. This should be about 2-5 % of the main flow ("bulk"), depending on number of Reynolds, Rokni (1998).

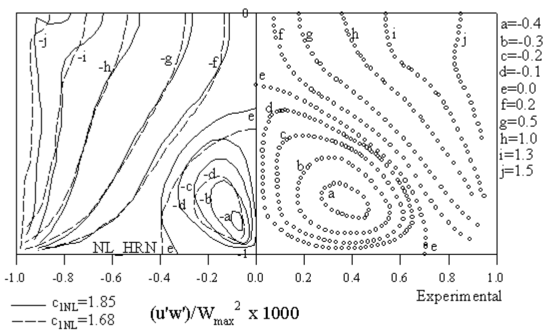

Figure 4. Reynolds shear stress contours $(u'w')/W_{\max}{}^{2}\,x1000$

Comparisons between the experimental work of Melling and Whitelaw (1976) with the computed values for the Reynolds shear stress τ_{xz} reported by Assato (2001) (NLEVM model) are shown in the Fig. 4.

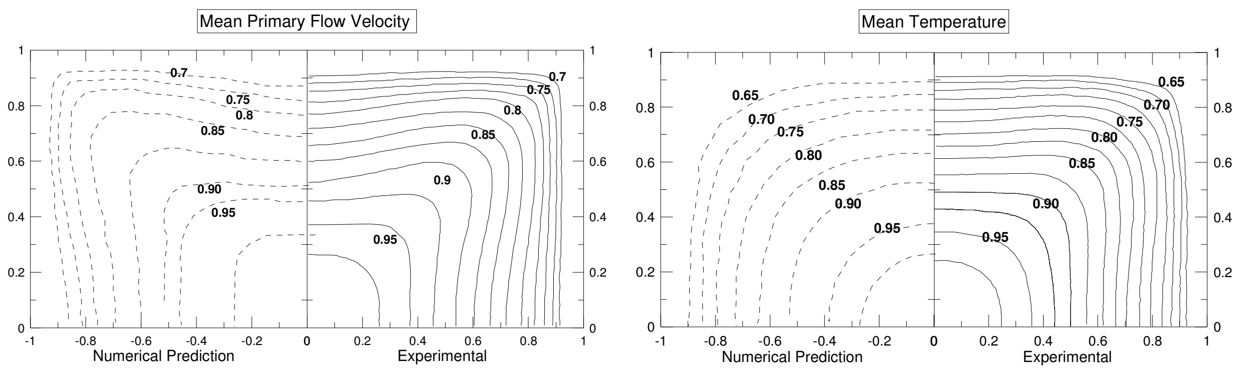

Figure 5. Streamwise velocity contours U/U_C (a) ; Mean Temperature $(T_W - T)/(T_W - T_C)$ (b)

Figure 5a presents the values of contours of velocities obtain experimentally by Hirota (1997) for Reynolds $6.5x10^4$. These are compared with RSM and U_C (velocity in the center duct). Figure 5b shows the mean temperature which has been utilized the SED model for air with constant wall temperature of 100 ºC. The mean temperature distribution is quite similar to primary flow velocity profile; however it is found those distortions of the contour temperatures are somewhat smaller than the mean velocity and in this way, the influences of the secondary flows in the heat transport are weaker than those in the momentum transport. Finally, we are able to observe that the results agree reasonably well with experimental data.

5. CONCLUSIONS

- The classical experimental researches of Gessner et al (1976), Hirota (1997) and Melling and Whitelaw (1976) show that in fully developed turbulent flow in straight square channels the condition of anisotropia appears in the flow.

- Figure 2-a shows what should take a special care in the prediction of the axial mean velocity. It is necessary to have sure that really we are dealing the zone of with fully developed flow. For example, the experimental work of Melling and Whitelaw shows comparisons in the position $z/d \cong 36.8$.

- The secondary velocity flow would be about 2-5 % of the main flow ("bulk"), depending on the number of Reynolds.

- The secondary flow distors the axial flow and reduces the volumetric flow rate. Likewise affects the wall shear stress and the heat transfer at walls.

- The prediction of the two important hydraulic parameters, from engineering point of view, are: friction factor and Nusselt number. The secondary flow generation is also very important.

- The equations of transport of the model RSM is of complex implementation and of high cost computational, in despite of that the models NLEVM demand lower cost. However the results of the model RSM predict much better at axial velocity contours than NELVM (qualitative form), but both models predict very well the turbulent quantities.

- The approach of the SED model represents reasonably the mean temperature (Fig. 5-b). Hirota (1997) considers the implementation of new models for turbulent heat fluxes in square ducts. The reason of this fact is that the properties of eddy thermal diffusivity and turbulent prandtl are only approached at square duct corner.

6. REFERENCES

Assato, M., 2001, “Análise numérica do escoamento turbulento em geometrias complexas usando uma formulação implícita”, Doctoral Thesis, Departamento de Engenharia Mecânica, Instituto Tecnológico de Aeronáutica - ITA, São José dos campos - SP, Brazil.

Kays and Crawford, 1980, “Convective Heat and Mass Transfer”, Second Edition, Ed. McGraw Hill, United States of America, 420 p.

Daly, B.J., and Harlow, F.H., 1970 “Transport equations in turbulence”, Phys.Fluid, Vol. 13, pp.2634-2649.

Fluent 6.1, 2003, user guide’s.

Emery, A.F., Neighbors, P.K. And Gessner, F.B., 1980, “The numerical prediction of developing turbulent flow and heat transfer in a square duct”, Vol. 102, pp. 51–57.

Gessner, F.B. and Emery, A.F., 1976, “A Reynolds stress model for turbulent corner flows – Part I: Development of the model”, J. Fluids Eng. 98, pp. 261-268.

Gessner, F.B. and Po, J.K., 1976, “A Reynolds stress model for turbulent corner flows – Part II: Comparison between theory and experiment”, J. Fluids Eng. 98, pp. 269-277.

Gibson, M.M. and Launder, B.E., 1978, “Ground effects on pressure fluctuations in the atmospherics boundary layer”, Journal Fluid Mechanical, Vol.86, pp. 491-511.

Hirota, M., Fujita, H. et al., 1997, “Turbulent heat transfer in a square duct”, International Journal Heat and fluid flow. Vol. 18, pp170-180.

Launder, B.E. and Ying, W.M., 1973, “Prediction of flow and heat transfer in ducts of square cross-section”, Proc. Inst. Mech. Eng. 187, pp. 455-461.

Launder, B.E. and Spalding, D.B., 1974, “The numerical computation of turbulent flows”, Comp. Meth. Appl. Mech. Eng. Vol. 3, pp. 269-289.

Launder, B.E., Reece, G.J. and Rodi, W., 1975, “Progress in the development of a Reynolds-Stress turbulence closure”, J.Fluid. Mech., Vol. 68, part.3, pp.537-566.

Launder, B.E., 1989, “Second - moment closure and its use in modelling turbulent industrial flows”, International Journal for Numerical Methods in Fluids, Vol. 9, pp. 963-985.

Lee K.B. and Jang, H.C., 1997. “A numerical prediction on the turbulent flow in closely spaced bare rod arrays by a nonlinear k- ε model”, Nuclear Engineering and Design, 172, pp. 351-357.

Lienand, F.S.and Leschziner, M.A., 1994, “Assessment of turbulent transport models including non linear RNG Eddy Viscosity formulation and second moment closure”, Computer and fluids, Vol. 23, part. 8, pp. 983-1004.

Melling, A. and Whitelaw, J.H., 1976, “Turbulent flow in a rectangular duct”, Journal Fluid Mechanical Vol. 78, pp. 289-315.

Myon, H.K., Kobayashi, T.,1991, “Numerical Simulation Of Three Dimensional Developing Turbulent Flow in a Square Duct with the Anisotropic κ-ε Model”, Advances in Numerical Simulation of Turbulent Flows ASME, Fluids Engineering Conference, Vol.117, Portland, United States of America, pp. 17-23.

Nakayama, A.; Chow, W.L. and Sharma, D., 1983, “Calculation of fully development turbulent flows in ducts of arbitrary cross-section”, J. Fluid Mech. Vol. 128, pp. 199-217.

Nikuradse, J., 1926, Untersuchung uber die Geschwindigkeitsverteilung in turbulenten Stromungen, Diss. Göttingen, VDI - forschungsheft 281.

Patankar, S.V., 1980, “Numerical Heat Transfer and Fluid Flow”, Ed. Hemisphere Publishing Corparation, Washington, United States, 311p.

Qin, Z.H.; Plecther, R.H.; 2006, “Large eddy simulation of turbulent heat transfer in a rotating square duct”, International Journal Heat Fluid Flow. Vol. 27, pp. 371-390.

Rokni, M., 1998, Numerical investigation of turbulent fluid flow and heat transfer in complex duct, Doctoral Thesis, Department of Heat and Power Engineering. Lund Institute of Technology, Sweden.

Speziale, C.G, Sarkar, S., and Gatski, T.B; 1991, “Modelling the Pressure-Strain Correlation of Turbulence: An Invariant Dynamical Systems Approach”. Journal Fluid Mechanical, Vol. 272, pp.227-245.

Speziale, C.G., 1987, “On Nonlinear k-l and k- ε Models of Turbulence”, J. Fluid Mech. Vol. 176, pp. 459-475.

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