

NUMERICAL ANALYSIS OF A TWO-PHASE SHOCK TUBE

Tobias R. Gessner, tobias@polo.ufsc.br

Jader R. Barbosa Jr., jrb@polo.ufsc.br

Polo – Laboratórios de Pesquisa em Refrigeração e Termofísica, Departamento de Engenharia Mecânica, Universidade Federal de Santa Catarina, Florianópolis, SC, 88040-900, Brasil.

Abstract. *The present paper advances a numerical model of a homogeneous two-phase flow in a shock tube. As the frictionless flow of a homogeneous two-phase mixture in a shock tube is amenable to analytical treatment, it represents an interesting benchmark for the evaluation of numerical schemes for two-phase transient problems. The hyperbolic nature of the mass, momentum and energy conservation equations for the two-phase flow gives rise to discontinuities in the solution domain, such as shocks and void waves. Finite differences solution algorithms based on the split coefficient matrix (SCM) method have been developed and validated for two different modeling approaches: the equilibrium homogeneous two-phase flow and the thermal non-equilibrium homogeneous two-phase flow models. Results obtained with both approaches are explored and compared with the analytical solution for a specific set of initial conditions in a two-phase shock tube.*

Keywords: *Shock-Tube, hyperbolic equations, homogeneous two-phase flow model, split coefficient matrix method.*

1. INTRODUCTION

Transient gas-liquid two-phase flows occur in several industrial and engineering applications and natural phenomena, such as nuclear reactors, heat exchangers, geothermal and oil-gas wells etc. As it is widely known, there are several modeling approaches to a two-phase flow problem, the most simple being the homogeneous formulation which assumes local hydrodynamic equilibrium (i.e., equal *in-situ* velocities) between the phases. It is also well established that the homogeneous model is more suitable for describing flows with very high volumetric fractions of the continuous phase due to the low relative velocity (i.e., slip) between the phases (Collier and Thome, 1994).

The shock-tube is a classical problem in gas dynamics which has been widely explored in the context of high velocity compressible flows. Under certain assumptions, the behavior of the flow properties (pressure, temperature, velocity etc.) in a single-phase shock tube problem can be described by simple algebraic relationships (Anderson, 1982), which can be used as a benchmark for numerical solution schemes (Gessner and Barbosa, 2008). With further assumptions, the analytical algebraic solution of the single-phase shock-tube problem can be extended in order to describe the flow of a homogeneous two-phase mixture (Städtke, 2006) in a shock-tube under severe transient conditions. For this reason, the two-phase shock tube can also be used as a standard problem for evaluating the ability of numerical schemes for two-phase transient flows.

This paper presents the development and application of a numerical procedure for solving two-phase homogeneous transient flows based on existing solution methods for hyperbolic systems of equations (Städtke, 2006). The Split Coefficient Matrix Method (SCM), proposed by Chakravarthy *et al.* (1980), is a variation of the Method of Characteristics. Both are upwind schemes based on an algebraic manipulation of the eigenvalues and eigenvectors of the governing system of hyperbolic PDEs which enables the evaluation of the influence of the different mechanisms by which perturbations are propagated in the flow field (shock and rarefaction waves, advection, etc.). The technique has been developed initially for compressible single-phase flows, but was subsequently extended to two-phase flow problems (Romstedt, 1987). It can be formulated as a function of conservative and non-conservative variables, implicitly or explicitly. At the present stage, the primary objective is to develop SCM-based routines for two variants of the two-phase flow homogeneous model, namely the equilibrium and the non-equilibrium formulation. While the equilibrium model assumes thermal and hydrodynamic equilibrium between the phases, the non-equilibrium model allows for different temperatures between the phases. Both formulations are described in detail and compared with the analytical solution of a two-phase shock-tube.

2. MODELLING

The two-phase shock-tube is composed of two horizontal chambers (1 and 2), separated by a diaphragm, as shown in Fig. 1. The working fluids are air (subscript *g*) and water (subscript *l*), which are initially at rest and have different homogeneous volumetric fractions as well as uniform temperatures and pressures in each chamber. As the diaphragm is instantaneously removed, a transient two-phase flow is initiated in the direction of the low pressure end of the shock-tube.

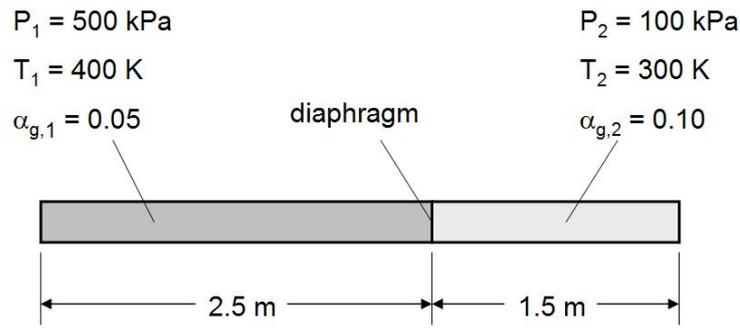


Figure 1. Problem geometry and initial conditions.

The flow is one-dimensional, frictionless and adiabatic. The gas and liquid phases are assumed to have the same velocity in every point of the domain (no slip) and no phase change takes place. The same hypotheses have been adopted in the analytical treatment of the two-phase shock tube reported by Städtke (2006). The hyperbolic system of equations which results from the application of the above hypotheses in a differential control volumes for each phase and for the two-phase mixture are presented next. The mass conservation equation for the gas and liquid phases are given by,

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \frac{\partial}{\partial x}(\alpha_g \rho_g U) = 0 \quad (1)$$

$$\frac{\partial}{\partial t}(\alpha_l \rho_l) + \frac{\partial}{\partial x}(\alpha_l \rho_l U) = 0 \quad (2)$$

where α_k is the volumetric fraction of phase k , ρ_k is the mass density of phase k and U is the homogeneous two-phase velocity. The momentum conservation equation for the two-phase mixture is given by,

$$\frac{\partial}{\partial t}(\rho U) + \frac{\partial}{\partial x}(\rho U^2) + \frac{\partial P}{\partial x} = -\rho g_x \quad (3)$$

where P is the thermodynamic pressure and ρ is the mixture density. The entropy balances in each phase are given by,

$$\frac{\partial}{\partial t}(\alpha_g \rho_g s_g) + \frac{\partial}{\partial x}(\alpha_g \rho_g U s_g) = \frac{Q_g^{\text{int}}}{T_g} \quad (4)$$

$$\frac{\partial}{\partial t}(\alpha_l \rho_l s_l) + \frac{\partial}{\partial x}(\alpha_l \rho_l U s_l) = \frac{Q_l^{\text{int}}}{T_l} \quad (5)$$

where s_k is the specific entropy of phase k and T_k is the temperature of phase k . The terms on the right hand side of equations (4) and (5) are the interfacial entropy transfer due to a finite temperature difference across the two-phase interface. The following definitions hold at every point of the domain,

$$\alpha_g + \alpha_l = 1 \quad (6)$$

$$\rho = \alpha_g \rho_g + \alpha_l \rho_l \quad (7)$$

$$Q_l^{\text{int}} = -Q_g^{\text{int}} \quad (8)$$

With the help of fundamental thermodynamic relations (Gyftopoulos and Beretta, 2005) for ρ_k as a function of P and s_k , equations (1) to (5) can be re-written in non-conservative form as follows,

$$\frac{\alpha_g}{\alpha_g^2} \left(\frac{\partial P}{\partial t} + U \frac{\partial P}{\partial x} \right) + \rho_g \left(\frac{\partial \alpha_g}{\partial t} + U \frac{\partial \alpha_g}{\partial x} \right) - \alpha_g \left(\frac{\rho_g \beta_g T_g}{c p_g} \right) \left(\frac{\partial s_g}{\partial t} + U \frac{\partial s_g}{\partial x} \right) + \alpha_g \rho_g \frac{\partial U}{\partial x} = 0 \quad (9)$$

$$\frac{\alpha_l}{\alpha_l^2} \left(\frac{\partial P}{\partial t} + U \frac{\partial P}{\partial x} \right) - \rho_l \left(\frac{\partial \alpha_g}{\partial t} + U \frac{\partial \alpha_g}{\partial x} \right) - \alpha_l \left(\frac{\rho_l \beta_l T_l}{c p_l} \right) \left(\frac{\partial s_l}{\partial t} + U \frac{\partial s_l}{\partial x} \right) + \alpha_l \rho_l \frac{\partial U}{\partial x} = 0 \quad (10)$$

$$\rho \left(\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \right) + \frac{\partial P}{\partial x} = -\rho g_x \quad (11)$$

$$\alpha_g \rho_g \left(\frac{\partial s_g}{\partial t} + U \frac{\partial s_g}{\partial x} \right) = \frac{Q_g^{\text{int}}}{T_g} \quad (12)$$

$$\alpha_l \rho_l \left(\frac{\partial s_l}{\partial t} + U \frac{\partial s_l}{\partial x} \right) = \frac{Q_l^{\text{int}}}{T_l} \quad (13)$$

Equations (9) to (13) correspond to the homogeneous thermal non-equilibrium model, since they naturally allow for different temperatures in the two phases. When the local thermal equilibrium condition is satisfied (homogeneous thermal equilibrium model), equation (13) is no longer needed, as s_l could be calculated from a thermodynamic relationship involving s_g and the homogeneous mixture temperature, which is determined from equation (12). The interfacial heat transfer rate per unit volume is given by,

$$Q_g^{\text{int}} = -h_{\text{int}} a_{\text{int}} (T_g - T_l) \quad (14)$$

where h_{int} and a_{int} are the interfacial heat transfer coefficient and interfacial area density, respectively. Considering that the gas is in the form of mono-dispersed spherical bubbles (a reasonable assumption in flowing systems given the low gas volume fraction on both sides of the diaphragm), the interfacial area density can be written as,

$$a_{\text{int}} = \frac{6\alpha_g}{d_b} \quad (15)$$

where d_b is a bubble diameter. Thus, equation (14) becomes,

$$Q_g^{\text{int}} = -\frac{6\alpha_g}{d_b} h_{\text{int}} (T_g - T_l) = -C_T \alpha_g (T_g - T_l) \quad (16)$$

where C_T is the so-called thermal constant (which is easier to specify in our model than the bubble diameter or the heat transfer coefficient). Equation (16) will be used in the thermal non-equilibrium formulation. In the homogeneous equilibrium model, it is more convenient to write the s_l derivatives in terms of the variations of P and s_g using thermodynamic (Maxwell) relationships. Thus, the interfacial heat transfer rate per unit volume is given by (Gessner, 2009),

$$Q_g^{\text{int}} = -\frac{\alpha_g \rho_g c p_g \alpha_l \rho_l c p_l}{\rho c p} \left(\frac{\beta_g}{\rho_g c p_g} - \frac{\beta_l}{\rho_l c p_l} \right) \left(\frac{\partial P}{\partial t} + U \frac{\partial P}{\partial x} \right) T \quad (17)$$

The interfacial heat transfer rate per unit volume in the above equation is such that it guarantees the thermodynamic equilibrium between the phases, i.e.,

$$T = T_g = T_l \quad (18)$$

The mixture specific heat capacity is given by,

$$c p = (\alpha_g \rho_g c p_g + \alpha_l \rho_l c p_l) / \rho \quad (18)$$

3. SYSTEMS OF CONSERVATION EQUATIONS

In terms of the thermal non-equilibrium model, it is possible to arrange Eqs. (9) to (13) in a compact format as follows,

$$\mathbf{E} \frac{\partial \vec{U}}{\partial t} + \mathbf{F} \frac{\partial \vec{U}}{\partial x} = \vec{B} \quad (20)$$

where the unknown vector \vec{U} is given by,

$$\vec{U} = [P \quad \alpha_g \quad U \quad s_g \quad s_l]^T \quad (21)$$

the matrices \mathbf{E} and \mathbf{F} are defined as,

$$\mathbf{E} = \begin{bmatrix} \frac{\alpha_g}{a_g^2} & \rho_g & 0 & -\alpha_g \left(\frac{\rho_g \beta_g T_g}{c p_g} \right) & 0 \\ \frac{\alpha_l}{a_l^2} & -\rho_l & 0 & 0 & -\alpha_l \left(\frac{\rho_l \beta_l T_l}{c p_l} \right) \\ 0 & 0 & \rho & 0 & 0 \\ 0 & 0 & 0 & \alpha_g \rho_g & 0 \\ 0 & 0 & 0 & 0 & \alpha_l \rho_l \end{bmatrix} \quad (22)$$

$$\mathbf{F} = \begin{bmatrix} \frac{\alpha_g U}{a_g^2} & \rho_g U & \alpha_g \rho_g & -\alpha_g U \left(\frac{\rho_g \beta_g T_g}{c p_g} \right) & 0 \\ \frac{\alpha_l U}{a_l^2} & -\rho_l U & \alpha_l \rho_l & 0 & -\alpha_l U \left(\frac{\rho_l \beta_l T_l}{c p_l} \right) \\ 1 & 0 & \rho U & 0 & 0 \\ 0 & 0 & 0 & \alpha_g \rho_g U & 0 \\ 0 & 0 & 0 & 0 & \alpha_l \rho_l U \end{bmatrix} \quad (23)$$

and the source-term vector \vec{B} is given by,

$$\vec{B} = \left[0 \quad 0 \quad -\rho g_x \quad \frac{Q_g^{\text{int}}}{T_g} \quad \frac{Q_l^{\text{int}}}{T_l} \right]^T \quad (24)$$

The system of equations given by Eq. (20) can also be written in the form,

$$\frac{\partial \vec{U}}{\partial t} + \mathbf{G} \frac{\partial \vec{U}}{\partial x} = \vec{C} \quad (25)$$

where the coefficient matrix \mathbf{G} is given by,

$$\mathbf{G} = \mathbf{E}^{-1} \mathbf{F} = \begin{bmatrix} U & 0 & \rho a^2 & 0 & 0 \\ 0 & U & -\alpha_g \alpha_l \left(\frac{1}{\rho_g a_g^2} - \frac{1}{\rho_l a_l^2} \right) \rho a^2 & 0 & 0 \\ \frac{1}{\rho} & 0 & U & 0 & 0 \\ 0 & 0 & 0 & U & 0 \\ 0 & 0 & 0 & 0 & U \end{bmatrix} \quad (26)$$

and the source-term vector \vec{C} is given by,

$$\vec{C} = \mathbf{E}^{-1} \vec{B} = \begin{bmatrix} \rho a^2 \left(\frac{\beta_g}{\rho_g c p_g} Q_g^{\text{int}} + \frac{\beta_l}{\rho_l c p_l} Q_l^{\text{int}} \right) \\ \rho a^2 \left(\frac{\alpha_l}{\rho_l a_l^2} \frac{\beta_g}{\rho_g c p_g} Q_g^{\text{int}} - \frac{\alpha_g}{\rho_g a_g^2} \frac{\beta_l}{\rho_l c p_l} Q_l^{\text{int}} \right) \\ -g_x \\ \frac{Q_g^{\text{int}}}{\alpha_g \rho_g T_g} \\ \frac{Q_l^{\text{int}}}{\alpha_l \rho_l T_l} \end{bmatrix} \quad (27)$$

The mixture sound velocity, a , introduced in Eqs. (26) and (27) is defined as,

$$a = \sqrt{\frac{\rho_g a_g^2 \rho_l a_l^2}{\rho (\alpha_g \rho_l a_l^2 + \alpha_l \rho_g a_g^2)}} \quad (28)$$

The coefficient matrix \mathbf{G} can be written in terms of its eigenvalue matrix $\mathbf{\Lambda}$ and of \mathbf{T} and \mathbf{T}^{-1} , which are the matrix of eigenvectors and its inverse, by using the following relationship,

$$\mathbf{G} = \mathbf{T} \mathbf{\Lambda} \mathbf{T}^{-1} \quad (29)$$

where

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \end{bmatrix} = \begin{bmatrix} U+a & 0 & 0 & 0 & 0 \\ 0 & U-a & 0 & 0 & 0 \\ 0 & 0 & U & 0 & 0 \\ 0 & 0 & 0 & U & 0 \\ 0 & 0 & 0 & 0 & U \end{bmatrix} \quad (30)$$

and

$$\mathbf{T} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -\alpha_g \alpha_l \left(\frac{1}{\rho_g a_g^2} - \frac{1}{\rho_l a_l^2} \right) & -\alpha_g \alpha_l \left(\frac{1}{\rho_g a_g^2} - \frac{1}{\rho_l a_l^2} \right) & 1 & 0 & 0 \\ \frac{1}{\rho a} & -\frac{1}{\rho a} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (31)$$

It should be noted that for subsonic flows (such as the present case) the eigenvalue λ_1 will always be positive, while the eigenvalue λ_2 will always be negative. The remaining eigenvalues, λ_3 , λ_4 and λ_5 will be either positive or negative, depending on the local value of the flow velocity. By decomposing the matrix $\mathbf{\Lambda}$ into parts which correspond to each eigenvalue, one has,

$$\mathbf{G} = \mathbf{T} \mathbf{\Lambda}_1 \mathbf{T}^{-1} + \mathbf{T} \mathbf{\Lambda}_2 \mathbf{T}^{-1} + \mathbf{T} \mathbf{\Lambda}_3 \mathbf{T}^{-1} + \mathbf{T} \mathbf{\Lambda}_4 \mathbf{T}^{-1} + \mathbf{T} \mathbf{\Lambda}_5 \mathbf{T}^{-1} = \mathbf{G}_1 + \mathbf{G}_2 + \mathbf{G}_3 + \mathbf{G}_4 + \mathbf{G}_5 \quad (32)$$

Next, a matrix \mathbf{G}^+ is constructed based on the coefficients involved in the propagation of a given quantity in the positive direction x . Similarly, a matrix \mathbf{G}^- is constructed in order to account for the propagation of a given quantity in the negative x direction. Thus,

$$\mathbf{G}^+ = \mathbf{G}_1 + \max\left(\frac{U}{|U|}, 0\right)(\mathbf{G}_3 + \mathbf{G}_4 + \mathbf{G}_5) \quad (33)$$

$$\mathbf{G}^- = \mathbf{G}_2 + \min\left(\frac{U}{|U|}, 0\right)(\mathbf{G}_3 + \mathbf{G}_4 + \mathbf{G}_5) \quad (34)$$

Therefore, it is possible to recast Eq. (25) in the form,

$$\frac{\partial \bar{U}}{\partial t} + \mathbf{G}^+ \frac{\partial \bar{U}}{\partial x} + \mathbf{G}^- \frac{\partial \bar{U}}{\partial x} = \bar{C} \quad (35)$$

For the equilibrium model, as can be easily verified, the system of equations is identical to that for the non-equilibrium model, except for the absence of terms associated with the derivatives of the liquid entropy.

4. NUMERICAL IMPLEMENTATION

4.1. Discretization of the system of governing equations

Since the coefficient matrix is divided into two parts, \mathbf{G}^+ and \mathbf{G}^- , which influence the propagation of the quantities in the positive and negative x directions, it is natural that a *backward differentiation scheme* (BDS) and a *forward differentiation scheme* (FDS) are employed in the evaluation of the spatial derivative associated with \mathbf{G}^+ and \mathbf{G}^- , respectively. Thus, by adopting a second-order discretization scheme in space and an implicit formulation, Eq. (35) can be approximated by

$$\frac{\bar{U}_i - \bar{U}_i^0}{\Delta t} + \mathbf{G}^+ \left[\frac{(\beta^2 - 1)\bar{U}_i - \beta^2\bar{U}_{i-1} + \bar{U}_{i-2}}{(\beta^2 - \beta)\Delta x_i} \right] - \mathbf{G}^- \left[\frac{(\varphi^2 - 1)\bar{U}_i - \varphi^2\bar{U}_{i+1} + \bar{U}_{i+2}}{(\varphi^2 - \varphi)\Delta x_{i+1}} \right] = 0 \quad (36)$$

where

$$\beta = \frac{\Delta x_i + \Delta x_{i-1}}{\Delta x_i} \quad (37)$$

$$\varphi = \frac{\Delta x_{i+1} + \Delta x_{i+2}}{\Delta x_{i+1}} \quad (38)$$

$$\Delta x_i = x_i - x_{i-1} \quad (39)$$

The system of equations (36) can be more conveniently written in the form

$$\mathbf{A}\bar{X} = \bar{K} \quad (40)$$

where \mathbf{A} is a block matrix whose non-zero elements are sub-matrices given by

$$A_{i,i-2} = \frac{1}{(\beta^2 - \beta)\Delta x_i} \mathbf{G}^+ \quad (41)$$

$$A_{i,i-1} = -\frac{\beta^2}{(\beta^2 - \beta)\Delta x_i} \mathbf{G}^+ \quad (42)$$

$$A_{i,i} = \frac{1}{\Delta t} \mathbf{I} + \frac{\beta^2 - 1}{(\beta^2 - \beta)\Delta x_i} \mathbf{G}^+ - \frac{\varphi^2 - 1}{(\varphi^2 - \varphi)\Delta x_{i+1}} \mathbf{G}^- \quad (43)$$

$$A_{i,i+1} = \frac{\varphi^2}{(\varphi^2 - \varphi)\Delta x_{i+1}} \mathbf{G}^- \quad (44)$$

$$A_{i,i+2} = -\frac{1}{(\varphi^2 - \varphi)\Delta x_{i+1}} \mathbf{G}^- \quad (45)$$

and the vectors containing the unknowns \vec{X} and the source-term \vec{K} are defined by,

$$X_i = \vec{U}_i \quad (46)$$

$$K_i = \vec{U}_i^0 + \vec{C}_i \quad (47)$$

4.2. Boundary conditions

Firstly, it is assumed that the properties (pressure, entropy and void fraction) at the left end are influenced only by the propagation associated with the negative eigenvalues. Likewise, the same properties at the right end are assumed to be influenced only by the propagation associated with the positive eigenvalues. At these points, the flow velocity is set to zero, which results in a modification of the coefficients associated with the momentum conservation equation.

4.3. Computational grid and solution procedure

The computational grid is composed of 4001 equally spaced (1 mm) nodes. The time step was set at 1μs. The solution procedure, at each time step, consists of the following stages:

1. All fields are set equal to the converged values obtained at the previous time step.
2. The vector \vec{K} is calculated.
3. The coefficients of the matrix \mathbf{A} are calculated.
4. The system of equations given by Eq. (40) is solved using the GBAND algorithm (Aziz e Settari, 1979; Ouyang, 1998), and new values of P , α_g , U , s_g are obtained. In the case of the homogeneous non-equilibrium formulation, s_l is also computed.
5. Based on the latest fields of P , s_g and, in the case of the homogeneous non-equilibrium model, s_l , the thermodynamic properties of each phase are calculated at each node using the equation of state physical properties subroutines available in the Fortran source code of the REFPROP 7.0 package (Lemmon et al., 2002).
6. The mixture properties ρ , cp and a are updated.
7. If the convergence criterion is not met, return to stage 2. The convergence criterion has been established based on the absolute difference between the values of each variable at two successive iterations (see Table 1 for the tolerances applied to each variable).

Table 1. Tolerances associated with each variable

Variable	Tolerance
P [kPa]	1×10^{-3}
α_g [-]	1×10^{-6}
U [m/s]	1×10^{-6}
s_g [kJ/kgK]	1×10^{-4}

5. RESULTS

By assuming that the gas phase behaves as an ideal gas, the liquid is incompressible and local hydrodynamic and thermal equilibrium exist between the phases, the two-phase shock-tube problem can be solved analytically (Anderson, 1982; Städtke, 2006). The analytical solution can be used to validate the numerical solution of the homogeneous equilibrium model and of the non-equilibrium model, if a high value is assigned to C_T .

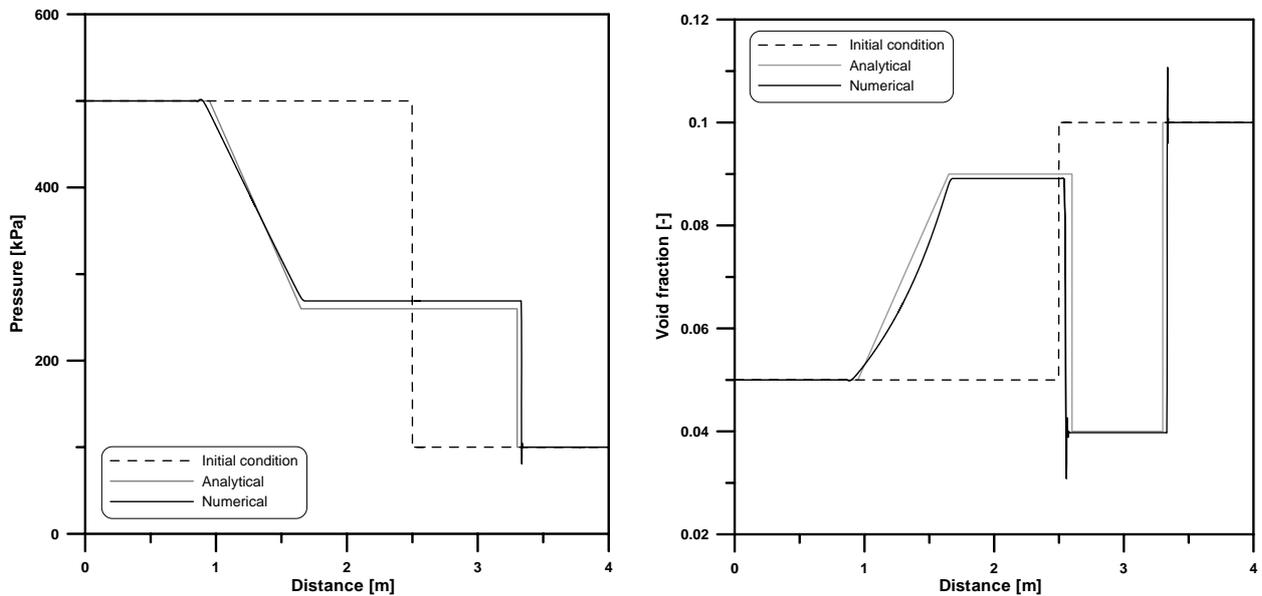


Figure 2. Pressure and void fraction fields at $t = 15ms$. Homogeneous equilibrium model.

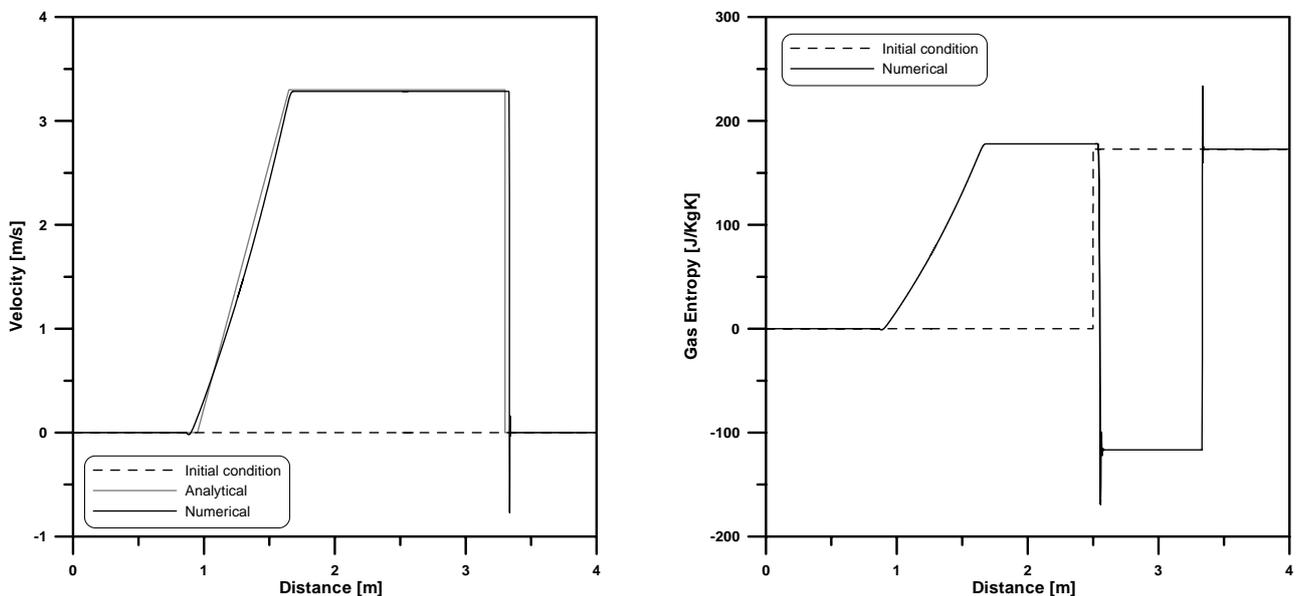


Figure 3. Velocity and gas entropy fields at $t = 15ms$. Homogeneous equilibrium model

Figures 2-5 illustrate the fields obtained for both models. As can be verified, both formulations were able to describe the analytical results satisfactorily. Evidently, the utilization of the homogeneous equilibrium model is more advantageous since it yields a simpler system of equations, thus requiring a lower computational effort. On the other hand, the homogeneous non-equilibrium model provides greater flexibility, since the thermal constant can reproduce the

unrealistic condition of absence of interfacial heat transfer ($C_T = 0$), the local thermal equilibrium condition ($C_T = \infty$) and every other intermediate condition. Figure 6 shows the gas temperature profiles obtained for different values of C_T .

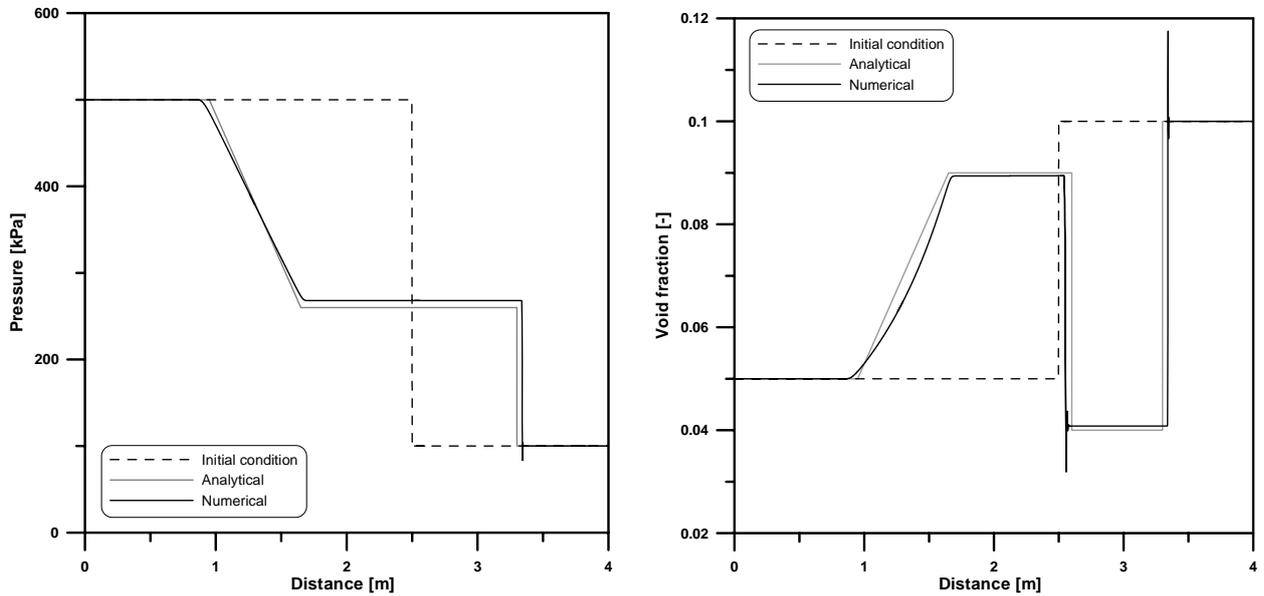


Figure 4. Pressure and void fraction fields at $t = 15ms$. Homogeneous non-equilibrium model.

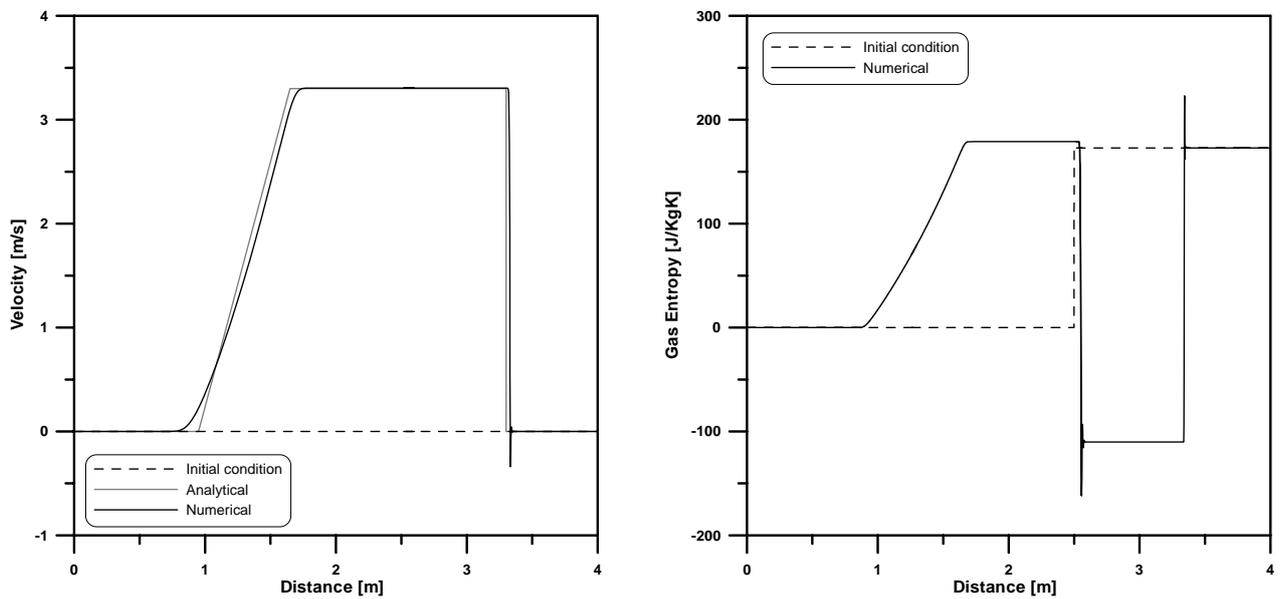


Figure 5. Velocity and gas entropy fields at $t = 15ms$. Homogeneous non-equilibrium model.

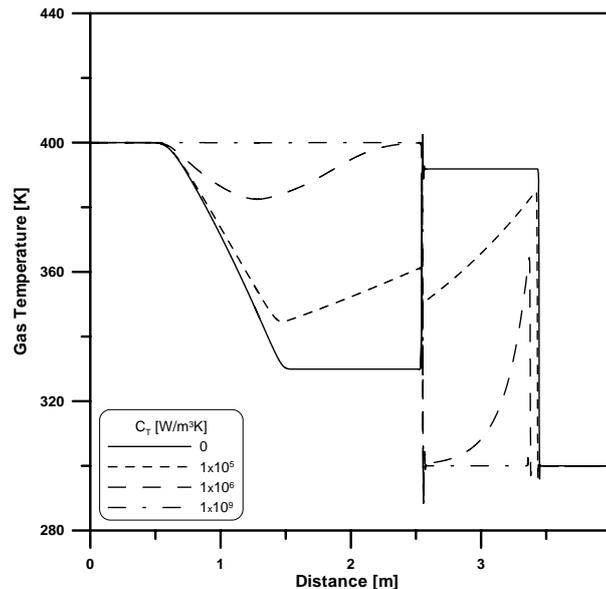


Figure 6. Gas temperature fields at $t = 15ms$. Homogeneous non-equilibrium model

6. CONCLUSIONS

In the present work, numerical routines based on the split coefficient matrix (SCM) method were proposed for two distinct gas-liquid two-phase flow models: the homogeneous equilibrium and the homogeneous non-equilibrium. The routines were evaluated and validated by a comparison with the analytical solution of a two-phase shock-tube. Both models gave similar and consistent results. The influence of the interfacial energy coupling in the non-equilibrium model was also investigated.

7. REFERENCES

- Anderson, J. D.; 1982, "Modern Compressible Flow". McGraw-Hill, NY.
- Aziz, K., Settari, A., 1979, "Petroleum Reservoir Engineering", Elsevier Applied Science, New York, NY.
- Chakravarthy, S. R., Andersen, D. A., Salas, M. D.; 1980, "The Split Coefficient Matrix Method for Hyperbolic Systems of Gasdynamic Equations". AIAA 18th Science Meeting, paper 80-0268, Pasadena, CA.
- Collier, J. G., Thome, J. R.; 1994, "Convective Boiling and Condensation", 3rd Ed., Oxford University Press.
- Gessner, T. R.; 2009, "Estudo Numérico de Escoamentos Bifásicos Transientes", M.Eng. dissertation (*in preparation*), Department of Mechanical Engineering, Federal University of Santa Catarina, Florianópolis, Brazil.
- Gessner, T. R., Barbosa, Jr., J. R.; 2008, "Numerical Solution of a Shock-Tube: A Comparison Between Formulations Based on the Split Coefficient Matrix Method", Proc. of the 12th Brazilian Congress of Thermal Sciences and Engineering (ENCIT 2008), Nov. 10-14, Belo Horizonte, MG.
- Gyftopoulos, E. P., Beretta, G. P., 2005, "Thermodynamics: Foundations and Applications", Dover, NY.
- Lemmon, E.W., McLinden, M.O, Huber, M.L., 2002, REFPROP V. 7.0, NIST.
- Ouyang, L.-B.; 1998, "Single Phase and Multiphase Flow in Horizontal Wells". PhD thesis, Dept. of Petroleum Engineering, Stanford University, Stanford, CA
- Städtke, H.; 2006, "Gasdynamic Aspects of Two-Phase Flow". Wiley-VCH.
- Romstedt, P.; 1987, "A Split-Matrix Method for the Numerical Solution of Two-Phase Flow Equations". Int. Top. Meeting on Advances in Reactor Physics, Mathematics and Computation, Paris, France.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.