ON THE USE OF HYBRID NEURO-FUZZY SYSTEMS FOR THE SOLUTION OF INVERSE RADIATIVE TRANSFER PROBLEMS

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Abstract. *Inverse problems related to the interaction of radiation with participating media have been attracting the attention of various researchers for many years due to the relevant applications not only in engineering, but also in astrophysics, physical oceanography (hydrologic optics), remote sensing and atmosphere/hydrosphere optics among many others. In this work the determination the single scattering albedo, optical thickness and diffuse reflectivities in one-dimensional homogeneous participating media is investigated using neuro-fuzzy networks, and a hybrid solution of neuro-fuzzy networks with artificial neural networks. Such neural networks are massively distributed and parallel structures, inspired upon the functioning of the human brain, able to implemented in software and hardware, and they try to reproduce the dynamics of biological networks. As they are developed through algorithmic numerals, the knowledge which is symbolic remains represented by the network in numerical form. Inverse problems are very susceptible to the error which is always present in experimental data, being therefore an essential part of its formulations and solutions. The fuzzy logic, which is based upon the ability of human beings to deal with inexact, imprecise and vague information, gives us a tool that may be helpful to handle with the experimental uncertainty. Here we use hybrid neuro-fuzzy systems for the solution of inverse problems in radiative transfer.*

Keywords*: Inverse Problems, Radiative Transfer, Neuro-Fuzzy.*

1. INTRODUCTION

The solution of direct and inverse problems of radiative transfer is an efficient tool for the estimation of optical properties in different ways. The analysis of inverse problems involving the interaction of different types of particles and radiation such as neutrons, gamma rays and photons, with a participating medium, i.e., an absorbing, emitting and scattering media, it has been widely used in the development of techniques for applications in engineering, medicine, geophysics, astrophysics, and in other areas (Alvarez Acevedo *et al*., 2002).

A classic example of participating medium is the earth's atmosphere which exerts a mitigating effect of solar radiation. Other examples of participating medium are the products of combustion engine - rockets and thermal protection for spacecraft (Pessoa Filho, 1998) and others.

In mathematical modeling of the direct problem of radiative transfer in a participating medium is the use of linear equation or Boltzmann equation of radiative transfer (Radiative Transfer Equation - RTE) which results from the application of the principle of conservation of energy in a way. The inverse problems can be expressed mathematically so explicit or implicit (Silva Neto, 2002), and several techniques have been developed to solve them (Silva Neto e Becceneri, 2009; Campos Velho, 2008; Silva Neto e Moura Neto, 2005; Beck *et al*., 1985).

In the present work is proposed to determine the estimate of the optical thickness, single scattering albedo and diffuse reflectivity coefficients at the inner side of the one-dimensional participant boundary surfaces a using for the solution of the inverse problem of radiative transfer with neuro-fuzzy networks and a method of neuro-fuzzy networks combined with an artificial neural network.

2. MATHEMATICAL FORMULATION AND SOLUTION OF THE DIRECT PROBLEM

The direct problem in radiative transfer in participating medium is to calculate the values of radiation intensity at any position of the medium and turn in any direction, when the properties of the medium and boundary conditions are known.

Consider the problem of transport of radiation in a participating medium one-dimensional, homogeneous (the coefficients of absorption and scattering do not depend on the position), without internal sources (the term emission is considered negligible in comparison external radiation incident in the medium), isotropic scattering, gray (the radiative properties of the medium independent of the wavelength of radiation) and is subject to the effect of isotropic radiation.

The physical situation is represented schematically in Fig. 1, where a one-dimensional homogeneous medium of optical thickness τ_0 (no spectral dependence), isotropic scattering with diffuse reflecting surfaces, is subject to the impact of external radiation in their boundary surfaces.

When the boundary conditions and the material properties are know, the direct problem of radiative transfer can be solved by providing the values of the radiation intensity for every point in the spatial and angular domains.

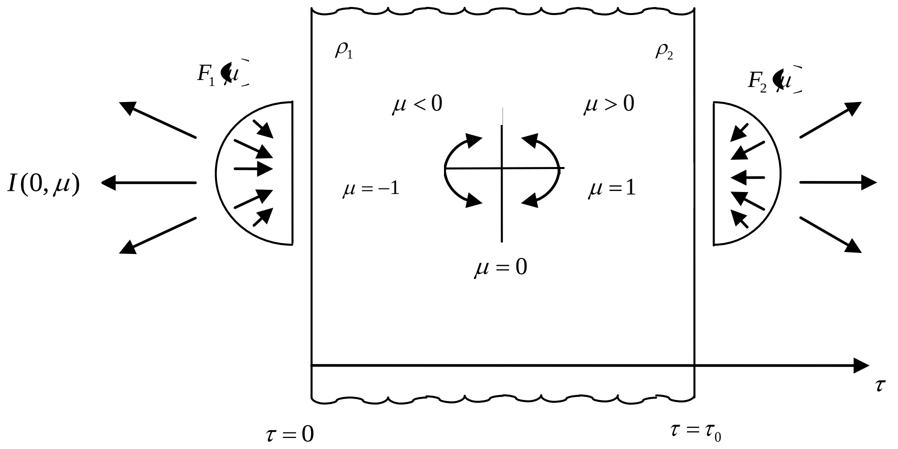

Figure 1 – Schematical representation of one-dimensional homogeneous participating, with inner diffusely reflecting boundary surfaces, subjected to isotropic external radiation

In the case of azymuthal symmetry and neglecting the term of issue, the linearized Boltzmann equation is written as

$$\mu \frac{\partial I(\tau,\mu)}{\partial \tau} + I(\tau,\mu) = \frac{\omega}{2}\int_{-1}^{1} p(\mu,\mu')I(\tau,\mu')d\mu' \text{ em } 0 < \tau < \tau_0, -1 \leq \mu \leq 1 \quad (1a)$$

with the following boundary conditions

$$I(0,\mu) = F_1(\mu) + 2\rho_1 \int_0^1 I(0,-\mu')\mu' d\mu' \,, \mu > 0 \quad (1b)$$

$$I(\tau_0,\mu) = F_2(\mu) + 2\rho_2 \int_0^1 I(\tau_0,\mu')\mu' d\mu' \,, \mu < 0 \quad (1c)$$

where p is the function of the phase of anisotropic scattering, $I(\tau,\mu)$ the intensity of electromagnetic radiation, τ the optical variable, τ_0 the optical thickness of the medium, μ the cosine of the polar angle, i.e., the cosine of the angle of the beam of radiation with the axis τ, μ the single scattering albedo, ρ_1 and ρ_2 the diffuse reflectivity of the inner side of the medium and the intensities of isotropic radiation sources are represented by $F_1(\mu)$ e $F_2(\mu)$ on the boundary surfaces $\tau = 0$ and $\tau = \tau_0$.

When the geometry, the radiative properties and the boundary conditions are know, problem as in Eq. (1), may be solved yielding the values of the radiation intensity $I(\tau,\mu)$ for $0 \leq \tau \leq \tau_0$ and $-1 \leq \mu \leq 1$.

The problem modeled by Eq. (1) is solved using the method of discrete ordinates for Chandrasekhar, in which the field of polar angle is discretized as shown in Fig. 2 and the integral on the right side of Eq. (1) is replaced by a Gauss quadrature.

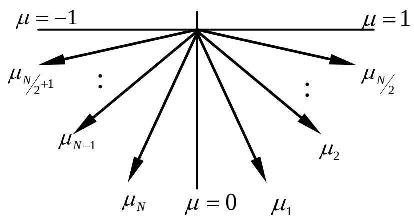

Figura 2 - Discretização do domínio do ângulo polar.

For the terms on the left side of Eq. (1) is used an approximation by finite differences, and making steps forward and backward, from $\tau = 0$ to $\tau = \tau_0$ and $\tau = \tau_0$ to $\tau = 0$, respectively, obtained is the value of the intensity of radiation $I(\tau,\mu)$ for all τ and all μ of discretized computational domain.

3. MATHEMATICAL FORMULATION AND SOLUTION OF THE INVERSE PROBLEM WITH NEURO-FUZZY NETWORK AND NEURO-FUZZY COMMITTEE

The Russian researcher in the field of inverse problems Oleg Mikailivitch Alifanov, "the solution of an inverse problem is to determine causes based on observation of its effects" (Campos Velho, 2008). Silva Neto and Moura Neto (2005) classify the inverse problems on the nature and extent of the problem:

Type I: Estimation of a finite number of parameters in a model of finite size;
Type II: Estimation of a finite number of parameters in a model of infinite dimension;
Type III: Estimation of an infinite number of parameters or a function in a model of infinite dimension.

This work has been a problem of type II related to the estimation of two parameters (constants) in a problem modeled by a differential- integration equation.

Knowing it measures the intensity of radiation leaving the participant medium for the various polar angles θ_i, i.e., $Y_i,\ i = 1,2,\cdots,N_e$, where N_e is the total number of experimental data available, and the radiative properties of the medium $\vec{Z} = \{\omega, \tau_{0,}\rho_1, \rho_2\}^T$ are unknown, seeks to solve the inverse problem from the values of intensities of radiation leaving the medium under study.

For convenience in the formulation of the inverse problem considered here is that the number of experimental data coincides with the number of directions used in the discretization of the angular domain, and furthermore, Y_i is measured in the polar angle corresponding to μ_i, where $\mu_i = cos\theta,$ as Fig. 2.

Several formulations can be used to solve the inverse problem in radiative transfer, but have been the object of much interests those that an optimization problem in which a cost functions is minimized. For this reason, deterministic methods, stochastic and hybrid have been used to minimize the function of Least-Mean-Square between intensities calculated and measures the radiation intensity that leaves the medium, and then determined with this procedure the radiative properties required.

As experimental data are not available, it built a set of summary data through a computational solution based on the direct problem.

Thus, the vector of unknowns $\vec{Z}$, where

$$\vec{Z} = \{\omega, \tau_0, \rho_1, \rho_2\}^T \tag{2}$$

is determined from the intensities of radiation

$$Y_i = I_{exp_i} = I_{calc_i}(\vec{Z}_{exact}) + \sigma r_i, i = 1,2,\cdots,N_e \tag{3}$$

where I_{calc_i} represents the values of radiation intensity by using the exact values of the parameters that a real application are not available, which are seeking to determine the solution of the inverse problem, σ the standard deviation of simulated errors of experimental data, and r_i is a pseudo-random number generated in the interval [-1,1].

Using the experimental data synthetic I_{exp_i}, will be presented in the next section the estimates for the diffuse reflectivities coefficients, optical thickness and scattering albedo, as a solution to the inverse problem of radiative transfer through neuro-fuzzy networks and neuro-fuzzy committee machines.

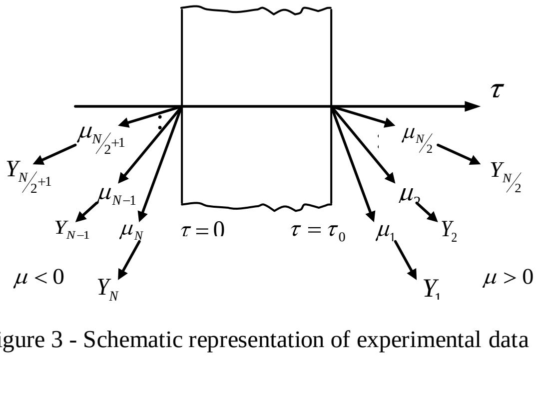

Figure 3 - Schematic representation of experimental data

3.1. Neuro-fuzzy network

The development of systems with ability to use knowledge to perform tasks or solve complex problems, which are similar to real problems is the key to developing intelligent systems. Some techniques in the development of computational systems have particular features that address only a specific set of problems. The artificial neural networks and fuzzy systems, for example, do not flee to this feature. While neural networks are optimal for detecting patterns, they are not efficient to explain how these standards are achieved. But the fuzzy systems that work with the imprecision and explain well its decision-making cannot automatically generate the rules that have taken that decision.

So many hybrid intelligent systems have been developed, where two or more techniques are combined to meet their individual limitations.

The neural networks and fuzzy systems provide the integration of this technology, called neuro-fuzzy systems. With the main objective of combining fuzzy systems to represent and process knowledge in a clear and easy to interpret, and use that learning ability in neural networks.

The neural networks are a good way to adjust the knowledge experts and automatically generate new fuzzy rules and new functions of relevance. Furthermore, fuzzy logic enhances the generalization ability of the systems of neural networks, promoting a more realistic output when extrapolation is needed beyond the limits of the training data.

A neuro-fuzzy system consists of various components of traditional fuzzy systems, except for the fact that each stage is composed of a layer of neurons and the learning capability of the neural network is used to obtain knowledge of the system.

Figure 5 represents the phase of training the neuro-fuzzy network, seeking to increase the performance of the system you can use a hybrid learning, combining the method of least squares with the method of gradient descending. To do this you must perform two steps: one direct and one reverse (Jang, 1997).

In phase direct the input signal will spread layer by layer until the layer where the consequent parameters of the rule are identified by the of least squares method.

In the reverse phase error signal is back propagated and the parameters of the background are updated by the gradient descending method.

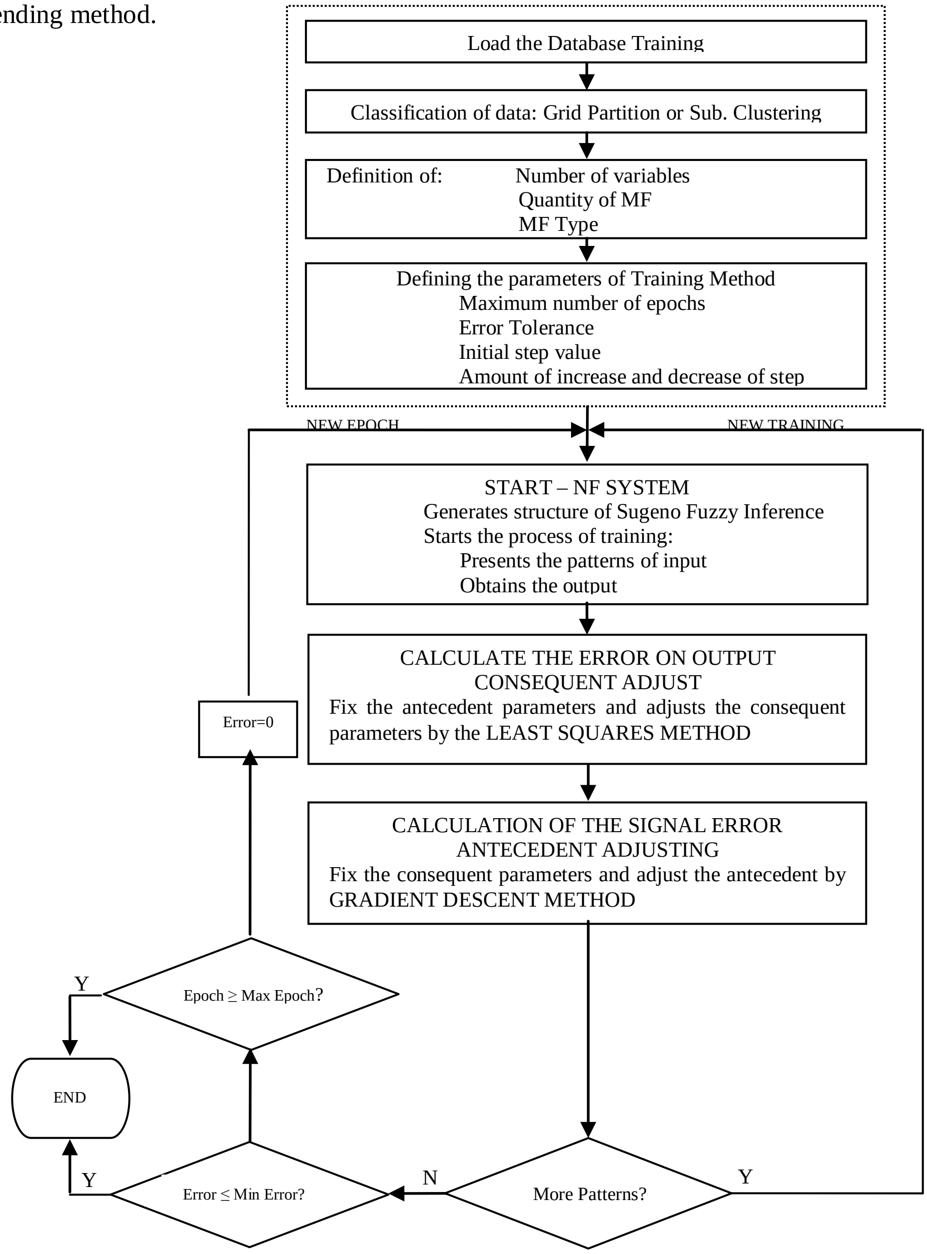

Figure 5 - Block diagram of the Neuro-fuzzy network training algorithm

As the neuro-fuzzy network developed for this work only allows the processing of an output, try to overcome this limit with the use of machinery committee. In this case, using neuro-fuzzy committees.

3.2. Neuro-fuzzy committees

The committee machine is a model based on a combination of techniques for recognition, based on a weighted mix of staff expertise (Haykin, 2001). A mixture of experts is composed of agents J and one neural network (called the combined system) which provides the weighting factors of the output of J agents. The J individual agents are trained in neuro-fuzzy networks and setting the weights of the neural network is performed based on backpropagation error of the committee, through an artificial neural network according to Fig 6.

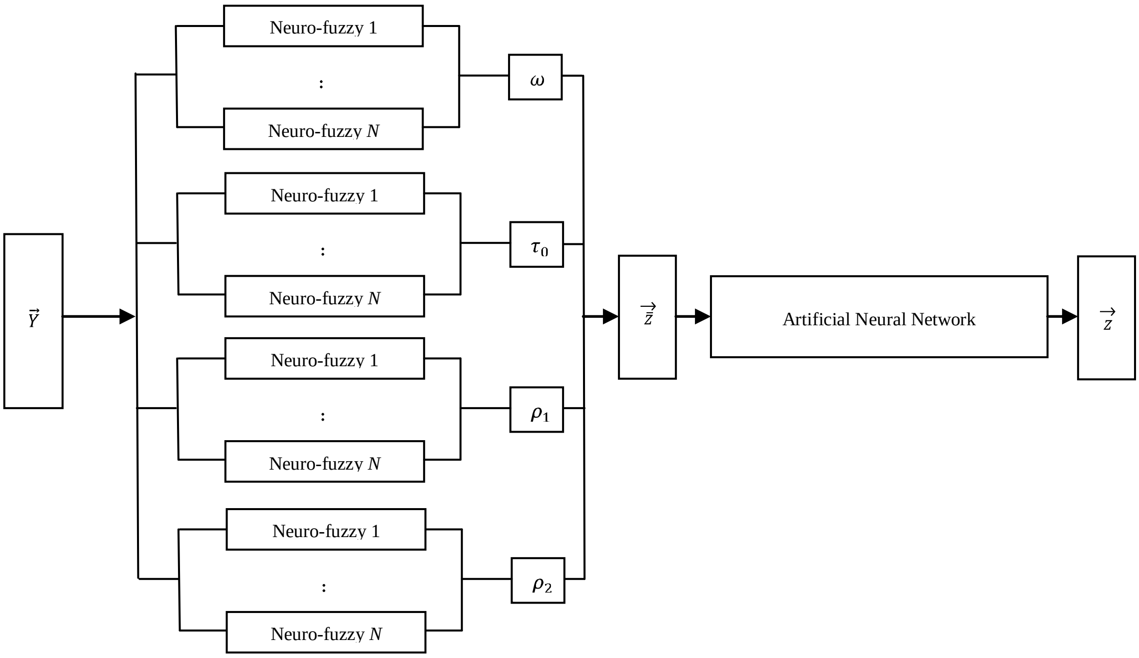

Figure 6: Schematic representation of the committee neuro-fuzzy

3.4. Inverse problem solution

To solve the inverse problem with the determination of optical thickness, the single scattering albedo and diffuse reflectivity coefficients of the boundary inner side of a participant one-dimensional homogeneous medium, using the measures of intensity of radiation leaving the medium participant. The modeling of the problem (direct and inverse) and the mathematical formulations in this section also described in section 2, as represented in Fig 7.

So in summary form, the direct problem is known and the geometry of the medium, the radiative properties and boundary conditions to determine the intensity of radiation that leaves the medium $I(\tau,\mu)$, where τ represents the variable optics (limits $\tau = 0$ and $\tau = \tau_0$) at $\mu = \cos\theta$,, and θ polar angle between the direction of the beam and the axis τ. In this case, ρ_1 and ρ_2 are the reflectivities in the limits $\tau = 0$ and $\tau = \tau_0$, respectively.

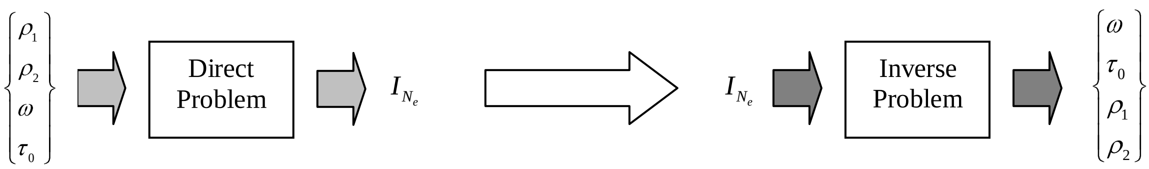

Figure 7: Schematic diagram - Problem with Direct and Inverse Problems

Assuming that the radiative properties of the environment are unknown, which is a severe restriction for the application of direct problem, but there are available experimental data of intensities of radiation leaving the participant medium at limits $\tau = 0$ and $\tau = \tau_0$, and that the number of experimental data is equal to the number of directions used in the discretization N_e the field of polar angle θ, using the method described in section 2 through sorted with discrete Gauss-Legendre quadrature and approximation by finite differences for discretization of the domain space, so that the intensities are measured for each corresponding value of $\mu = \cos\theta$.

Suppose further that the experimental data can be generated by simulation and thus created many pairs as are necessary for use in any technique for determining these parameters, the format is appropriate for their application to that is made possible in inverse form the determination of radiative properties.

4. RESULTS AND DISCUSSION

To validate the solution of the inverse problem of radiative transfer through neuro-fuzzy network, and the committee of the neuro-fuzzy network, the results of these solutions will be compared with results obtained with the direct solution of the problem. Please note that it is necessary to solve the direct problem several times, which requires a high computational effort.

From experimental data of intensities of radiation in this case obtained by simulation, which means leave the participant medium in the limits $\tau = 0$ and $\tau = \tau_0$, and that the number of experimental data is equal to the number of directions N_e used in discretization of the field angle polar θ, so that the intensities are measured for each corresponding value of $\mu = \cos\theta$. May be certain values optical thickness τ_0, single scattering albedo ω and diffuse reflectivity coefficients of the inner side in the participating medium ρ_1 and ρ_2.

Due to restrictions imposed by the neuro-fuzzy system used, as its ability to produce an output only in the architecture of the neuro-fuzzy network, and the need to determine together the diffuse reflectivity (ρ_1 and ρ_2), was chosen to use the combination a hybrid method of neuro-fuzzy networks for each of the two radiative properties to be determined, and a method that will act as the combiner of the machine results of committee.

We constructed four sets of different neuro-fuzzy networks for the determination of each variable radiative, single scattering albedo ω, optical thickness τ_0 and the diffuse reflectivity (ρ_1 and ρ_2) according to the characteristics described in Tab. 1 and represented in Fig. 8.

Table 1: Neuro-fuzzy networks committee parameters.

Neuro-fuzzy	Maximum number of epochs	100
	Tolerance of mean square error	10^{-3}
	Number of membership functions Q_μ by NF network	3
	Number of experimental data N_e by neuro-fuzzy network	6 per sample
	Number of samples of data	100
	Number of rules Q_R	729
	Number of neuro-fuzzy networks	4 for each variable
	Type of membership function	bell shape and triangular
	Training method	hybrid and backpropagation

The results of neuro-fuzzy networks have been used on a machine with a combined committee of Artificial Neural Network (ANN), i.e., the results obtained using neuro-fuzzy networks are submitted to the ANN previously trained, constructed according to the characteristics described in Tab . 2.

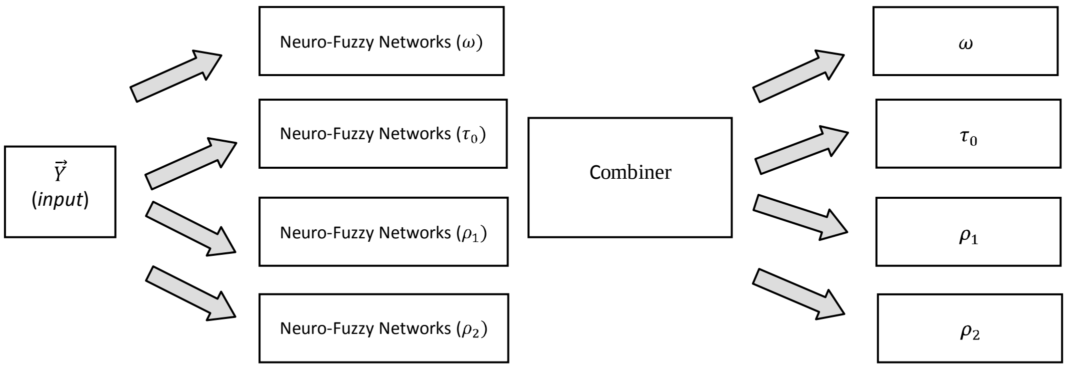

Figure 8: Schematic diagram - Solution of Inverse Problems with Network NF Committees

Table 2: Combiner ANN parameters.

Neural Network	Type of artificial neural network	MLP (Cascade Multi-layer Perceptron)
	Number of hidden layers	2
	Number of neurons in hidden layers	40 e 30
	Number of neurons in output layer	20
	Activation function	Sigmoid
	Training method	Levenberg-Marquardt
	Maximum number of epochs	5000
	Minimum error	10^{-3}
	Learning rate	0,05

For the committee training of neuro-fuzzy networks are used the same 100 samples of data for all networks.

At the end of this phase of neuro-fuzzy networks training, the same data used during this phase are presented to input layer of neural network, producing a second phase of training for the artificial neural network.

Table 3 shows the values of the radiative properties: calculated by the direct method, determined by the neuro-fuzzy networks and the committee and their respective percentage errors for values calculated by the direct method.

Table 3: Results of direct method, of neuro-fuzzy networks and neuro-fuzzy committee from the input data without noise.

	Direct method value	Neuro-fuzzy		Committee	
		output	%error	output	%error
ω	0,1713	0,1789	4,38	0,1647	3,88
τ_0	0,8157	0,6676	18,16	0,8128	0,36
ρ_1	0,9469	0,8393	11,36	0,9481	0,13
ρ_2	0,9354	0,4704	49,72	0,9095	2,77

Looking to evaluate the generalization ability of neuro-fuzzy network, were included in the random noise values of radiative intensities of the data generated according to Tab. 2, with variations between ± 3%, ± 5% and ± 7% and these data were submitted to the network.

The results of the values of the radiative properties: the scattering albedo, optical thickness and coefficient of diffuse reflectivity of the participating medium determined by the neuro-fuzzy networks and also the committee of neuro-fuzzy networks, from the data of the radiative intensities that leave medium with the additive noise of ± 3%, ± 5% ± 7% are presented in Table 4, 5 and 6, respectively. As well, the error percentage of each of these data compared to results calculated by the direct method.

Table 5: Results of: direct method, neuro-fuzzy networks and neuro-fuzzy committee from the input data with noise of ± 3%.

	Direct method value	Neuro-fuzzy		Committee	
		output	%error	output	%error
ω	0,1713	0,1945	13,51	0,1690	1,38
τ_0	0,8157	0,6568	19,49	0,8105	0,64
ρ_1	0,9469	0,8478	10,47	0,9499	0,33
ρ_2	0,9354	0,4699	49,76	0,9104	2,68

Table 6: Results of: direct method, neuro-fuzzy networks and neuro-fuzzy committee from the input data with noise of ± 5%.

	Direct method value	Neuro-fuzzy		Committee	
		output	%error	output	%error
ω	0,1713	0,1888	10,17	0,1703	0,63
τ_0	0,8157	0,7724	5,32	0,8138	0,24
ρ_1	0,9469	0,9998	5,52	0,9623	1,63
ρ_2	0,9354	0,9519	1,77	0,9195	1,70

Table 7: Results of: direct method, neuro-fuzzy networks and neuro-fuzzy committee from the input data with noise of ± 7%.

	Direct method value	Neuro-fuzzy		Committee	
		output	%error	output	%error
ω	0,1713	0,1582	7,68	0,1652	3,59
τ_0	0,8157	0,6253	23,35	0,8521	4,46
ρ_1	0,9469	0,8663	8,51	0,9369	1,05
ρ_2	0,9354	0,4757	49,15	0,9044	3,32

Tables 4 to 7 are presented some comparisons of the results obtained by the neuro-fuzzy networks committee for known values of the radiative properties. Noting that in some cases the values determined by the NF committee combiner with ANN as present values very close to expected values, indicating a good ability to generalize from the host committee of the neuro-fuzzy networks combined with the ANN.

5. CONCLUSIONS

The results of the neuro-fuzzy networks and the committee machines, regardless of their limitations were satisfactory, a goal achieved is the holding of an important characteristic that is the speed of response, because once done the training of the NF committees, results for a new sample is a simple product of the weights of the networks. Unlike other methods for the solution of inverse problems in which a new sample carries a large amount of direct assessments of the problem and a large computational effort.

6. ACKNOWLEDGEMENTS

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