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CHARACTERIZATION OF POLYMER POLYDIMETHYLSILOXANE (PDMS) UNDER SIMPLE SHEAR TEST

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Abstract. *Polydimethylsiloxane (PDMS) is a commercially elastomer used as both a structural component in micro-electromechanical system devices and a stamping material for creating micro- and nanoscale features on surfaces. This paper describes and analyzes the mechanical behavior of polymer polydimethylsiloxane (PDMS) submitted to large deformations without destruction. The goal is to estimate the angular distortion associated with different applied forces, considering a simple shear test. The experimental procedure is carried out using the digital image correlation (DIC) method, which is an optical-numerical experimental approach developed for full-field and non-contact measurements. The material parameters, associated with classical Mooney-Rivlin model, are estimated from experimental data by means of Levenberg-Marquardt method. In addiction, it is proposed a new non-linear model and three new material parameters are determined in the same way.*

Keywords: *Polymer, Hyperelasticity, PDMS, Full-field displacement*

1. INTRODUCTION

Polydimethylsiloxane (PDMS) is the most common and commercially available silicone rubber. Due to important characteristics, such as flexibility and stability, this material has a wide range of applications in mechanical sensors (Kim *et al.*, 2008; Lin *et al.*, 2009), electronic products (Tiercelin *et al.*, 2006; Lee *et al.*, 2009) and medical devises (Lawrence *et al*., 2009). There are many different types of mechanical tests for determining proprieties of polymeric materials (Schneider *et al.*, 2009; Ward and Sweeney, 2004; Mujika *et al.*, 2006; Brown, 2002). There is some special attention to determine the shear modulus. Shear modulus is usually measured at small strains where the stress–strain relationship is essentially linear. There are a number of loading systems which give rise to shear stresses including lap shear, punch shear, torsion and four point loading. Recently, Khan *et al.* (2008) presented a discussion about the characterization of shear deformation inshape memory polymers. A test method for determining the shear modulus of elastomeric bearing was proposed by Topkaya and Yura (2002). Nunes (2009) proposes to analyze the adhesive deformation in the single lap joint specimen and to estimate the shear modulus of Polydimethylsiloxane (PDMS) using the DIC method.

The polymer PDMS presents hyperelastic behavior at room temperature condition. In the work developed by Yu *et al*. (2009), an analysis of the role of vertical component of surface tension of a water droplet on the deformation of membranes and microcantilevers was made using PDMS.

There are many proposed strain energy density expressions in the literature. Some forms of strain-energy functions, which are well tried within the constitutive theory of finite elasticity and frequently employed in the literature can be found in Holzapfel (2000). The most widely cited strain-energy formulations are the Mooney-Rivlin and Ogden models (1997). Treloar (1943) presented a well-developed experimental work. Sasso *et al.* (2008) and Meunier *et al.* (2008) proposed mechanical characterization tests of hyperelastic rubber-like materials using optical methods.

The aim of this work is characterize of the polymer Polydimethylsiloxane (PDMS) submitted to large deformations using a method known as digital image correlation (DIC). In order to do this, an arrangement based on single lap joint was used to generate a simple shear behavior. The angular distortion is associated with different applied load. The material parameters, taking into account classical Mooney-Rivlin model and experimental data, are estimated by means of Levenberg-Marquardt method. In addiction to that, a new model for strain-energy function is proposed to estimate some mechanical parameters.

2. CONSTITUTIVE MODEL OF RUBBER ELASTICITY

A material element $d\mathbf{X}$ in the reference configurations can be transformed, into a material element $d\mathbf{x}$ in the current configuration, using the deformation gradient tensor **F**. The relation between this elements is given by $d\mathbf{X} = \mathbf{F}d\mathbf{x}$. Let us consider the case of simple shear deformation, illustrated in Fig. 1, which the rectangular Cartesian coordinate of any point of deformed element can be written as

$$x_1 = X_1 + \gamma X_2; \quad x_2 = X_2; \quad x_3 = X_3 \tag{1}$$

Using the Eq. (1), the deformation gradient tensor **F** can be expressed as

$$\mathbf{F} = \nabla \mathbf{x} = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

From Eq. (2), the left Cauchy-Green deformation tensor, **B** can be written as

$$\mathbf{B} = \mathbf{F}\mathbf{F}^{\mathbf{T}} = \begin{bmatrix} \gamma^2 + 1 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

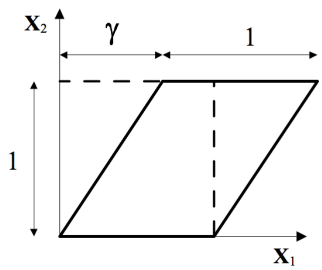

Figure 1. Schematic of simple shear deformation

There are many tensor invariants that can be written in terms of the metric tensor in the undeformed and deformed, as well as the relative stretches. The principal scalar invariants of the left Cauchy-Green deformation tensor can be determined as

$$\begin{aligned} I_1 &= \operatorname{tr} \mathbf{B} = \gamma^2 + 3; \\ I_2 &= \frac{1}{2}\left[(\operatorname{tr} \mathbf{B})^2 - \operatorname{tr} \mathbf{B}^2 \right] = \gamma^2 + 3 \\ I_3 &= \det \mathbf{B} = 1 \end{aligned} \tag{4}$$

In this case of simple shear deformation, we take $I_1 = I_2 = \gamma^2 + 3$. For an incompressible rubber-like material I_3 is equal to 1. The principal physical Lagrangian or engineering stresses is used to establish the constitutive relationship for a hyperelastic material and it is given by

$$\tau = \frac{\partial W}{\partial \gamma} \tag{5}$$

where the strain energy function W can be expressed as a function of the shear strain. Mooney-Rivlin observed that rubber response is linear under simple shear loading conditions. The strain energy function depends on the first two invariants and the first order model is defined by

$$W(I_1, I_2) = c_{10}(I_1 - 3) + c_{01}(I_2 - 3) \tag{6}$$

where c_{10} and c_{01} are the material parameters.

The purpose here is to find a close form strain-energy function, which present a simple mathematical structure to describe the elastic behavior of elastomers. For this, the proposed model is given by

$$W(I) = c_1(I - 3) + c_2(I - 3)^{1/2} + c_3(I - 3)^{3/4} \tag{7}$$

where $$I(\gamma) = I_1(\gamma) = I_2(\gamma) = \gamma^2 + 3 \\ I_3(\gamma) = 1 \qquad (8)$$

Substituting Eqs. (6) and (7) into Eq. (5), we have the constitutive equation using Mooney-Rivlin and the proposed models, respectively

$$\tau(\gamma) = 2(c_{10} + c_{01})\gamma \qquad (9)$$

$$\tau(\gamma) = 2c_1\gamma + c_2 + \frac{3}{2}c_3\sqrt{\gamma} \qquad (10)$$

where the shear modulus, μ, is $2(c_{10} + c_{01})$

3. EXPERIMENTAL PROCEDURE

The experimental procedure was conducted to determine the angular distortion associate to different loads using a simple shearing mechanism and DIC method. The images of single lap joint specimen were captured and processed by means of DIC program to obtain full-field displacements. This is well established experimentally (Nunes, 2010).

3.1. Material and methods

In this work, a single lap joint is used to transfer load from one adherend to another by a simple shearing mechanism. The stiffness of the adherends is much greater than the adhesives, implying that the adherends do not deform and the adhesives only deform in shear. The geometry is schematically illustrated in Fig. 2.

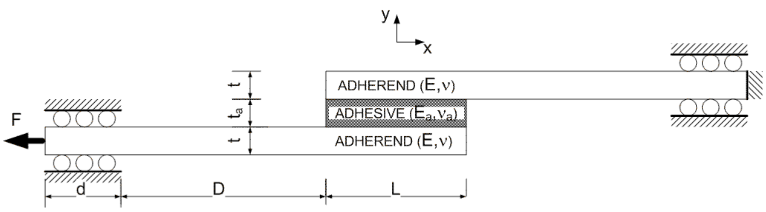

Figure 2. Schematic of simple shear deformation

In order to obtain the experimental results, the geometric shape for the single lap joint, schematically illustrated in Fig. 2, was considered with the following data: (a) Different applied forces, F, from 0 to 290 N; (b) length of restraint against transversal motion, d = 25 mm; segment of length, D = 50 mm; joint length, L = 30 mm; joint width, w = 25 mm; adherend and adhesive thickness, t = 1.9 mm and t_a = 1.6 mm, respectively. The upper and lower adherends have the same characteristics, i.e., Steel A36 and the material of adhesive is Silicone rubber (Polydimethylsiloxane).

The bonded region of the adherend received a superficial treatment. The procedure consisted of abrading the adherend surface at the overlap region with fine sandpaper and cleaning with acetone before the application of the adhesive. In order to control and to guarantee adhesive thickness, the test specimen (single-lap joint) was manufactured in a mold (apparatus). The applied cure cycle was 48 h at room temperature.

The displacement measurements were carried out using digital image correlation (DIC) method. This method is an optical-numerical full-field surface displacement measurement (Nunes, 2010). It is based on a comparison between two images of a specimen coated by a random speckled pattern in the undeformed and in the deformed states. Its special merits encompass non-contact measurements, simple optic setups, no special preparation of specimens and no special illumination.

3.2. Experimental set up

The experimental arrangement for conducting shear testing involves an apparatus developed for applying strain in a single lap joint, a CCD camera set perpendicularly to the specimen and a computer for capturing and processing the images, as shown in Fig. 3. The single lap joint, fixed in the strain apparatus, was covered with painted speckles (random black and white pattern). It is in agreement with the geometrical model, as seen in Fig. 2. The CCD camera (Sony XCD-SX910) used to record the images of the specimen has a resolution of 1376x1024 pixels. In this experimental configuration, one pixel of the CCD camera corresponds to an area approximately equal to 4.65×4.65 μm^2 on the specimen. The basic idea of experimental procedure is to take the images of specimen in the undeformed and deformed states.

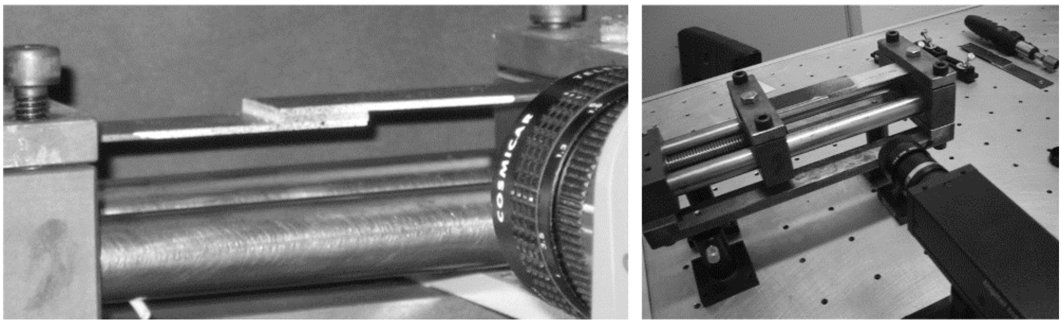

Figure 3. Experimental arrangement

4. RESULTS AND DISCUSSION

Figure (4) shows the results of full-field displacements u(x,y) and v(x,y) obtained when a load equal to 50 N is applied. These fields represent a surface area at adhesive region. This example was chosen to show that the displacement v(x,y) can be neglected when compared with the displacement u(x,y).

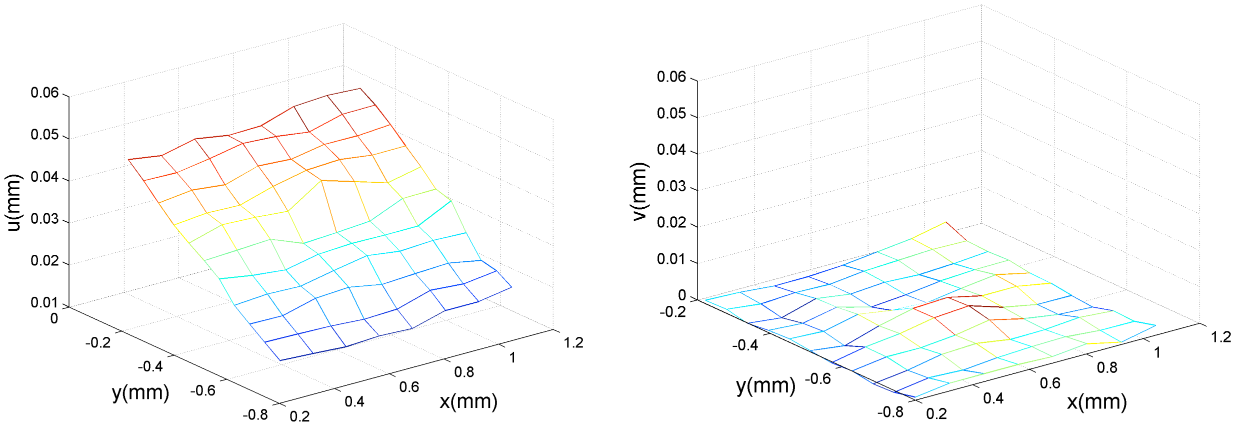

Figure 4. Full-field displacements u(x,y) and v(x,y) for applied load equal to 50 N.

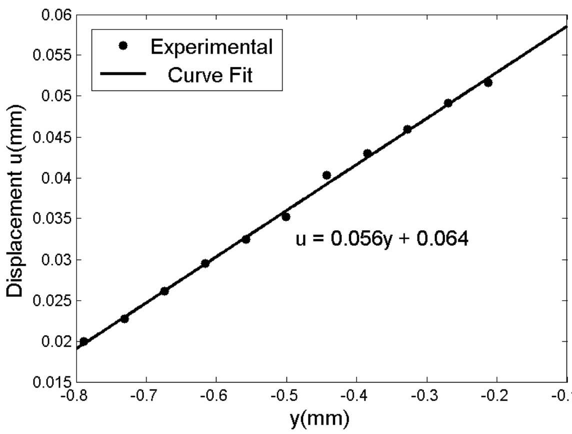

Figure 5. Average value of displacement u(x,y) along direction x as a function of y for applied load equal to 50 N.

In order to estimate the angular distortion, the average value of displacement u(x,y) along direction x as a function of y was considered, as illustrated in Fig. (5). Curve fit data were taken into account to evaluate the angular distortion, i.e., $\partial u/\partial y$. Due to smaller value of displacement v(x,y) the component $\partial v/\partial x$ can be neglected. Thus the shear strain can be defined as $\partial u/\partial y$. In Fig. (5), this value is 0.056.

Figure (6) shows the results of the shear stress vs. shear strain. It is possible to observe a non-linearity behavior in shear stress-strain curve. This behavior was also observed by Lahellec *et al.* (2004).

The objective is to find values for the four material parameters μ, c_1, c_2 and c_3 in Eqs. (9) and (10), associate to Mooney-Rivlin and proposed models, that give the best fit to the experimental data. These parameters are estimated using Levenberg-Marquardt method (Nunes *et. al.*, 2007), which is a well-known and powerful iterative method for solving nonlinear least squares problems of parameter estimation. These parameters are shows in Tab. 1

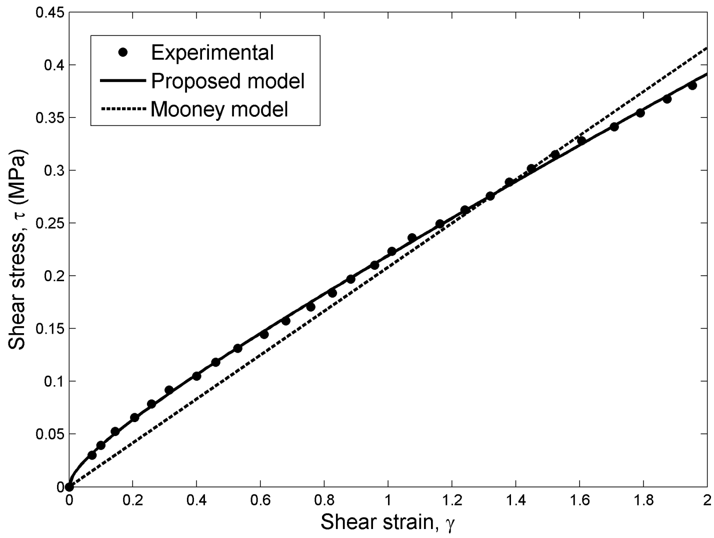

Figure 6. Comparison between experimental data, Mooney model and proposed model.

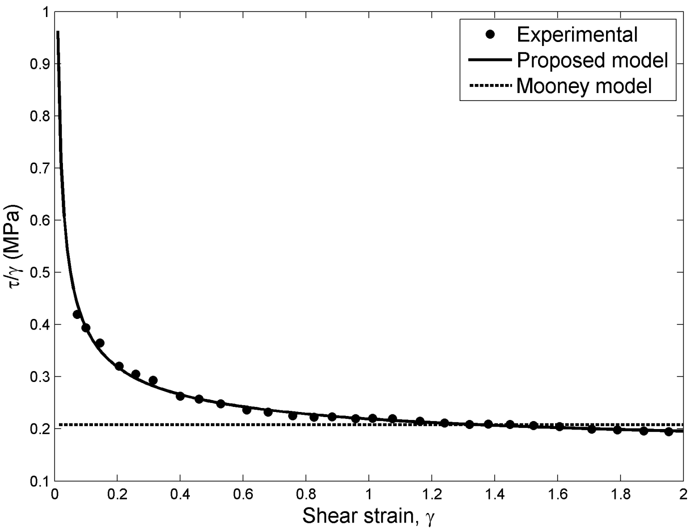

Figure 7. Shear stress divided by shear strain vs. shear strain.

The shear stress divided by shear strain is shown in Fig. (7). It can be see the good agreement between the experimental data and proposed model. However, there is a little discordance between experimental dada and Mooney-Rivlin model.

After fitting data using Mooney-Rivlin and proposed models by means of Levenberg-Marquardt method, the goodness of fit is evaluated. Table 1 shows the goodness of fit statistics for parametric models: R-square, the sum of squares due to error (SSE) and root mean squared error (RMSE).

Table 1. Comparison between Mooney-Rivlin and proposed model

Model	Parameters (MPa)	R-square	SSE	RMSE
Mooney-Rivlin	$\mu = 2(c_{10}+c_{01}) = 0.21$	0.9746	$9\text{x}10^9$	$1.8\text{x}10^4$
Proposed	$c_1 = 0.07$; $c_2 = 295.5\text{x}10^{-6}$ and $c_3 = 0.05$	0.9994	$0.2\text{x}10^9$	$0.3\text{x}10^4$

A better understanding of the results can be obtained plotting the residue of shear stress value. The residuals from a fitted model are defined as the differences between the response data and the fit to the response data at each predictor value. The results of residue are illustrated in Fig. 8. Clearly, the calculate residue of proposed model is smaller than the Mooney-Rivlin model.

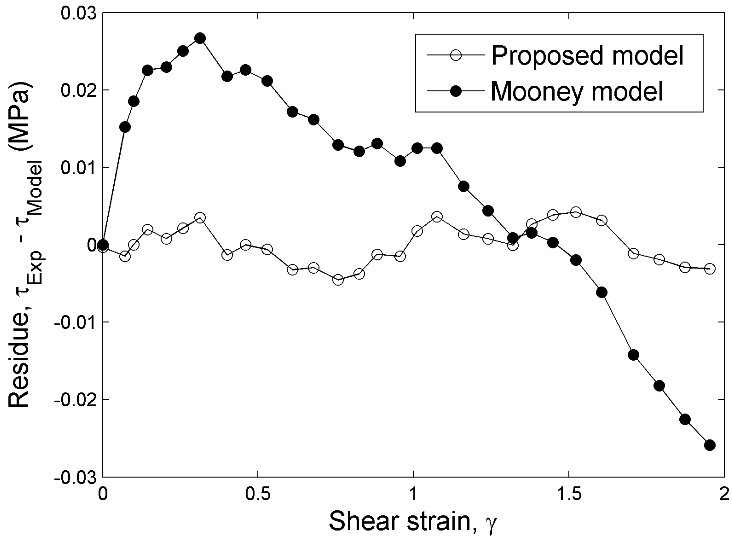

Figure 8. Residue, $\tau_{Exp} - \tau_{Model}$, for different shear strain: comparison between Mooney and proposed model

5. CONCLUSIONS

In the present work, the experimental data of shear strain was estimated by means of the Digital Image Correlation method. The purpose was taken the value of angular distortion associate with different applied forces, considering a simple shear test. In literature, the shear stress-strain behavior is linear. However, observing the results, it is possible to note a non-linear behavior in the experimental data. In order to find a best model to the experimental data, the classical Mooney-Rivlin and proposed models was investigated. As can be observed, there is a significant difference between the experimental data and the classic Mooney model, whereas the proposed model is much more closer. For future work the authors aim to generate more experimental data, varying geometrical parameters.

6. ACKNOWLEDGEMENTS

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