



THERMOELASTIC STRUCTURES IN TOPOLOGY OPTIMIZATION PROCESS

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Abstract: *This work proposes a formulation for optimization of 2D-structure layouts submitted to mechanic and thermal shipments and applied an h-adaptive filter process which conduced to computational low spend and high definition structural layouts. The main goal of the formulation is to minimize the structure mass submitted to an effective state of stress of von Mises's, with stability and lateral restriction variants. A criterion of global measurement was used for intents a parametric condition of stress fields. To avoid singularity problems was considerate a release on the stress constraint. On the optimization was used a material approach where the homogenized constructive equation was function of the material relative density. The intermediary density effective properties were represented for a SIMP-type artificial model. The problem was simplified by use of the method of finite elements of Galerkin using triangles with linear Lagrangian basis. On the solution of the optimization problem, was applied the Augmented Lagrangian Method, that consists on minimum problem sequence solution with box-type constraints, resolved by a 2nd order projection method which uses the method of the quasi-Newton without memory, during the problem process solution. This process reduces computational expends showing be more effective and solid. The results materialize more refined layouts with accurate topologic and shape of structure definitions. On the other hand formulation of mass minimization with global stress criterion provides to modeling "ready" structural layouts, with violation of the criterion of homogeneous distributed stress.*

Keywords: *Topology optimization; Thermoelasticity; Plane Stress; FEM; Augmented Lagrangian.*

1. INTRODUCTION

The reduction of the cost of manufacturing a given component or product may be obtained by applying some optimization tool. In the particular case of components or products obtained by an injection process (plastic or metal powder), the possibility to consider complex geometry allow us to explore the flexibility of the process by designing optimized molds with an optimum topology for the domain of the component.

One of the most difficult decisions in the designing phase is to specify the layout of the geometry of the component. Once the layout or topology of the component is defined we may concentrate in the definition of the optimum shape of the domain, sizing of some additional geometric parameters used to define the model and some material properties, (Suzuki and Kikuchi, 1990; Suzuki and Kikuchi, 1991; Bendsoe and Kikuchi, 1998; Bendsoe, 1995).

In general, the appropriate choice of the layout is strongly dependent of the designer, what implies in the necessity of a designer with a large practical experience. The decision process associated with the definition of the optimum layout of component may be done automatically by employing a topology optimization software, (Bendsoe and Sigmund, 2003; Bendsoe and Rodrigues, 1991; Bendsoe *et al.* 1993).

In this work, the layout optimization is done by considering a Solid Isotropic Microstructure with Penalization (SIMP). The material density function ρ are the design parameters and varies continuously from 0 to 1, taking the

value of 1.0 for a solid material and 0.0 for a void material, Costa Jr. and Alves (2003). To avoid numerical singularity, the lower bound of material, ρ_{\min} , is introduced as

$$0 < \rho_{\min} \leq \rho \leq 1. \quad (1)$$

2. FORMULATION OF THE PROBLEM

2.1. Determination of the Thermo Mechanical Problem

The thermal problem considered in this work is illustrated in Fig. 1.

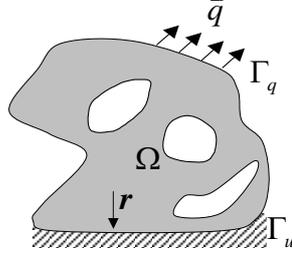


Figure 1. Definition of the Thermal Problem.

Here $\partial\Omega = \Gamma_T \cup \Gamma_q$; $\Gamma_T \cap \Gamma_q = \emptyset$. Denoting by Γ_T and Γ_q the part of the boundary where the temperature and the heat flux are prescribed respectively. At this point, we define the set of admissible temperatures, W_T , and the set of the temperature variations, Var_T , to be given as: $W_T = \{T \mid T \in H^1(\Omega) \text{ and } T = \bar{T} \text{ at } \Gamma_T\}$ and $Var_T = \{\hat{T} \mid \hat{T} \in H_0^1(\Omega)\}$. We consider the source/sink to be given by a convection heat transfer from the body to a fluid, *i. e.*, $r = -h(T - T_f)$ where T_f denotes the temperature of the fluid and h the convection heat transfer coefficient. Notice that, if $T > T_f$ then heat is removed from the body and if $T < T_f$ heat is given to the body, (Alves and Alves, 1999; Silva, 2007).

The weak formulation of the thermal problem may be stated as: Let $T = T^* + T_p$, where $T_p \in W_T$ is a known field. The problem consists in the determination of $T^* \in Var_T$ such that,

$$a_T(T^*, \hat{T}) = l_T(\hat{T}), \quad \forall \hat{T} \in Var_T \quad (2)$$

where

$$a_T(T^*, \hat{T}) = \int_{\Omega} \mathbf{K}^H \nabla T^* \cdot \nabla \hat{T} d\Omega + \int_{\Omega} h T^* \hat{T} d\Omega \quad (3)$$

and

$$l_T(\hat{T}) = \int_{\Omega} h(T_f - T_p)\hat{T} d\Omega + \int_{\Gamma_q} \bar{q} \hat{T} d\Gamma - \int_{\Omega} \mathbf{K}^H \nabla T_p \cdot \nabla \hat{T} d\Omega \quad (4)$$

Here, the conductivity matrix $\mathbf{K}^H = k^H \mathbf{I}$, where k^H is the homogenized thermal conductivity of the material that is porous material dependent, such that

$$k^H = \rho^\eta k \quad (5)$$

where k is the conductivity parameter for the fully density material, ρ the relative density material and η is the SIMP penalty parameter, (Cho and Choi, 2005; Rodrigues and Fernandes, 1993).

The mechanical problem considered in this work is illustrated in Fig. 2.

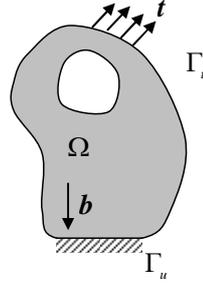


Figure 2. Definition of the Mechanical Problem.

Here $\partial\Omega = \Gamma_u \cup \Gamma_t$; $\Gamma_u \cap \Gamma_t = \emptyset$. Here, Γ_u and Γ_t represent the part of the boundary where the displacement and the traction are prescribed respectively. At this point we define the set of admissible displacements, W_u , and the set of the displacement variations, Var_u , to be given as: $W_u = \{ \mathbf{u} \mid \mathbf{u} \in (H^1(\Omega))^3 \text{ and } \mathbf{u} = \bar{\mathbf{u}} \text{ at } \Gamma_u \}$ and $Var_T = \{ u \mid u \in (H_0^1(\Omega))^3 \}$. Here, for simplicity we consider $\bar{\mathbf{u}} = 0$. As a result, $W_u = Var_u$.

The weak formulation of the mechanical problem may be stated as: Let $T \in W_T$ be the solution of the problem. Then, the problem consists in the determination of $\mathbf{u} \in W_u$ such that

$$a_u(\mathbf{u}, \mathbf{v}) = l_u(\mathbf{v}), \quad \forall \mathbf{v} \in Var_u. \quad (6)$$

where

$$a_u(\mathbf{u}, \mathbf{v}) = \int_{\Omega} \boldsymbol{\sigma}(\mathbf{u}) \cdot \boldsymbol{\varepsilon}(\mathbf{v}) d\Omega, \quad (7)$$

$$l_u(\mathbf{v}) = \int_{\Omega} \mathbf{b} \cdot \mathbf{v} d\Omega + \int_{\Gamma_t} \mathbf{t} \cdot \mathbf{v} d\Gamma, \quad (8)$$

and

$$\boldsymbol{\sigma}(\mathbf{u}) = \mathbf{D}^H \boldsymbol{\varepsilon}(\mathbf{u}) - (T - T_0) \mathbf{B}^H \quad (9)$$

Now, since we consider the material (matrix) to be isotropic, we have:

$$B_{ij} = B_0 \delta_{ij}, \quad B_0 = \frac{E\alpha}{1-2\nu}. \quad (10)$$

Here, α is the linear thermal expansion coefficient, $(T - T_0) B_{ij}$ is the thermal stress tensor and \mathbf{D} is the generalized Hooke's law for a linear elastic body, (Brahim-Otsmane *et al.*, 1989; Francfort, 1983). Moreover,

$$D_{ijrs} = \lambda \delta_{ij} \delta_{rs} + \mu (\delta_{ir} \delta_{js} + \delta_{is} \delta_{jr}) \quad (11)$$

with

$$\lambda = \frac{\nu E}{(1-\nu)(1-2\nu)}, \quad \mu = G = \frac{E}{2(1-\nu)}. \quad (12)$$

Where λ and μ are the Lamé's constants, ν is the Poisson's ratio and E is the Young modulus. The temperature T_0 is the reference temperature of the body.

The constitutive matrix \mathbf{D}^H adapted to intermediary density material, proposed by Cho and Choi (2005) is given

$$\mathbf{D}_{ijkl}^H = \rho^n \mathbf{D}_{ijkl} \quad (13)$$

2.2. Formulation of the Problem

The objective of this work is to determine the optimum layout of the structure obtained as solution to an optimization problem. The optimization problem consists in the minimization of the mass of the structure subjected to an effective von Mises equivalent stress and side constraints. The design variable is the relative density of the material,

represented by ρ , for dealing with the problem of stress criteria in mass minimization was used SIMP exponential penalty system to describe the constitutive relation of the material, which is used $\eta = 3$, with this choice, proposed by Sigmund and Petersson (1998), we get a description of feasible microstructure material.

The problem may than be formulated as:

$$\min_{\rho(\mathbf{x})} \int_{\Omega} \rho d\Omega, \quad (14)$$

such that

$$\sigma_{eq}^*(\rho(\mathbf{x}), \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T), T) - \sigma_y \leq 0 \quad (15)$$

$$\rho_{\inf} - \rho \leq 0 \quad (16)$$

$$\rho - \rho_{\sup} \leq 0, \quad \forall \mathbf{x} \in \Omega \quad (17)$$

where $\mathbf{u}(\mathbf{a})$ and $T(\mathbf{a})$ are obtained as a solution to the problem:

$$a_T(T^*, \hat{T}) = l_T(\hat{T}), \quad \forall \hat{T} \in Var_T \quad (18)$$

$$a_u(\mathbf{u}, \mathbf{v}) = l_u(\mathbf{v}), \quad \forall \mathbf{v} \in Var_u \quad (19)$$

and $T = T^* + T_p$, for a given $T_p \in W_T$. The effective von Mises stress, for this microstructure is considered to be given as:

$$\sigma_{eq}^* = \frac{\sigma_{eq}}{\rho^\eta}. \quad (20)$$

The Karush-Kuhn-Tucker necessary optimality conditions associated with this problem is given by: Let $L(\bullet)$ denote the lagrangian functional associated with the problem, *i. e.*,

$$L(\rho, \mathbf{u}, T^*, \eta, \lambda_i, \lambda_s) = \int_{\Omega} \rho d\Omega + \int_{\Omega} \eta (\sigma_{eq}^*(\rho, \mathbf{u}(\rho, \mathbf{x}, T^*), T^*) - \sigma_y) d\Omega + \int_{\Omega} \lambda_i (\rho_{\inf} - \rho) d\Omega + \int_{\Omega} \lambda_s (\rho - \rho_{\sup}) d\Omega, \quad (21)$$

where η , λ_i , and λ_s are the Lagrange multipliers associated with the inequality constraints, Costa Jr. and Alves (2003). Then, the optimality conditions are given by:

$$\eta \geq 0, \quad \lambda_i \geq 0, \quad \lambda_s \geq 0, \quad (22)$$

$$\eta (\sigma_{eq} - \sigma_y) = 0, \quad \lambda_i (\rho_{\inf} - \rho) = 0, \quad \lambda_s (\rho - \rho_{\sup}) = 0, \quad (23)$$

$$\sigma_{eq} - \sigma_y \leq 0, \quad \rho_{\inf} - \rho \leq 0, \quad \rho - \rho_{\sup} \leq 0 \quad \text{and} \quad (24)$$

$$1 + \eta \frac{\partial \sigma_{eq}^*}{\partial \rho} - \lambda_i + \lambda_s = 0, \quad \forall \mathbf{x} \in \Omega. \quad (25)$$

2.3. Stress Singularity Problem

In order to open the degenerated parts of the design space with the possibility of creating or removing holes without violating the effective stress constraint we apply the κ - relaxation technique (Duysinx, 1998; Duysinx and Sigmund, 1998; Duysinx and Bendsoe, 1998). In this work, we implement an automatic and systematic strategy to reduce the initial perturbation parameter κ . The stress relaxation parameter is decremented as we get closer to the solution. Now, let $\rho_o = 1 - a_{\inf}^2$ be the relative density associated with the full material condition. Then, the relaxed adimensionalized effective stress constraint may be written as:

$$g(\rho(\mathbf{x}), \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T), T) = \rho(\mathbf{x}) \left(\frac{\sigma_{eq}^*(\rho(\mathbf{x}), \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T), T)}{\sigma_y} - 1 \right) + \kappa(\rho_{sup} - \rho(\mathbf{x})) \leq 0. \quad (26)$$

From this consideration, the relaxed minimization problem may be formatted as:

$$\min_{\rho} \int_{\Omega} \rho d\Omega \quad (27)$$

such that

$$\rho \left(\frac{\sigma_{eq}^*}{\sigma_y} - 1 \right) + \kappa(\rho_{sup} - \rho) \leq 0 \quad (28)$$

$$\rho_{inf} - \rho \leq 0 \quad (29)$$

$$\rho - \rho_{sup} \leq 0, \quad \forall \mathbf{x} \in \Omega \quad (30)$$

3. DISCRETIZATION OF THE PROBLEM

In order to solve the thermo-mechanical problem we apply the Galerkin Finite Element Method. Moreover, we consider the material density associated with each finite element linearly distributed. Consequently, the material properties related to given element are characterized by a single microstructure. Thus, for each element we have a design variable “a” which represents the size of the void of the microstructure that fully represents the given finite element material properties. From this consideration, the number of design variables is given by the number of finite elements in the mesh.

Furthermore, we make use of the slope-constrained conditions proposed by Petersson and Sigmund (1998). These conditions are employed in order to ensure the existence of a solution to the layout optimization problem and to eliminate the well-known checkerboard instability problem (Bendsoe, 1995; Sigmund and Petersson, 1998), that occur in the Galerkin finite element discretization, when using a low order interpolation base function, in the approximation space. Thus,

$$\left(\frac{\partial \rho}{\partial x} \right)^2 \leq C_x^2 \quad \text{and} \quad \left(\frac{\partial \rho}{\partial y} \right)^2 \leq C_y^2 \quad (31)$$

Here, the constants C_x and C_y define the bounds for the components of the gradient of the relative density. These bounds are imposed component wise with the objective of properly imposing a symmetry condition, which may be used in some particular cases.

The discretized problem may be formulated as:

$$\min_{\rho} \int_{\Omega} \rho d\Omega \quad (32)$$

such that

$$\frac{\sigma_{eq}^*(\rho(\mathbf{x}), \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T), T)}{\sigma_y} + \kappa \left(\frac{1}{\rho_0} - \frac{1}{\rho} \right) - 1 \leq 0, \quad \forall \mathbf{x} \in \Omega \quad (33)$$

and

$$\rho_{inf} - \rho_i \leq 0, \quad (34)$$

$$\rho_i - \rho_{sup} \leq 0, \quad i=1, \dots, n \quad (\text{number of nodes in the mesh}) \quad (35)$$

Notice that, the effective stress constraint is a parametric constraint that must be satisfied for $\forall \mathbf{x} \in \Omega$. In order to handle this parametric constraint we relax the pointwise criteria and consider a global integrated constraint. This can be done by replacing a parametric constraint of the type

$$g(\rho(\mathbf{x}), \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T), T^*(\rho)) \leq 0, \quad \forall \mathbf{x} \in \Omega \quad (36)$$

by the following associated global constraint:

$$\bar{g}(\rho(\mathbf{x}), \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T), T^*(\rho)) = \left\{ \frac{1}{\Omega} \int_{\Omega} \langle \rho(\mathbf{x}), \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T), T^*(\rho) \rangle^p d\Omega \right\}^{1/p} = 0. \quad (37)$$

Here, in order to enforce the point wise constraint we must consider $p \rightarrow +\infty$. However, for practical purposes we consider $p = 2$ where $\langle g(\bullet) \rangle$ denotes the positive part of the function $g(\bullet)$, i. e., $\langle g(\bullet) \rangle = \max\{0, g(\bullet)\}$.

3.1. Formulation of the Discretized Problem

The discretized formulation of the relaxed problem may be stated as: Determine $\rho \in \wp$, with $\wp = \{\rho \in \mathbb{R}^n \mid \rho_i^{\text{inf}} \leq \rho_i \leq \rho_i^{\text{sup}}, i = 1, \dots, n\}$, where n is the total number of nodes, so that it is the solution of:

$$\min_{\rho} \int_{\Omega} \rho d\Omega. \quad (38)$$

such that

$$\bar{g}(\rho(\mathbf{x}), \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T), T^*(\rho)) = \left\{ \frac{1}{\Omega} \int_{\Omega} \langle g(\rho(\mathbf{x}), \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T), T^*(\rho)) \rangle^p d\Omega \right\}^{1/p} = 0, \quad (39)$$

$$\rho_{\text{inf}} - \rho_i \leq 0, \quad (40)$$

$$\rho_i - \rho_{\text{sup}} \leq 0, \quad i = 1, \dots, n. \quad (41)$$

$$g_{2e-1}(\rho) = \frac{1}{\beta} \left[\left(\frac{\partial \rho}{\partial x} \right)^2 - C_x^2 \right] \leq 0, \quad (42)$$

$$g_{2e}(\rho) = \frac{1}{\beta} \left[\left(\frac{\partial \rho}{\partial y} \right)^2 - C_y^2 \right] \leq 0, \quad e = 1, \dots, n_e, \quad (43)$$

denoting by n_e is the total number of elements. The values $\mathbf{u}(\rho)$ and $T^*(\rho)$ are the solution of:

$$a_T(T^*, \hat{T}) = \rho_T(\hat{T}), \quad \forall \hat{T} \in \text{Var}_T^n \subset \text{Var}_T \quad \text{and} \quad (44)$$

$$a_u(\mathbf{u}, \mathbf{v}) = l_u(\mathbf{v}), \quad \forall \mathbf{v} \in \text{Var}_u^n \subset \text{Var}_u \quad (45)$$

4. ALGORITHM

We are using a bound constrained Truncated-Newton method. The Truncated-Newton method is preconditioned by a limited-memory Quasi-Newton method with a further diagonal scaling.

5. NUMERICAL APPLICATIONS

5.1. Problem Case (1)

Here we consider the problem illustrated in Fig. 3, which the material properties (stainless steel AISI 304) to be given as: Young Modulus $E = 193 \text{ GPa}$, Poisson's ratio $\nu = 0.29$. The distributed load, $t = 207,000 \text{ kN/m}$. The reference temperature of the body is $T_r = 20^\circ\text{C}$, the temperature of the fluid is $T_f = 25^\circ\text{C}$ and the prescribed temperature at the clamped edge is $T_p = 100^\circ\text{C}$. The yield stress, $S_y = 207 \text{ MPa}$. The heat conductivity of the material, $k = 16.6 \text{ W/m}^\circ\text{C}$. The coefficient $\beta = 17 \times 10^{-6} \text{ m/m}^\circ\text{C}$. The convection heat transfer coefficient, $h = 5 \text{ W/m}^2\text{C}$.

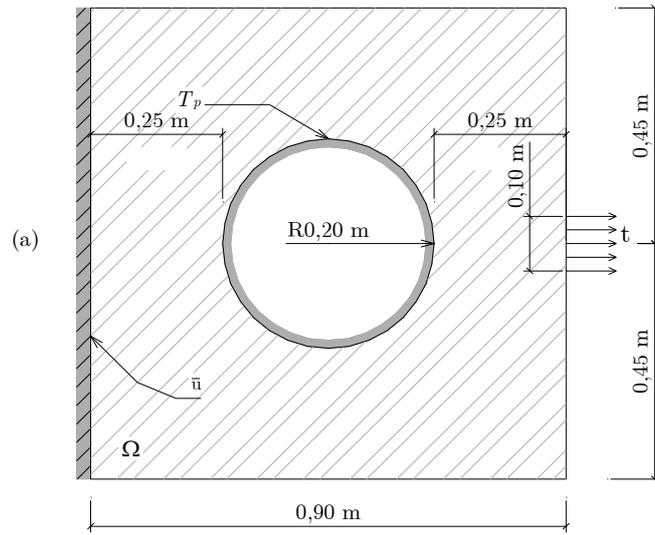


Figure 3. Definition of the Problem Case (1).

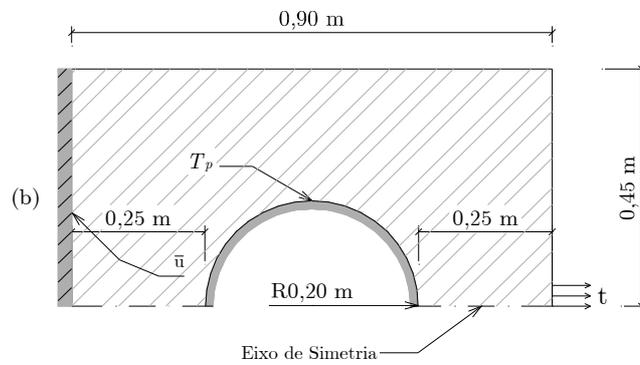


Figure 4. Symmetrical Model of the Problem Case (1) to be solved.

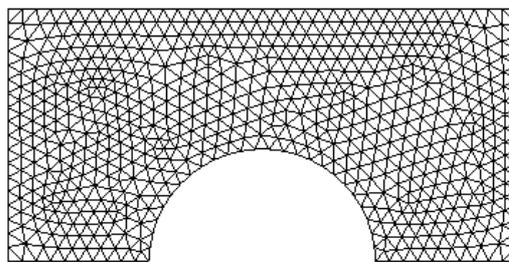


Figure 5. Mesh with 1201 elements and 660 nodes.

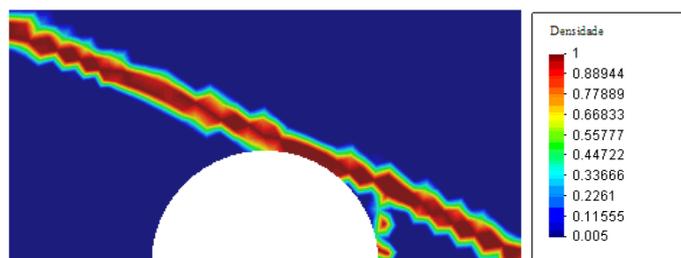


Figure 6. Optimum mass distribution.

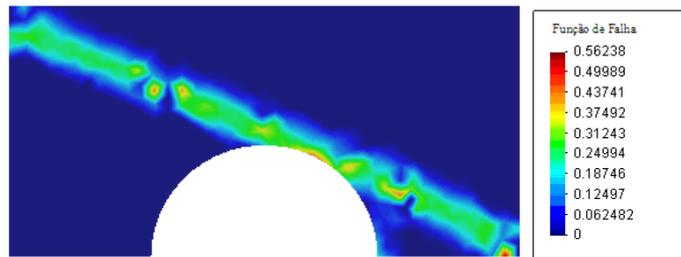


Figure 7. Failure function.

5.2. Problem Case (2)

Here we consider the problem illustrated in Fig. 8, which the material properties (stainless steel AISI 304) to be given as: Young Modulus $E = 193\text{GPa}$, Poisson's ratio $\nu = 0.29$. The distributed load, $t = 207,000\text{kN/m}$. The reference temperature of the body is $T_R = 20^\circ\text{C}$, the temperature of the fluid is $T_f = 25^\circ\text{C}$ and the prescribed temperature at the clamped edge is $T_p = 100^\circ\text{C}$. The yield stress, $S_y = 207\text{MPa}$. The heat conductivity of the material, $k = 16.6\text{W/m}^\circ\text{C}$. The coefficient $\beta = 17 \times 10^{-6}\text{m/m}^\circ\text{C}$. The convection heat transfer coefficient, $h = 5\text{W/m}^2^\circ\text{C}$.

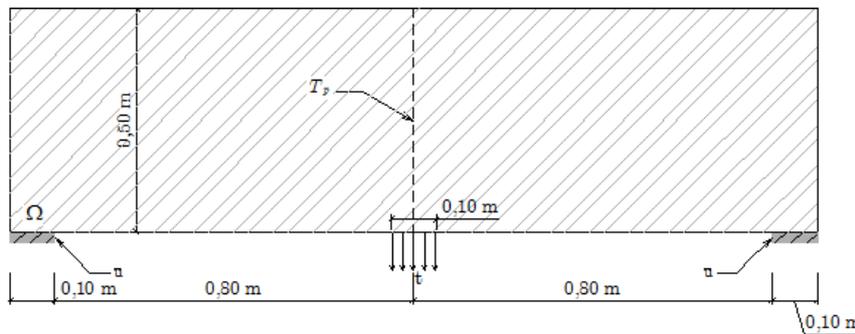


Figure 8. Definition of the Problem Case (2).

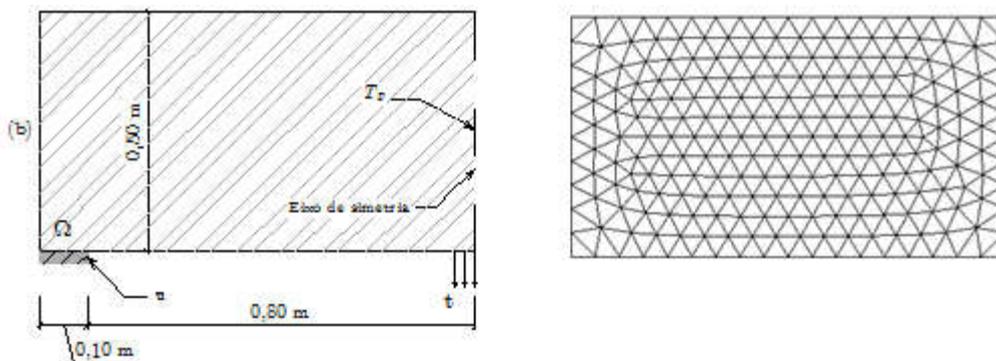


Figure 9. Symmetrical Model of the Problem Case (2) to be solved and mesh with 416 elements and 237 nodes.

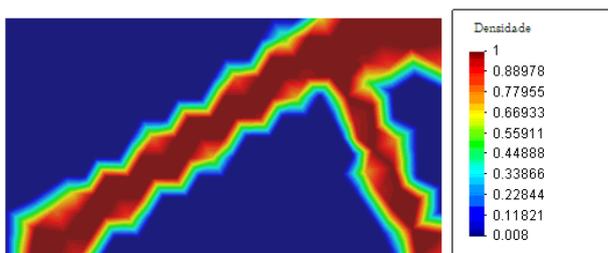


Figure 10. Optimum mass distribution.

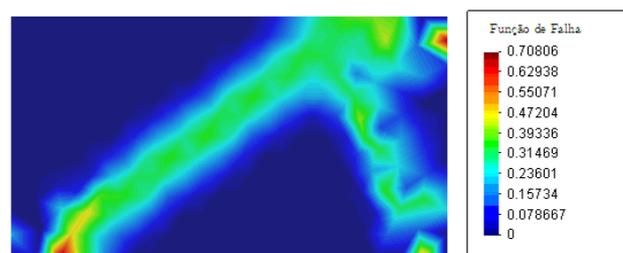


Figure 11. Failure function.

5.3. Problem Case (3)

Here we consider the problem illustrated in Fig. 12, which the material properties (stainless steel AISI 304) to be given as: Young Modulus $E = 193 \text{ GPa}$, Poisson's ratio $\nu = 0.29$. The distributed load, $t = 207,000 \text{ kN/m}$. The reference temperature of the body is $T_R = 20^\circ\text{C}$, the temperature of the fluid is $T_f = 25^\circ\text{C}$ and the prescribed temperature at the clamped edge is $T_p = 100^\circ\text{C}$. The yield stress, $S_y = 207 \text{ MPa}$. The heat conductivity of the material, $k = 16.6 \text{ W/m}^\circ\text{C}$. The coefficient $\beta = 17 \times 10^{-6} \text{ m/m}^\circ\text{C}$. The convection heat transfer coefficient, $h = 5 \text{ W/m}^2^\circ\text{C}$.

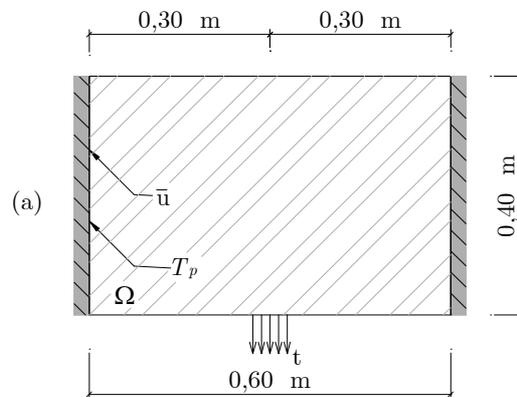


Figure 12. Definition of the Problem Case (3).

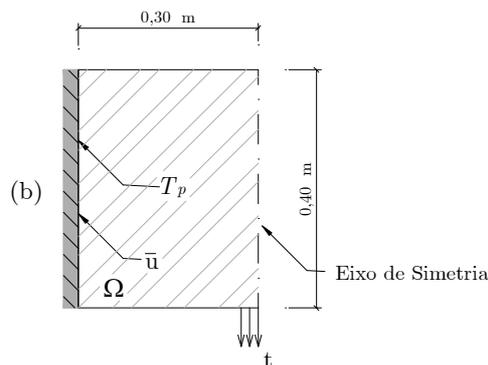


Figure 13. Symmetrical Model of the Problem Case (3) to be solved.

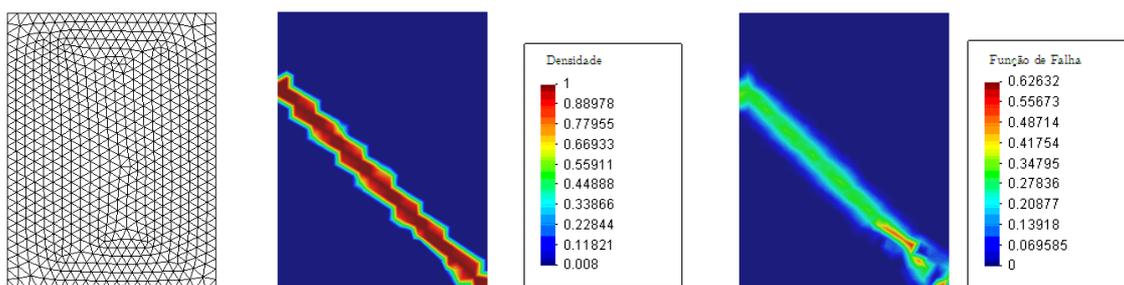


Figure 14. Mesh with 316 elements and 185 nodes, optimum mass distribution and failure function.

6. CONCLUSION

The usage of a multigrid approach or remeshing procedure is important to increase the rate of convergence to the optimum layout of the problem. With this approach, we are able to handle problem with a large number of design variables.

The usage of a non-uniform refinement has shown to decrease the number of design variables and this decrease becomes even more relevant if we refine the problem with a very large number of elements, Carey (1997).

The results were promising, given the stress constraint and tested in meshes with few elements, but for a sharp optimum layout require a very refined mesh, suggesting high computational cost. Hence, the implementation of an adaptive process of refinement would be very prudent.

One of the disadvantages of the adopted approach is that we need to determine the element matrices and vectors what increase the computational cost when compared with the pixel type of strategy employed by many Authors. However, the pixel approach requires a refined mesh in order to describe the material boundary with some precision.

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