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ON THE IMPORTANCE OF GEOMETRIC DAMPING IN DYNAMIC MODELING OF SIMPLY SUPPORTED ROTORS

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Abstract. *The purpose of this paper is to present a simulation of rotating machinery using numerical methods to correctly simulate the geometric damping that takes place when flexible models of the foundation are used. The supporting soil is treated here as an homogeneous and isotropic viscoelastic media modeled as a semi-infinite half-space where Sommerfeld´s radiation condition must be taken into account. A substructuring technique is employed to use the dynamic behavior of the foundation/soil sub-system directly into rotor´s equations of motion in frequency domain. The main advantage of a substructured analysis is that the dynamic response of the unbounded domain, which frequently requires more DOF´s then the structure itself , can be obtained in a separated analysi using boundary or finite/infinite elements. Numerical simulations demonstrating the importance of the soil influence in the unbalance response of simply supported rotors grounded on rigid, non-massive foundations are also presented.*

Keywords*: rotordynamics, finite elements, boundary elements*

1. Introduction

The development of sophisticated numerical methods for studying several problems concerning the vibration study of rotating machinery has suffered a dramatic increase in the last ten years. Indeed, several phenomena that were put aside due to the non existence of a proper methodology in rotordynamics can now be included in the equations of motion of such dynamic systems. Rotating machines have many applications in today's industrial plants and are viewed as complex machines made primarily of a rotating set where one can find shafts, discs and bearings and of a heavy steel or iron supporting structure which is fixed upon a concrete or steel foundation resting on the surface of a supporting soil. Cases of rotating machinery where the foundation is partially buried in the surrounding soil are not uncommon due to the better dynamic isolation of such alternatives.

The shape, materials and techniques involved in making such foundations are complex and demand sophisticated numerical methods for a detailed dynamic analysis (Bonello, 2001). These foundations are connected to and supported by soils that have also a complex dynamic behaviour mainly due to the dissipative effects of these almost infinite media. The modelling procedures involved in the description of the dynamic behaviour of such media can be very time-consuming, depending on the level of precision required for the analysis. Also, methods based on integral formulations such as Boundary Element Methods (BEM) or Green´s Functions are unfamiliar to the vast majority of the structural engineers and researchers of rotordynamics. A review of the main aspects involved in modelling the dynamic soil-structure interaction (DSSI) shows that until very recently engineers did not recognise the role of the geometric damping of the soil in the description of the dynamic behaviour of structures connected to it (Gazetas, 1983). Such damping or dissipative mechanism arises in the rotating machinery modelling due to the propagation of the rotor energy throughout the soil via mechanical waves. These non-reflected waves carry away part of the vibration energy of the machine in an attenuation mechanism similar to the material (viscous or hysteretic) damping, hence the name geometric damping (Wolf, 1985). Due to the presence of these attenuation phenomenon, vibration levels of structures connected to the soil are quite distinct than those isolated by trenches or rigid piles (Barros, 1996).

Many numerical and analytic techniques have been created to model the geometric damping in DSSI: Mesquita Neto (1989) and Romanini (1995) used specially developed Green´s Functions with the indirect version of BEM and Pontes (1992) used the direct version of BEM with fundamental solutions of full-space. Barros (1996) used a hybrid model of finite (FEM) and infinite elements which granted greater flexibility since allowed modelling of several imperfections in the region close to the foundation such as buried structures or layered soils with no additional difficulty.

A common characteristic of all this methods based on boundary or domain discretization is that the dynamic response of the soil-foundation set is obtained directly in the frequency domain via the so-called soil impedance matrix. Mathematically this is due to the absence of a modal basis in which the movement of the soil-foundation set can be represented since one cannot find separated mass, damping and stiffness matrices for the soil that characterise its dynamics. An important characteristic of the soil-foundation system is that its impedance matrix is obtained through direct inversion of the foundation flexibility matrix which is calculated in turn via FEM or BEM. Besides, the terms of such flexibility matrix usually present a distinct dependence of the frequency. Such dependence does not permit a modal decomposition of the coupled machine-soil-foundation and therefore the procedures of modal analysis cannot be employed.

2. Analysis of the rotor-foundation system in frequency domain

The mathematical model of the coupled machine-soil-foundation presented above is split in two distinct parts:

- machine or rotor system, represented by mass [M], damping [C] and stiffness [K] matrices of the shaft, disc and bearings elements together with any elements (ex. plates, beams or rods) representing the machine structure. In the rotor system one can also include possible stiffness and damping effects of the oil film in the journal bearings. One can also included stiffening effects on the shaft due to the presence of axial or shear forces (Lalanne, 1990).
- auxiliary or supporting system, represented by the soil-foundation impedance matrix [S(ω)] also called complex stiffness matrix which is obtained directly in frequency domain.

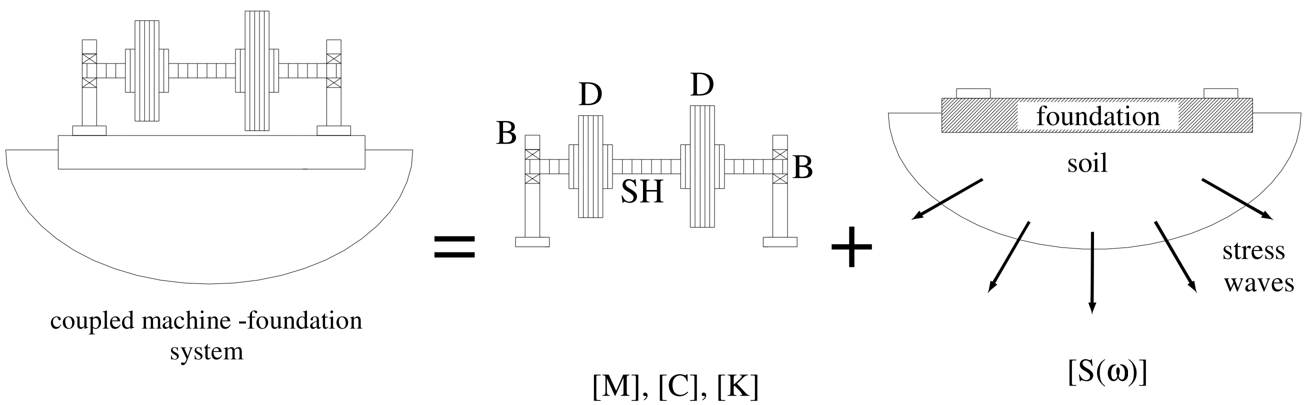

Figure 1: decomposition of the original coupled system into machine and support sub-systems
(B = bearings, D = discs, SH = shaft elements)

One can define the displacement vector {x(t)} containing the degrees of freedom of the rotor system. The vector $\{x_B(t)\}$ contains a partion of {x(t)} with all the DOF´s of the connecting nodes lying on the machine-foundation interface and $\{x_S(t)\}$ contains the remaining nodes of {x(t)}. Therefore the rotor equation of motion can be represented as (Lalanne, 1990):

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{F(t)\} \quad (1)$$

where:

$$\{x(t)\}=\begin{Bmatrix}\{x_S(t)\}\\ \{x_B(t)\}\end{Bmatrix} \quad \text{e} \quad \{F(t)\}=\begin{Bmatrix}F_S(t)\\ F_B(t)\end{Bmatrix}$$

The term {F (t)} represents the vector of external generalised forces and the partition $\{F_S(t)\}$ contains all the forces acting on the rotor system. The partition $\{F_B(t)\}$ represents the interaction forces between the machine structure and the supporting system. In the case of the rotor system, [M], [C] and [K] are obtained via summation of each finite element matrix corresponding to the shaft, disk and bearing elements of the rotor plus any DOF representing the machine structure following the procedures of structural analysis. Partitioning of mass, damping and stiffness matrices of the rotor system results:

$$\begin{bmatrix}[M]_{SS} & [M]_{SB}\\ [M]_{BS} & [M]_{BB}\end{bmatrix}\begin{Bmatrix}\{\ddot{u}(t)\}_S\\ \{\ddot{u}(t)\}_B\end{Bmatrix}+\begin{bmatrix}[C]_{SS} & [C]_{SB}\\ [C]_{BS} & [C]_{BB}\end{bmatrix}\begin{Bmatrix}\{\dot{u}(t)\}_S\\ \{\dot{u}(t)\}_B\end{Bmatrix}+\begin{bmatrix}[K]_{SS} & [K]_{SB}\\ [K]_{BS} & [K]_{BB}\end{bmatrix}\begin{Bmatrix}\{x(t)\}_S\\ \{x(t)\}_B\end{Bmatrix}=\begin{Bmatrix}\{F(t)\}_S\\ \{F(t)\}_B\end{Bmatrix} \quad (2)$$

where the B-index indicates those nodes of the structure lying on the machine-foundation interface and the S-index indicates all the remaining nodes in the rotor system. The Fourier transform of equation (2) gives

$$\begin{bmatrix}[S]_{SS} & [S]_{SB}\\ [S]_{BS} & [S]_{BB}\end{bmatrix}\begin{Bmatrix}\{X\}_S\\ \{X\}_B\end{Bmatrix}=\begin{Bmatrix}\{F\}_S\\ \{F\}_B\end{Bmatrix} \quad (3)$$

where $[S]_{ab}$ for a,b = S,B represents a partition of the rotor impedance matrix, given by:

$$[S]_{ab} = -\omega^2 [M]_{ab} + i\omega [C]_{ab} + [K]_{ab} \tag{4}$$

and $\{X_S\}$ e $\{X_B\}$ represent displacement amplitudes in the rotor system. The interaction forces arising between the machine and the supporting systems are calculated as follows:

$$\{F\}_B = [S]^G_{BB} \left(\{X\}^G_B - \{X\}_B \right) \tag{5}$$

where $\{X\}^G_B$ represents the displacement amplitudes of the interface nodes which are calculated without the machine influence and $[S]^G_{BB}$ is the impedance matrix of the supporting system. In cases where external forces exist only on the rotor (such as mass unbalance), the term $\{X\}^G_B$ vanishes and the equation of motion of the coupled system is simplified as:

$$\begin{bmatrix} [S]_{SS} & [S]_{SB} \\ [S]_{BS} & [S]_{BB} + [S]^G_{BB} \end{bmatrix} \begin{Bmatrix} \{X\}_S \\ \{X\}_B \end{Bmatrix} = \begin{Bmatrix} \{F\}_S \\ \{0\} \end{Bmatrix} \tag{6}$$

The soil-foundation impedance matrix $[S]^G_{BB}$ is obtained using the concept of flexibility matrix as given as follows.

2.1 Impedance matrix of the supporting system

The equation of motion of the supporting system is obtained through direct inversion of the flexibility matrix for a foundation-soil group excited by an harmonic force for each frequency step. The foundation can be treated as flexible or rigid since there are no differences in the formulation. The equation of motion in frequency domain for the supporting system is:

$$[N(\omega)]_{BB} \{F_B\} = \{X_B\} \tag{7}$$

where $[N(\omega)]$ represents the flexibility matrix for each structural DOF on the interface between the machine and the supporting system. The soil-foundation impedance matrix is:

$$[S]^G_{BB} = [N]^{-1}_{BB} . \tag{8}$$

In order to consider the soil dissipative effects on the rotor behaviour, only the case of a rigid foundation will be considered here. In this case the supporting system has only three DOF´s which are related to the foundation rigid body modes in the plane of figure 2.

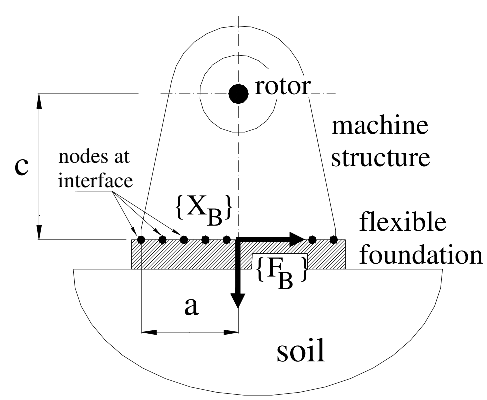

Figure 2: supporting system of a flexible foundation

Neglecting axial forces on the rotor system and considering a flexurally rigid foundation, the equation of motion (7) is given by (Romanini, 1995):

$$\begin{bmatrix} N_{wz} & 0 & 0 \\ 0 & N_{ux} & N_{um} \\ 0 & N_{\varphi x} & N_{\varphi m} \end{bmatrix} \begin{Bmatrix} F_Z \\ F_X \\ M/a \end{Bmatrix} = \begin{Bmatrix} w \\ u \\ \varphi.a \end{Bmatrix} \tag{9}$$

where displacements *w, u* and *φ* are the foundation DOF´s. Neglecting the machine structure flexibility in the plane of figure 2 and considering the forces F_Z and F_X acting directly on the rotor axis, equation (9) can be changed as:

$$\begin{bmatrix} N_{wz} & 0 & 0 \\ 0 & N_{ux} + \frac{c}{a} N_{\varphi x} & N_{um} + \frac{c}{a^2} N_{\varphi m} \\ 0 & \frac{N_{\varphi x}}{a} & \frac{N_{\varphi m}}{a^2} \end{bmatrix} \begin{Bmatrix} F_Z \\ F_X \\ M \end{Bmatrix} = \begin{Bmatrix} w \\ u \\ \varphi \end{Bmatrix} \tag{10}$$

where *a* and *c* are, respectively, foundation half-bandwidth and distance from rotation axis to the foundation-machine interface. One can see from equation (10) that there is no dynamic coupling between displacement in vertical direction and the other two. Hence, the supporting impedance matrix $[S]^G_{BB}$ is given by:

$$[S]^G_{BB} = \begin{bmatrix} k^*_{wz} & 0 & 0 \\ 0 & k^*_{ux} & k^*_{um} \\ 0 & k^*_{\varphi x} & k^*_{\varphi m} \end{bmatrix} \tag{11}$$

where $k^*_{ij}(\omega)$ are complex-valued dynamic stiffness coefficients.

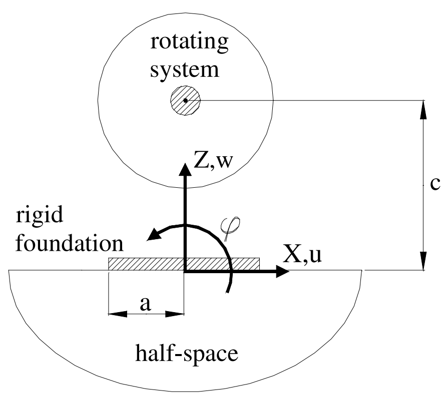

Figure 3: modelling of a rigid foundation for machine supporting

Considering small displacements in a way that sen(φ) ≈ φ and neglecting torcional vibrations due to the rotor-shaft movement, rocking moment M is given simply by $c.F_x$ and, therefore, the supporting system behaviour can be described using only translation DOF´s *w* and *u* available on the rotor bearings. Therefore:

$$\begin{bmatrix} k_{zz} & 0 \\ 0 & k_{xx} \end{bmatrix} \begin{Bmatrix} w \\ u \end{Bmatrix} = \begin{Bmatrix} F_z \\ F_x \end{Bmatrix} \tag{12}$$

where equivalent stiffness k_{wz} and k_{ux} are given by:

$$k_{wz} = k^*_{wz} \quad \text{and} \quad k_{ux} = \frac{k^*_{ux} - \dfrac{k^*_{um} k^*_{\varphi x}}{k^*_{\varphi m}}}{1 - c\left(\dfrac{k^*_{um}}{k^*_{\varphi m}}\right)} \tag{13}$$

The base rocking displacement φ is, therefore:

$$\varphi = \left(\frac{k^*_{\varphi x} - ck^*_{ux}}{ck^*_{um} - k^*_{\varphi m}} \right) u \qquad (14)$$

2.2 Methods for obtaining flexibility coefficients for soil-foundation system

An important aspect in obtaining flexibility functions for the supporting system that must be considered is the correct simulation of the geometric damping through one-directional wave propagation away from the rotating machinery. The finite element method is a domain discretization method and cannot simulate the radiation condition because Dirichlet's or Neumann's boundary conditions (or a linear combination of both) must be applied to the FEM mesh for the problem solution (Zienkiewicz, 1989). In addition to that, the FEM mesh must be truncated at some point away from the source to limit the number of DOF´s in the solution. This mesh truncation imposes an artificial reflecting boundary where conditions have to be applied for the sake of uniqueness of solution. This boundary, when posed close to the source, causes wave reflection toward the source and violation of the radiation condition (Barros, 1996). Some alternatives like absorbing boundaries (essentially the usage of calibrated dampers on the boundary) didn't work well and were abandoned shortly after their proposition. In this paper three alternatives for obtaining the foundation-soil impedance matrix, namely the hybrid finite/infinite element method, the boundary element method and the simplified cone model method will be investigated.

2.2.1 Cone Models

Cone models, as described by Wolf (1985), present a simple and efficient method for obtaining some half- or layered-space flexibility functions. However, since they ignore large portion of the half space very close to the machine, such models does not permit simulation of Rayleigh (or surface) wave propagation which carries more than half of the vibration energy as seen in half-space models. Therefore, such models underestimate the soil damping capacity, particularly in the case of buried foundation or piles.

For the case of vertical flexibility function $N_{wz}(\omega)$, the exponential cone model permits some results that indeed allow modal techniques to be applied in the coupled soil-foundation-machine system. For a cone with exponentially increasing transversal area, the vertical flexibility function for rigid foundation is:

$$N_{wz}(\omega) = \frac{f}{2EA_0\left(1 + \sqrt{1 - 4a_0^2}\right)} \quad \text{where} \quad a_0 = \frac{\omega f}{c_p} \qquad (15)$$

where E is the soil stiffness, A_0 is the soil-foundation contact area and f is a parameter that defines how fast the cone transversal area grows. In practice, parameter f is chosen in a way that N_{wz} calculated using equation (15) is a good approximation of N_{wz} calculated using BEM or hybrid FEM models.

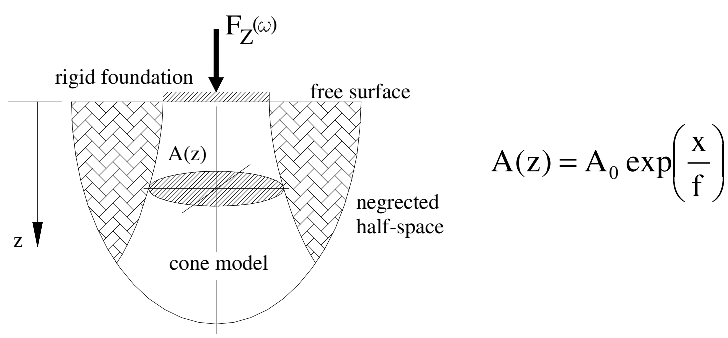

Figure 4: model of exponential cone for $N_{wz}(\omega)$

In equation (15), the term a_0 is referred as "non-dimensional frequency' , c_p is the propagation speed of body waves, defined as $c_p = (E/\rho)^{1/2}$, and ρ is the medium density. Figure 5 illustrates the behaviour of the real and imaginary parts of $k_{wz}(\omega)$ when f = 1.10, E = $1.25\text{x}10^7$ N/m^2 m, ρ = 3800 kg/m^3 and A_0= 0.50 m^2 .

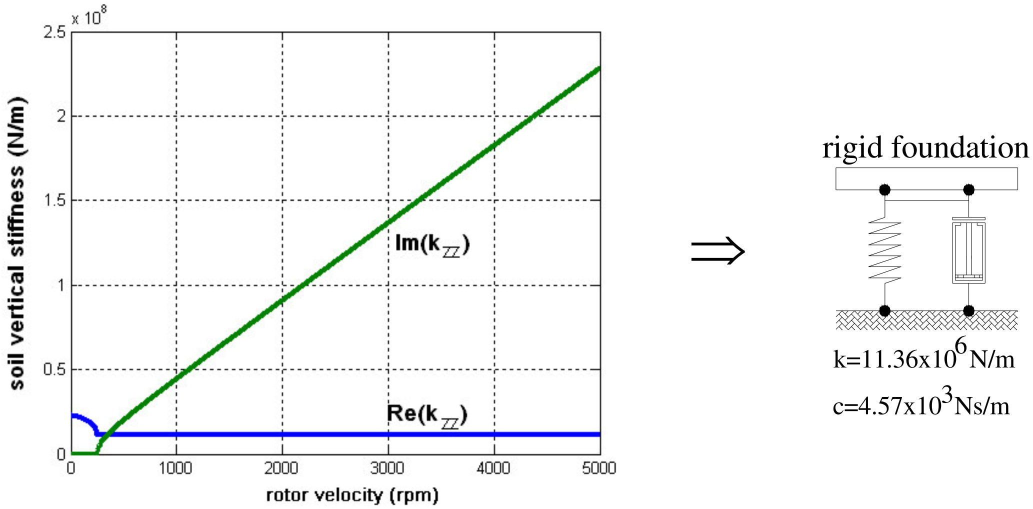

Figure 5: behaviour of $k_{wz}(\omega)$ for a exponential cone model

An important fact of cone models usage is that, excluding the low-frequency band, the dynamic behaviour of this model can be translated into a simple 1 DOF spring-dashpot model and, therefore, be readily incorporated into available codes of rotordynamics. For the model described above, one can have soil parameters which are frequency-independent and given by k_{wz} = 11.36x10^6 N/m and c_{wz} = 4.57x10^3 N.s/m which cannot be neglected in a realistic dynamic analysis.

2.2.2 Boundary element methods

The direct version of BEM (Romanini, 1995) is an alternative for obtaining soil-foundation impedance matrices since the so-called fundamental solutions (usually Green's functions of full-space) automatically fulfil the radiation condition. Besides, when specially-defined Green's functions are employed, only the interface between soil and foundation (rigid or flexible) need discretization and, consequently, very few DOF´s are required as compared to FEM solutions. The main disadvantage of the BEM is that fundamental solutions employed in the method have to be formulated directly in the frequency domain, allowing identification of an approximate modal base only via parameter identification techniques of the rotor's FRFs (when the system is solved in the frequency domain directly).

2.2.3 Infinite elements

The usage of hybrid models of finite/infinite elements has been proved to be very efficient in the numerical evaluation of soil-foundation dynamic matrices (Barros, 1996). In fact, since 1998, commercial packages like ABAQUS® or ANSYS® have infinite elements in their element library. The modelling technique consists of using higher-order (quadratic or cubic) finite elements to model the region close to the foundation (known as *near field*) and using exponential decrease or mapped infinite elements to model the region away from the foundation (known as *far field*) .

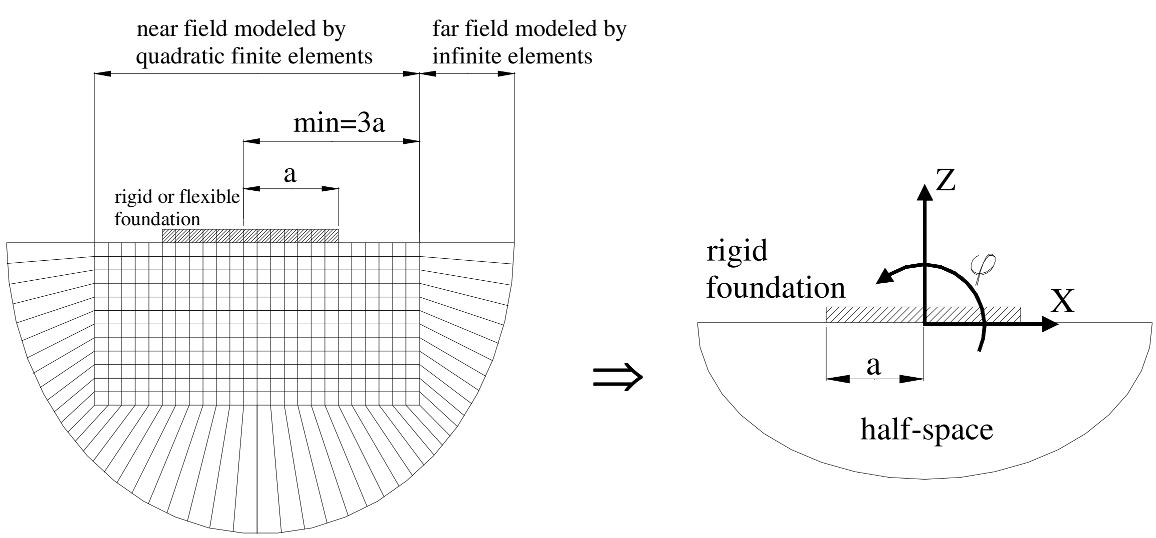

Figure 6: Modelling technique of a homogeneous half-space by a hybrid finite/infinite element mesh

This technique allows the modelling of any geometric irregularity (as buried foundation or layered soils) in the near field very easily due to the versatility of the current mesh generator programs. Medina (1982) recommended a near field distance of no less than three times the foundation half-bandwidth *a* for a precise modelling of the stress gradients near the soil-foundation interface. However this technique results in a large number of DOF´s (typically 3000 or 4000 DOF) only for obtaining the support dynamic response, which has to be solved for each frequency step. Barros (1996) obtained impedance matrices for a rigid foundation resting on a surface of a viscoelastic isotropic half-space by stiffening the foundation elasticity modulus at least 20 times more than the soil's.

The figures 7 and 8 illustrates the behaviour of the impedance matrix coefficients $N_{ux}(\omega)$ and $N_{um}(\omega)$ for a half-space with ν=0.25 and hysteretic damping factor of 10% obtained through the technique described above and a comparison with the BEM results obtained by Romanini (1995) and some Luco and Westman (1972) analytical results .

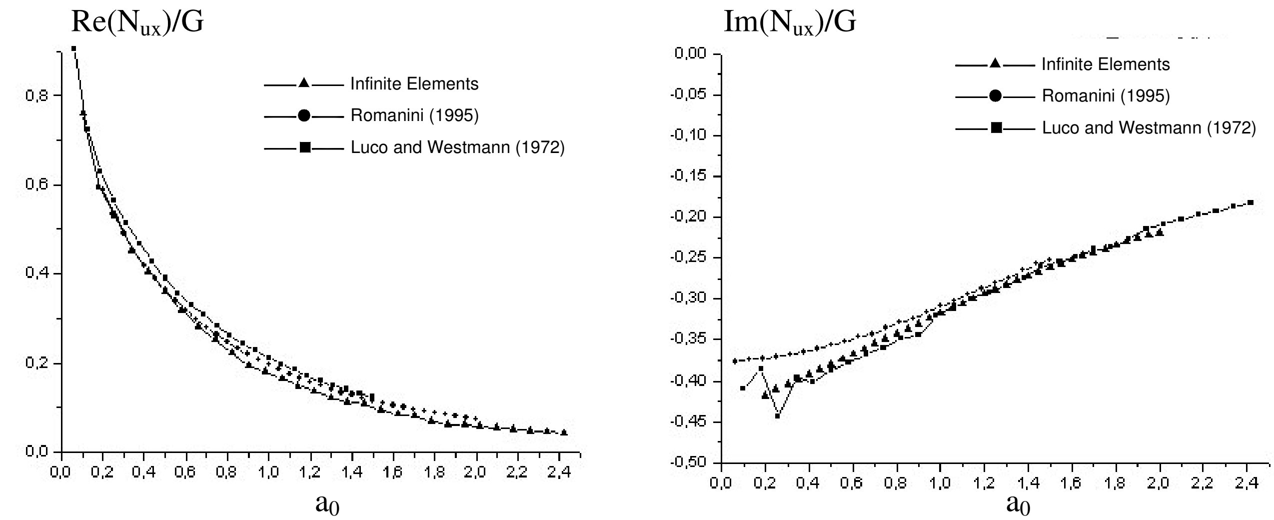

Figure 7: Flexibility function $N_{ux}(\omega)$ for η=0.10 and ν=0.25 .

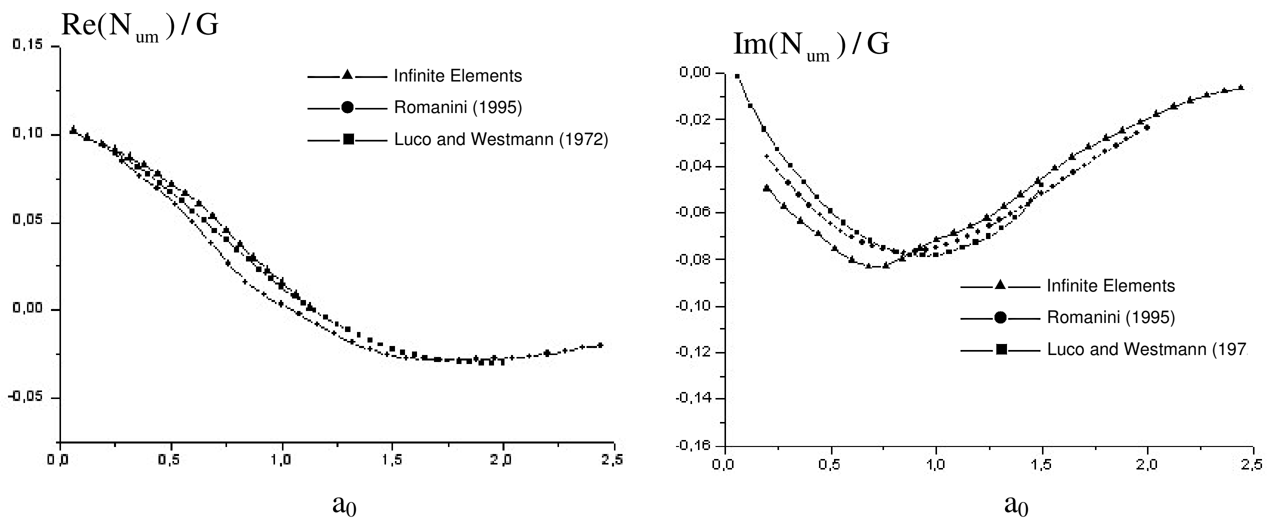

Figure 8: Flexibility function $N_{um}(\omega) = N_{\varphi x}(\omega)$ for η=0.10 and ν=0.25.

One can realise a good agreement between the analytical results and the numerical FEM and BEM simulations mainly in the high frequency band, for $a_0 \geq 1.00$.

2.3. Numerical Implementation: FRF´s of an asymmetric rotor supported by a viscoelastic half-space

In order to determine the influence of the soil dissipative parameters on the dynamic behaviour of rotating machinery, the flexibility functions described above were used together with the motion equations of rotordynamics to determine some FRF curves of an asymmetric rotor supported by a viscoelastic half-space. A comparison illustrating the influence of the foundation mass in the results was also obtained. Figure 9 illustrates the geometry of the present problem.

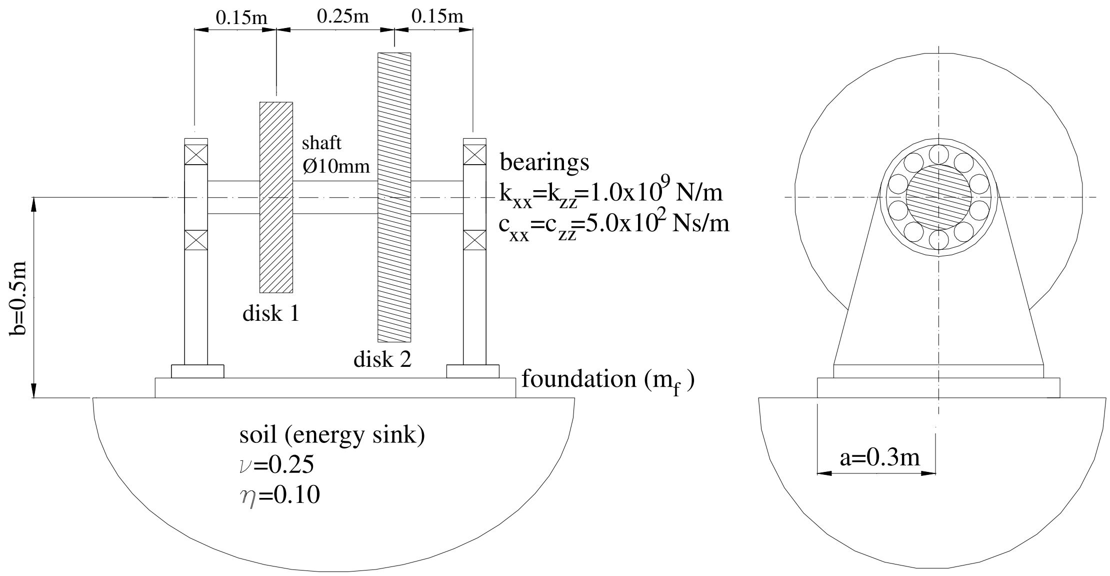

Figure 9: Geometry of the rotating machinery

The simulation was carried out considering the shaft sections modelled by FEM as beam elements and neglecting the effect of transverse shear on shaft stiffening. The discs were modelled considering only their inertial effect upon matrices [M] and [C] and also neglecting possible stiffening effects on the shaft sections. Foundation mass (m_f) was equally split between the two supports and modelled as concentrated mass points. No foundation rotational inertia was modelled. Dynamic coupling between orthogonal directions in the bearings elements was also neglected, so $k_{xz} = k_{zx}= 0$ and $c_{xz} = c_{zx} = 0$ for these elements. Table 1 summarises the physical properties of each element.

Table 1: physical properties of the several elements used in the model

bearings	$K_{xx} = K_{zz} = 1.0x10^9$ N/m , $K_{xz} = K_{zx} = 0$ $C_{xx} = C_{zz} = 5.0x10^2$ N.s/m , $C_{xz} = C_{zx} = 0$
disks	m_d= 0.5 kg, $I_x = I_z = 0.0029$kgm^2 e Iy=0.056 kgm^2
shafts	E = 2.09x10^{11} N/m2 I = 7887 mm^4 ρ = 7920 kg/m^3 , m_{shaft} = 1.38 kg
lateral supports plus bearing (each)	m = 3.5 kg
total rotating machine mass including structure	m_{total} = 9.98 kg

The system was excited by an unbalancing force corresponding to 0.1 kgm on disc 1. To demonstrate the influence of the foundation mass on the displacements, the soil impedance matrix $[S]_{BB}^{G}$ was calculated via finite/infinite modelling considering three distinct mass relations of m_f /m_{total} = 0, m_f /m_{total} = 10 and m_f /m_{total} = 1000 were m_{total} is the approximate machine mass. The results shown in Figure 10 refers to the displacement of the left support. As expected, foundation mass plays a major role in obtaining the dynamic response of high-speed rotating machinery.

To demonstrate the influence of the modelling technique, Figure 11 shows the same system response but now considering the three alternatives of modelling of the supports: rigid base, finite/infinite method and cone model for a foundation mass of 100 kg that corresponds to $m_f /m_{total} \approx 10$. Results demonstrate that the modelling technique is also important when considering the response of rotating machinery but there is no appreciable difference between the results obtained via MEF and via cone models. This suggests that an economical analysis can be performed simply by considering an equivalent spring-dashpot model for soil attenuation.

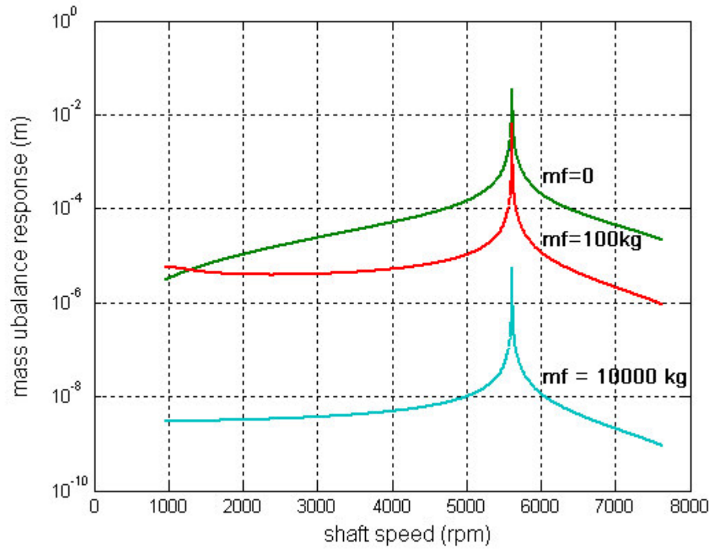

Figure 10: Influence of the foundation mass in dynamic response of rotating machinery

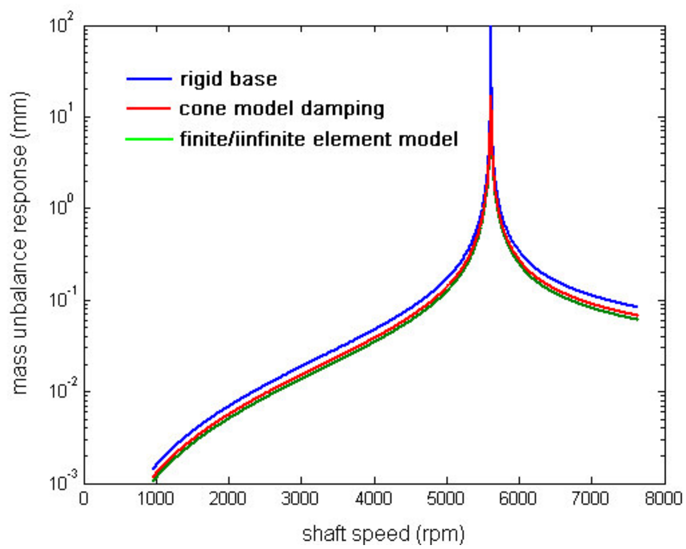

Figure 11: Influence of the modelling technique

2.4. Conclusions

In this paper several topics concerning the modelling of the radiation condition in rotating systems were addressed. A substructuring technique that permits the foundation-soil impedance matrix to be coupled with the rotor equations of motions were presented. The more important techniques for the modelling of the radiation condition, namely simplified cone models, finite/infinite elements and boundary elements were also discussed.

Results of the simplified, lumped parameters, model implemented here gives indication that both foundation mass and modelling technique are very important for the numerical simulation of vibration in rotating machinery. Also, that the simplified cone models appears to be an economical alternative for the foundation simulation if the flexural rigidity of the foundation does not include elastic foundation models in the frequency range being analysed.

The methodology presented here can be extended, with very little modifications, for the analysis of the dynamics of grounded structures subjected to non-cyclic loadings through spectral decomposition directly in time domain.

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