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EXTREME DISTURBANCE PROBLEMS IN DYNAMICS

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Abstract. *Often, in testing protective devices that are to be subjected to dynamic loading, such as child seats in automobiles, a corridor is prescribed into which a loading pulse must fit. That is, a regulatory agency sets an impact corridor for dynamic pulses to test the success of a device in providing protection for a fragile object. An extreme disturbance analysis can be performed to determine the worst and best responses of the device. For an extreme disturbance analysis, a system response, such as an acceleration, is selected as an objective function to be minimized or maximized. The prescribed upper and lower bounds of the loadings, such as the defining bounds on corridors or allowable values for an impulse, form the constraints for a mathematical programming problem. Maximizing and minimizing the objective function leads to two time histories of loading functions, the* worst *and* best disturbances*. The worst disturbance corresponds to the maximum value of the objective function and the best disturbance generates the minimum value. The response of the device to any other loading within the defined loading bounds will fall between the extreme values of the responses of concern.*

Keywords*: extreme disturbance analysis, best disturbance, worst disturbance, uncertain input.*

1. Introduction

There is a variety of structural analysis problems where the loading is not fully defined. Normally, such problems are treated as stochastic problems utilizing the theory of probability. An alternative deterministic approach is to perform analyses utilizing the available information to calculate bounds on possible responses. The lower bound would be the response that must occur. This corresponds to the *best disturbance*. The upper bound is the peak possible response. This corresponds to the *worst disturbance*.

One example occurs if an explosive environment is characterized in terms of a total impulse. This is sufficient information to determine what must happen to some equipment, e.g., military equipment, and the maximum response of this equipment.

Other examples occur in the testing of safety devices for crashing vehicles. For example, tie-down systems for wheelchairs are tested on high-speed sleds for which the crash pulse must adhere to standards that are defined in terms of an envelope in which the crash pulse must lie. An even more common example occurs in the sled testing of child seats, where the sled pulse must lie within prescribed time-varying bounds. The proposed scheme provides upper and lower bounds for critical responses. These responses can be used to help gauge whether the prescribed pulse corridor is too tight or too broad to provide effective standards. Thus, the procedure here develops a technology to study the sensitivity of dynamic responses to prescribed test conditions.

We choose to use motor vehicle impacts to illustrate the principles of this sensitivity technology.

2. Computational Formulation

A single-degree-of-freedom system (Fig. 1) will be used to illustrate the computational procedure of an extremal disturbance analysis. Suppose that the input pulse is required to lie between upper and lower bound prescribed wave forms. Such a pulse corridor is often specified for testing components. Of all of the input pulses that lie between the bounding waveforms, the *best disturbance* is the one that leads to a minimum response and the *worst disturbance* is the one corresponding to the maximum response.

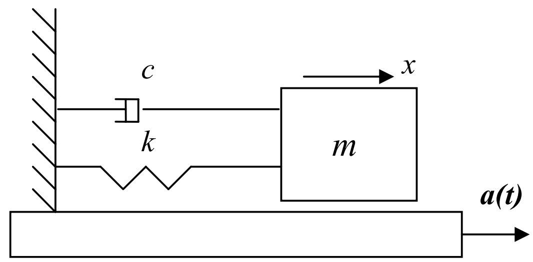

Figure 1. Single-degree-of-freedom model.

Suppose the response of interest is the peak displacement of the mass relative to the base. In the model *m* is the mass and *x* is the displacement of *m* with respect to the base which is undergoing the input acceleration pulse *a(t)*.

The differential equation of motion for this system is

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = a(t) \tag{1}$$

where *a(t),*.is the input pulse and

$$\omega_n^2 = \frac{k}{m} \qquad \zeta = \frac{c}{2\sqrt{km}} \tag{2}$$

With zero initial conditions, $\dot{x}(0) = 0$ and $x(0) = 0$, the solution is

$$x(t) = \int_0^t a(\tau) g(t-\tau)\, d\tau \tag{3}$$

where *g(t),* the impulse response, depends only on ω_n and ζ:

$$g(t) = \frac{1}{\omega_D} \exp(-\zeta\omega_n t) \sin \omega_D t \tag{4}$$

where $\omega_D = \omega_n \sqrt{1-\zeta^2}$ is the damped natural frequency.

The problem is to find the best and the worst pulses belonging to a certain pre-selected pulse corridor. Thus the control function to be determined is *a(t),* which may be discretized as

$$a_k = a(t)[(k-1)\Delta t] \qquad 1 \le k \le n \tag{5}$$

The performance index is taken here as the peak relative displacement of the mass

$$J(a(t)) = J(a_1, a_2, ..., a_n) = \max_{1 \le k \le n} \left| x[(k-1)\Delta t] \right| \tag{6}$$

The constraints of the problem place all allowable input pulses in the prescribed corridor by specifying upper and lower bounds:

$$L_k \le a_k \le U_k \qquad 1 \le k \le n \tag{7}$$

If desired, other response constraints can be imposed. For example, the area under the acceleration curve could be specified:

$$\Delta V = \int_0^T a(\tau) d\tau = \sum_{i=1}^{n} W_i a_i \tag{8}$$

where the weights W_i depend on the numerical integration method used to calculate ΔV . The quantity ΔV is the difference between the base velocity before (0) and after (T) the dynamic occurrence takes place.

The best disturbance problem is to minimize

$$\Psi = \begin{bmatrix} 0^T & 1 \end{bmatrix} \begin{bmatrix} a \\ J \end{bmatrix} = J \tag{9}$$

where $\boldsymbol{a}$ is the column vector whose i^{th} element is a_i. The constraints of the problem are the inequality of (7) and the equality of (8), as well as

$$-J \le x[(k-1)\Delta t] \le J \qquad\qquad k = 1,2,...., n \tag{10}$$

In this example, the function to be minimized and the constraints are linear in the variables $a_1, a_2, ..., a_n, J$. The methods of linear programming are therefore applicable.

In general, constraints are readily included to supplement the formulation. If it becomes desirable to add constraints, this is done simply by expanding the constraint matrix. Additional constraints are sometimes useful for controlling the smoothness of best and worst disturbance functions. For a crash problem, it may be physically meaningful to impose restrictions on rise time or on the rate at which *a(t)* can change in a certain time interval. It is also possible to use the best and worst disturbances obtained from limited degree-of-freedom models as inputs in large-scale dynamic simulators to calculate the best and worst responses.

3. Numerical Example

This section presents an application of best and worst disturbance analyses to a sled test under crash pulse loading. The test vehicle model is the single-degree-of-freedom system shown in Fig. 1. The mass *m* is taken to be equal to the total mass of the occupant and his seat in this example, and the spring-damper force represents the total restraint force acting on the occupant. This restraint force is chosen as the critical response, the peak value of which is to be minimized or maximized. This contrasts somewhat with the formulation of the previous section, where the peak relative displacement was used as the critical response. The numerical values of the system parameters are $m = 162.5$ kg, $k = 7$ MN/m and $\zeta = 0$ or $\zeta = 0.10$. With these values, the natural frequency of the system is

$$f_c = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 33.0 \text{ Hz}$$

The deceleration corridor, within which all allowable crash pulses are required to stay, is shown in dashed lines in Fig. 2. This is a corridor proposed for wheelchair testing by the International Standards Organization (ISO).

The velocity change for the crash is nominally 48 km/h, but considering the uncertainty in this value, the velocity change constraint of Eq. (6) is replaced with the inequality constraint

$$47.2 \text{ km/h} \le \int a(\tau)\, d\tau \le 48.8 \text{ km/h} \tag{11}$$

A sensitivity index R defined by

$$\text{R} = \frac{peak\ restraint\ \ force\ \ for\ worst\ disturbance}{peak\ restraint\ \ force\ \ for\ best\ disturbance}$$

gives a measure of how far apart any two test results may be expected to lie when they are confined to the same sled deceleration corridor with the same velocity change.

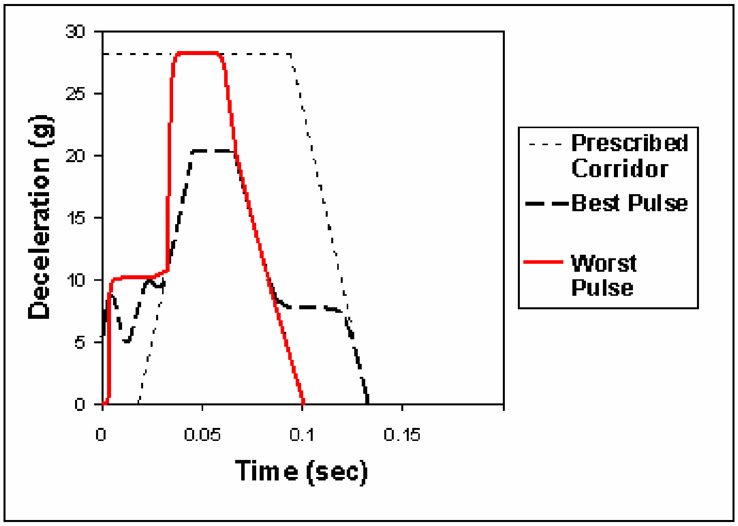

Figure 2. Best and worst disturbance

Figure 2 shows the best sled deceleration, which, among all other crash pulses satisfying the constraints stated above, is the one that produces the minimum peak restraint force possible. The worst sled deceleration, corresponding to the maximum peak force, is also shown in Fig. 2. It is seen from Fig. 2 that the best disturbance analysis tends to drive the deceleration curve down to its smallest allowable value while keeping the area under the curve within the bounds specified by the inequalities (11). The worst disturbance in Fig. 2 has the tendency to drive the deceleration curve up to its largest allowable value.

Figure 3 shows the best and the worst displacement responses corresponding to the best and the worst crash pulses of Fig.2. It can be seen that the worst response has a main frequency component of about 33 Hz, which is the natural frequency of the system model.

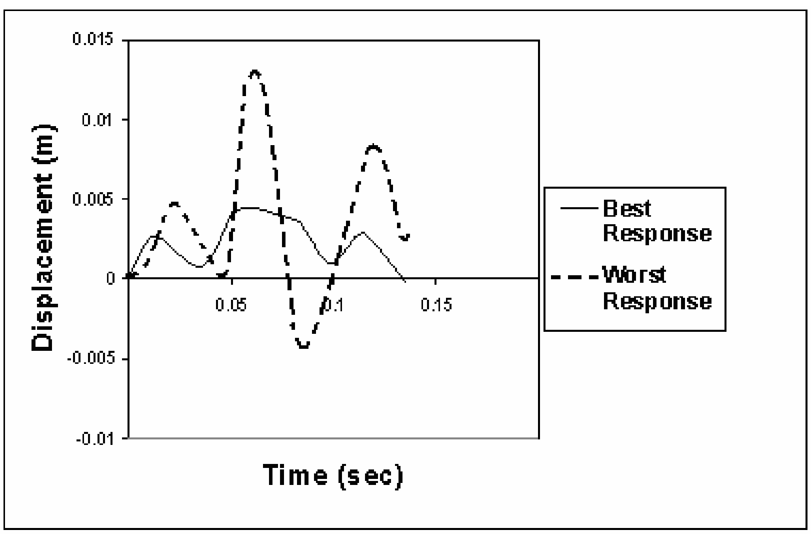

Figure 3. Best and worst responses

The sensitivity indices R are found to be

$$R = 87{,}510.9\ \text{N}/32{,}707.6\ \text{N} = 2.68 \qquad \text{for} \qquad \zeta = 0.00$$

$$R = 75{,}791.4\ \text{N}/32{,}163.6\ \text{N} = 2.36 \qquad \text{for} \qquad \zeta = 0.10$$

Thus, with this standard ISO corridor, the peak restraint forces measured in two different tests can vary by a factor of 2.68 for an undamped system or by a factor of 2.36 with a damping ratio of 0.10. This corridor would, therefore, probably be considered to be too wide for practical sled test research, and should be made narrower to reduce the sensitivity index R.

4. Conclusions

The extreme disturbance analyses are useful in the dynamic analysis of structures subject to incompletely described loading. In particular, the best and worst disturbance analysis is a useful tool in evaluating the effectiveness of parameters for defining crash pulses that can be considered comparable or equivalent, that is, this methodology can be applied to the problem of defining deceleration corridors for impact tests. Practical guidelines for the selection of the test corridors are provided in Crandall, et al.

5. Acknowledgements

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6. References

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