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THREE METHODS TO STUDY THE DYNAMIC BEHAVIOR OF A VERTICAL AXIS WASHING MACHINE DURING THE SPINNING STAGE

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Abstract. *This work presents and compares three methods to study the dynamic behavior of a vertical axis washing machine during the water extraction process, also called spinning stage. The three methodologies used to study the dynamic behavior of a vertical axis washing machine are: analytical, experimental and simulation using MBS (Multi Body System). The interest in this stage is due to the main vibration problems of an automatic washing machine occur in the spinning cycle. Since the unbalance force that causes the vibration problems is centrifugal in nature, this makes intuitive sense as the rotational speed of the basket is the greatest during the spin extraction cycle. Thus, modeling the washing machine during the spinning cycle allow us to get its dynamic behavior as well as the main characteristics of this behavior. It is assumed for this analysis that the spin speed and the unbalanced mass are constant over the time interval of interest.*

Keywords: *washing machine, vibration, simulation using MBS*

1. Introduction

Basically, two tasks are realized by a washing machine, the washing process and the water extraction process or the spinning. During the washing process the machine stay almost static. The spinning is the stage that needs more effort from the engineer to reach an acceptable dynamic behavior (low level of noise and vibration). During the spinning the basket of the washing machine (as well as the clothes inside the basket) rotates at speeds somewhere between 600 and 1200 rpm. The centrifugal forces generated by the unbalanced mass (due to the clothes distribution) are transmitted to the cabinet through the suspension system. Hence, a suspension system must be so designed to avoid the machine "walk" and limit the internal mobility of the washing unit to avoid striking the cabinet. The obtainment of a configuration that attends the above requirements, with a lower cost, is the main challenge for an engineer of suspension and balance system for a washing machine.

The aim of this work is to study and to compare three methodologies to study the dynamic behavior of a vertical axis washing machine: analytical, experimental and simulation using MBS.

2. Review of literature

Since the invention of the first washing machine by Schaefer in 1766 (Türkey et al, 1998) there had been a continuous development in washing machine technology.

Today, there are two basic types of washing machines (classified according to the direction of the basket rotational axis): horizontal axis washing machines and vertical axis.

The horizontal axis washing machines are mostly utilized in the European countries and have the advantages of using less water, detergent and electrical energy.

The vertical axis washing machines are mostly utilized in Asia and America, including Brazil. They posses the advantages of loading convenience and load capacity.

The vibration problems of the washing machines are caused by the rotating unbalanced laundry mass. Thus, the aims of suspension design are to minimize vibration amplitudes of the tub, avoiding cabinet sliding with quiet operation together with possible maximum laundry mass at a possible maximum spin speed.

The publications in this specific are limited. The researches of Bagepalli (1987) and Zuoxin (1991) are the few examples of vertical axis machine analysis. Bagepalli carried out a comparative theoretical and experimental analysis of two different designs. Zuoxin introduced the concept of liquid equilibrium ring and proposed a solution to the problem of poor stability at spin-up.

Conrad and Soede (1998) discusses the general approach of controlling an unbalanced rotating system through the use of auxiliary mass dampers like balance rings.

The number of researches for horizontal axis machine is bigger but not enough. Sümer (1992), Türkey et al (1992, 1993) worked on the horizontal axis machines for the analysis and optimization of the suspension system dynamics.

Conrad and Soede (1995) discussed the sliding problem of vertical axis washers using simple models without suspension.

Türkey et al (1998) reports a detailed modeling and vibration analysis of horizontal axis washing machines.

Bae et al (2002) shows a analytical formulation of a balance ring applied in a washing machine. They also discuss the dynamic behavior of a vertical axis washing machine using the balance ring. The results are validated using experimental analysis.

3. The study of the dynamic behavior of a vertical axis washing machine

To realize this work it was used the washing machine BWQ24 from Brastemp trade mark (this washing machine is the leader in the Brazilian market). This washing machine is a vertical axis and uses a hang suspension system, Fig. 1.

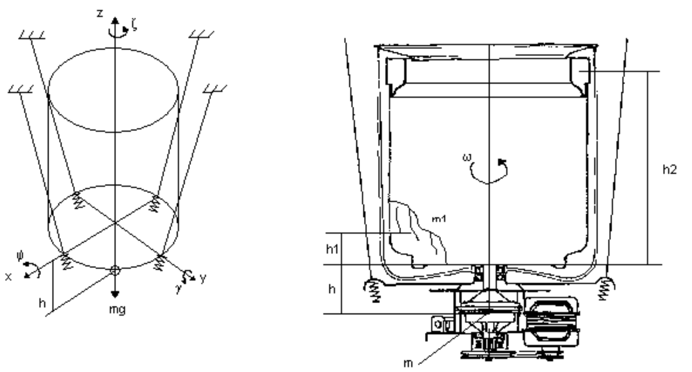

Figure 1. A schematic of the machine used in this work.

The numerical values of the physical and geometrical parameters used in the studies are: m = 26kg; h = 80m; l = 450mm; θ_0 = .14486; k = 2N/mm; d = 460mm; w = 78.54 rd/s; m_1 = 1kg; h_2 = 380mm; h_1 = 200mm.

To get the dynamic behavior of a washing machine during the spinning, some results are very important:

- Upper tub displacement during the steady state and transient (upper orbit): as lower as better, during the steady state these displacements give an idea of the level of noise and vibration. During the transient these displacements can’t be so high to avoid the tub hits the cabinet.
- Lower tub displacement during the steady state and transient (lower orbit): the same as the prior. During the steady state the displacements can’t be so high to avoid the harness to get broken.
- Cabinet vibration: as lower as better, to avoid customer complaint and high noise.
- Amplitude of the rod suspension force: it gives an idea of the cabinet vibration level since the vibration is generated by the forces transmitted from the suspension to the cabinet.

To get the results listed above, three methods can be used: the analytical, the experimental and simulation using MBS technique. However, it is very difficult to get all the above results using only one method. The use of two or three methods together will allow the obtainment of all the results.

The following will describe a summary of the three methods used to get the dynamic behavior of a vertical axis washing machine during the spinning. For all studies the excitation frequency used in the steady state was 12.5Hz (750rmp).

3.1. Analytical method

In this method, we will use an energy method to determine the response of the washing machine tub assembly.

To write the vibration equation of the system, we need to take out from the Fig. 1 a vertical coordinate plane XOZ (Fig. 2).

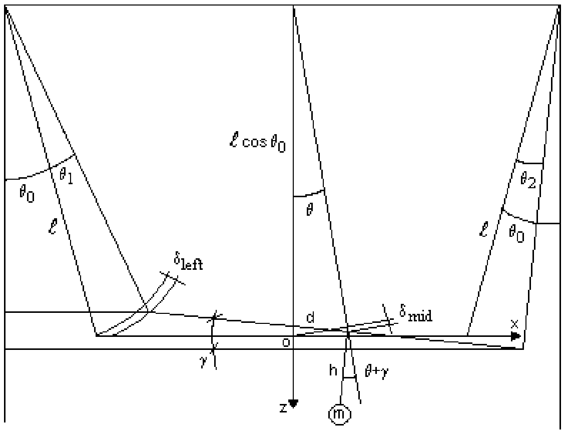

Figure 2. Plane XOZ.

In the Fig. 2, l is the length of the rod suspension, d is the distance between the two tub hanging points, h is the distance between the system's mass center and the hanging plane. If the pendulum angle in the center is θ, then θ_1, θ_0 e θ_2 are assured to be almost equal and since $x \cong l\theta\cos\theta_0$, we can obtain:

$$\delta_{left} = 2\delta_{mid} = \frac{2sin(\theta_0)x}{cos^2(\theta_0)} - \frac{d\gamma}{cos(\theta_0)} \tag{1}$$

Where δ is the elastic deformation of a suspension spring and θ_0 is the static angle of a hanging point. Eq. 1 indicates the relation of δ and the displacements vectors. It is one of the keys to derive the equations of motion of the system.

In this study the generalized coordinates are x, γ, y and ψ (see Fig. 1).

The equations of motion can be determined from Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} = Q_j^{(nc)} \tag{2}$$

where:

qj are the generalized coordinates;

$Q_j^{(nc)}$ are the non-conservative forces;

T is the kinetic energy of the system;

U is the potential energy of the system.

The kinetic energy T is given by:

$$T = \frac{m\left(\dot{x}^2 - 2\dot{x}\dot{\gamma}h + \dot{\gamma}^2h^2\right) + m\left(\dot{y}^2 - 2\dot{y}\dot{\psi}h + \psi^2h^2\right)}{2} + \frac{J_a\dot{\gamma}^2 + J_a\dot{\psi}^2}{2} \tag{3}$$

The potential energy U is given by:

$$\begin{aligned} U = {} & \frac{mgx^2}{2l\,cos(\theta_0)} + \frac{3k\,tan^2(\theta_0)x^2}{cos^2(\theta_0)} - \frac{3kd\,tan(\theta_0)\gamma x}{cos^2(\theta_0)} + \frac{4k\,tan^2(\theta_0)yx}{cos^2(\theta_0)} - \frac{2kd\,tan(\theta_0)\psi x}{cos^2(\theta_0)} + \frac{mgh\gamma^2}{2} \\ & + \frac{3kd^2\gamma^2}{4\,cos^2(\theta_0)} - \frac{2kd\,tan(\theta_0)y\gamma}{cos^2(\theta_0)} + \frac{kd^2\psi\gamma}{cos^2(\theta_0)} + \frac{mgy^2}{2l\,cos(\theta_0)} + \frac{3k\,tan^2(\theta_0)y^2}{cos^2(\theta_0)} - \frac{3kd\,tan(\theta_0)y\psi}{cos^2(\theta_0)} \\ & + \frac{mgh\psi^2}{2} + \frac{3kd^2\psi^2}{4\,cos^2(\theta_0)} \end{aligned} \tag{4}$$

Thus, the equation of motion become:

$$[M]\{\ddot{x}\} + [K]\{x\} = \{Q^{(nc)}\} \tag{5}$$

where:

$$[M] = \begin{bmatrix} m & -mh & 0 & 0 \\ -mh & J_a + mh^2 & 0 & 0 \\ 0 & 0 & m & -mh \\ 0 & 0 & -mh & J_a + mh^2 \end{bmatrix}$$

$$[K] = \begin{bmatrix} \frac{mg}{l\cos(\theta_0)} + \frac{6k\tan^2(\theta_0)}{\cos^2(\theta_0)} & -\frac{3kd\tan(\theta_0)}{\cos^2(\theta_0)} & \frac{4k\tan^2(\theta_0)}{\cos^2(\theta_0)} & -\frac{2kd\tan(\theta_0)}{\cos^2(\theta_0)} \\ & mgh + \frac{3kd^2}{2\cos^2(\theta_0)} & -\frac{2kd\tan(\theta_0)}{\cos^2(\theta_0)} & \frac{kd}{\cos^2(\theta_0)} \\ & & \frac{mg}{l\cos(\theta_0)} + \frac{6k\tan^2(\theta_0)}{\cos^2(\theta_0)} & -\frac{3kd\tan(\theta_0)}{\cos^2(\theta_0)} \\ & SIM & & mgh + \frac{3kd^2}{2\cos^2(\theta_0)} \end{bmatrix}$$

$$\{x\} = \begin{Bmatrix} x \\ \gamma \\ y \\ \psi \end{Bmatrix} \qquad \{\ddot{x}\} = \begin{Bmatrix} \ddot{x} \\ \ddot{\gamma} \\ \ddot{y} \\ \ddot{\psi} \end{Bmatrix} \qquad \{Q^{(nc)}\} = \begin{Bmatrix} -(m_1 - \rho\pi H\,Re)Rw^2\sin(wt) \\ -(m_1 - \rho\pi HRh_2e)Rw^2\sin(wt) \\ -(m_1 - \rho\pi H\,Re)Rw^2\cos(wt) \\ -(m_1 - \rho\pi HRh_2e)Rw^2\cos(wt) \end{Bmatrix}$$

Where m is the overall mass of the system, J_a is the moment of inertia around the horizontal axis through the system mass center, k is the stiffness of a suspension spring and e is the eccentricity generated by the unbalanced mass m_1.

To solve the Eq. 5 first we need to get the value of e from the $Q^{(nc)}$ vector. According to Conrad (1994) the eccentricity can be determined as:

$$e = \frac{m_1 rw^2}{(m + \rho H\pi R^2)w^2 - k} \tag{6}$$

Where r is radius of the unbalanced mass position, w is the spin speed of the machine, p is density of fluid inside the balance ring, R and H are respectively the radius and height of the balance ring.

Now, we will use the mode superposition method or normal mode method to solve the Eq. 5, by which such a set of coupled equations can be transformed into a set of uncoupled equations through use of the normal modes of the system.

The first step in a mode superposition solution is to obtain the natural frequencies and natural modes of the system.

Assuming the values cited previously for the machine considered in this study, we have:

w1=0.3145rd/s w2=0.5449rd/s
w3=5.054rd/s w4=6.306rd/s

$$\Phi = \begin{bmatrix} -0.1501 & 0.4070 & -0.7071 & 0.7071 \\ -0.691 & 0.5782 & 0.0012 & -0.0053 \\ 0.1501 & 0.4070 & 0.7071 & 0.7071 \\ 0.691 & 0.5782 & -0.0012 & -0.0053 \end{bmatrix}$$

With the modal matrix we can get the modal mass and modal stiffness matrix.

The next step in this procedure is to do the coordinate transformation:

$$\{u(t)\} = [\Phi]\{\eta(t)\} = \sum_{r=1}^{N} \Phi_r \eta_r(t) \tag{7}$$

where $\eta_r(t)$ are the modal coordinates.
Now, consider the steady state response of an undamped MDOF system with harmonic excitation given by:
$\{p(t)\} = \{P\} cos(\Omega t)$

In this case, according to Craig (1981), the response is given by:

$$\{u(t)\} = \sum_{r=1}^{N} \Phi_r \left(\frac{F_r}{K_r}\right)\left(\frac{1}{1-(\omega/\omega_r)^2}\right) cos(\omega t) \tag{8}$$

where:
$\{F_r\} = [\Phi]^t \{P\}$

$$u(t) = \begin{Bmatrix} x \\ \gamma \\ y \\ \psi \end{Bmatrix}$$

Substituting the correct values into Eq. 8, we obtain the vectors of displacement of the system.

$$x(t) = \Phi_{11}\left(\frac{F_1}{K_1}\right)\left(\frac{1}{1-(\omega/\omega_1)^2}\right) sin(\omega t) + \Phi_{12}\left(\frac{F_2}{K_2}\right)\left(\frac{1}{1-(\omega/\omega_2)^2}\right) sin(\omega t) + \Phi_{13}\left(\frac{F_3}{K_3}\right)\left(\frac{1}{1-(\omega/\omega_3)^2}\right) sin(\omega t)$$
$$+ \Phi_{14}\left(\frac{F_4}{K_4}\right)\left(\frac{1}{1-(\omega/\omega_4)^2}\right) sin(\omega t)$$

$$\gamma(t) = \Phi_{21}\left(\frac{F_1}{K_1}\right)\left(\frac{1}{1-(\omega/\omega_1)^2}\right) sin(\omega t) + \Phi_{22}\left(\frac{F_2}{K_2}\right)\left(\frac{1}{1-(\omega/\omega_2)^2}\right) sin(\omega t) + \Phi_{23}\left(\frac{F_3}{K_3}\right)\left(\frac{1}{1-(\omega/\omega_3)^2}\right) sin(\omega t)$$
$$+ \Phi_{24}\left(\frac{F_4}{K_4}\right)\left(\frac{1}{1-(\omega/\omega_4)^2}\right) sin(\omega t)$$

$$y(t) = \Phi_{31}\left(\frac{F_1}{K_1}\right)\left(\frac{1}{1-(\omega/\omega_1)^2}\right) cos(\omega t) + \Phi_{32}\left(\frac{F_2}{K_2}\right)\left(\frac{1}{1-(\omega/\omega_2)^2}\right) cos(\omega t) + \Phi_{33}\left(\frac{F_3}{K_3}\right)\left(\frac{1}{1-(\omega/\omega_3)^2}\right) cos(\omega t)$$
$$+ \Phi_{34}\left(\frac{F_4}{K_4}\right)\left(\frac{1}{1-(\omega/\omega_4)^2}\right) cos(\omega t)$$

$$\psi(t) = \Phi_{41}\left(\frac{F_1}{K_1}\right)\left(\frac{1}{1-(\omega/\omega_1)^2}\right) cos(\omega t) + \Phi_{42}\left(\frac{F_2}{K_2}\right)\left(\frac{1}{1-(\omega/\omega_2)^2}\right) cos(\omega t) + \Phi_{43}\left(\frac{F_3}{K_3}\right)\left(\frac{1}{1-(\omega/\omega_3)^2}\right) cos(\omega t)$$
$$+ \Phi_{44}\left(\frac{F_4}{K_4}\right)\left(\frac{1}{1-(\omega/\omega_4)^2}\right) cos(\omega t)$$

Figure 3 below shows a plotting of the vector x along the time. It was used a unbalanced mass of 1.0kg.

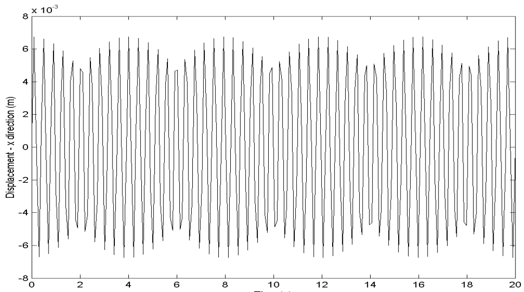

Figure 3. Plotting of the displacement in the x direction.

If we plot the x displacement x versus y displacement we will get the orbit of the machine.

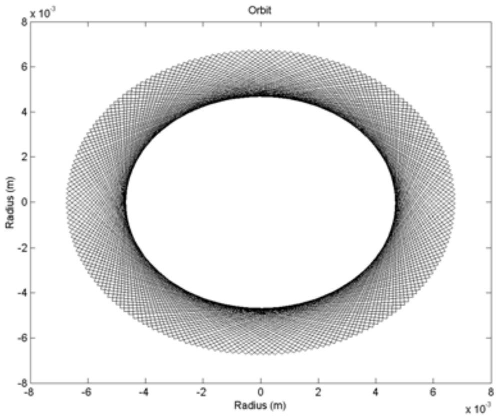

Figure 4. Orbit of the machine.

Substituting x and γ into Eq. 1 and multiplying it by the stiffness k, we get the response force of a suspension spring. See Fig. 5 bellow.

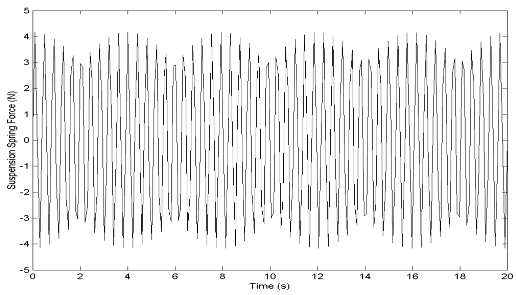

Figure 5. Response force of a suspension spring.

3.2. Experimental method

To study the dynamic behavior of a washing machine during the spinning experimentally, it was used a vibration analyzer and some accelerometers connected to the washing machine, in four points according to Fig. 6.

The measurement of the displacements in the point 1 (x and y direction) gives the upper orbit (during the steady state and transient). The direction z gives the vertical displacement of the tub.

In the point 2 (x and y) gives the lower orbit (during the steady state and transient).

The point 3 was used to get the vertical displacement of the top of the tub, and in the point 4 the measurement of the velocity gives the vibration level of the cabinet.

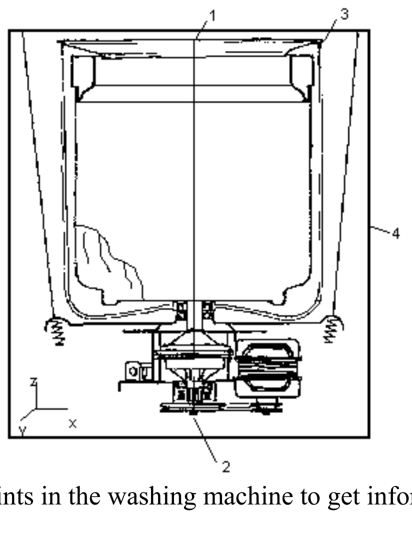

Figure 6. Points in the washing machine to get information

The figures below show some results gotten with this approach. It was used a unbalanced mass of 1.0kg.

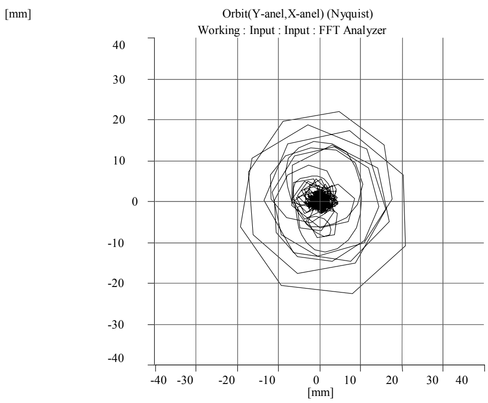

Figure 7. Upper orbit.

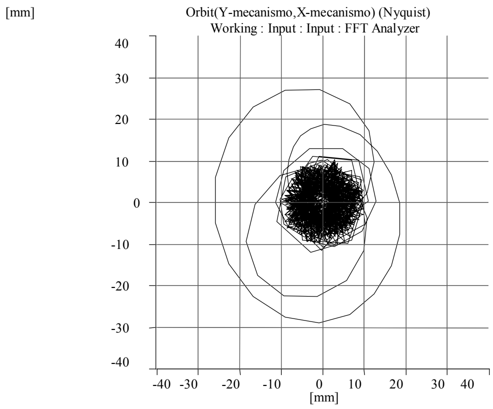

Figure 8. Lower orbit.

Figure 7 shows the behavior of the upper orbit. Figure 8 shows the behavior of the lower orbit. During the transient the radius of the orbit is bigger than during the steady state. With these data we can check if the tub will hit the cabinet and have an idea of the vibration level during the steady state.

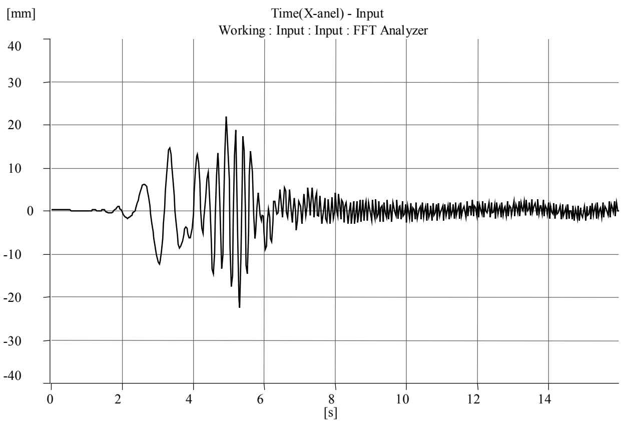

Figure 9. Upper Tub displacement (x direction).

Figure 9 shows the upper displacement of the tub in the x direction on the horizontal plane. The same evaluation done using Fig. 7 and 8 can be done here.

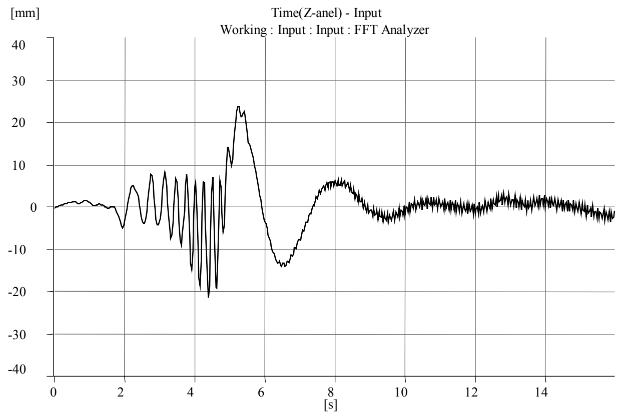

Figure 10. Vertical Tub displacement.

Figure 10 shows the vertical displacement of the tub in the point 3. This kind of result is useful to check hitting of the tub with the top of the machine or components assembled on the top.

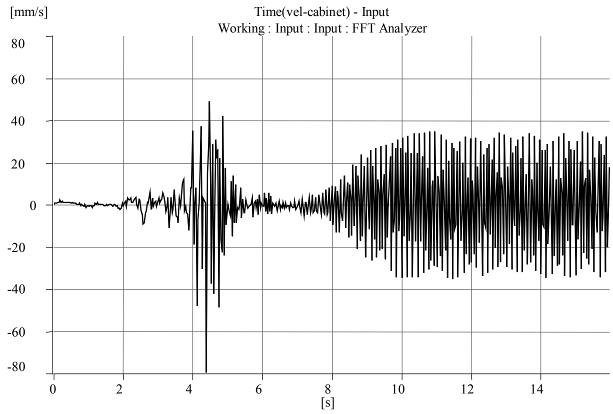

Figure 11. Vibration of the lateral panel of the cabinet (point 4).

The cabinet vibration is the main generator of noise during the spinning. So, to reduce noise level of the machine and consequently the customer complain, a low level of cabinet vibration must be achieved.

3.3.Simulation Using MBS

The multi body system (MBS) are mechanical systems composed by rigid parts that have relative motion among them. These parts are interconnected by joints that are put in action by movements pre set or forces and are submitted to constraints.

In this work it was used the software ADAMS from Mechanical Dynamics Inc. to simulate the dynamic behavior of the washing machine during the spinning.

The figure below shows the model used in this work.

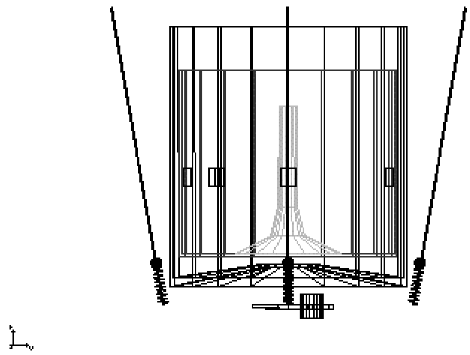
Figure 12. Model used in this work.

In the model above, the tub, the basket, the drive system and the rod suspension were modeled as rigid parts, and were connected among them using joints.

Using this method and the model showed in the figure above is possible to get some results that help the obtainment of the dynamic behavior of the washing machine during the spinning. Some of them are: upper and lower orbits during the steady state and transient, amplitude of the rod suspension forces.

The figures below show some results gotten with this approach. It was used a unbalanced mass of 1.5kg.

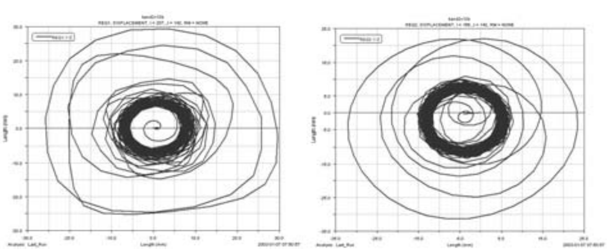
Figure 13. Upper orbit and Lower orbit.

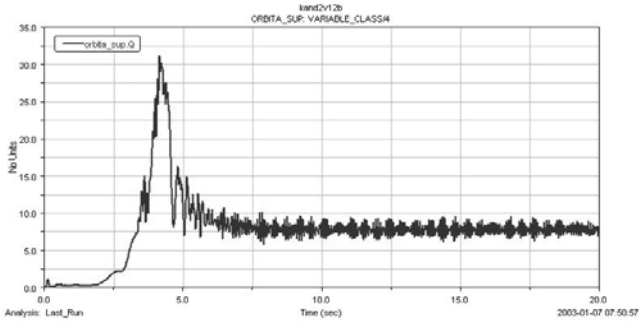

Figure 14. Upper orbit radius.

Figures 13(a) and 14 are two ways to show the behavior of the upper orbit. Figure 13(b) shows the behavior of the lower orbit. During the transient the radius of the orbit is bigger than during the steady state. With these data we can check if the tub will hit the cabinet and have an idea of the vibration level during the steady state.

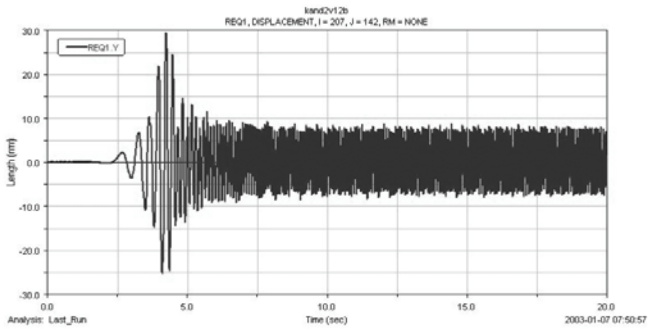

Figure 15. Tub displacement (top).

Figure 15 shows the upper displacement of the tub in the y direction on the horizontal plane. The same evaluation done using Fig. 13, 14 can be done here.

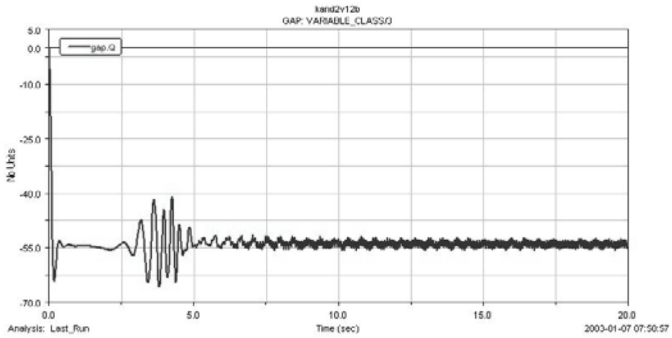

Figure 16. Vertical Tub displacement.

Figure 16 shows the vertical displacement of the tub. This kind of result is useful to check hitting of the tub with the top or components assembled on the top.

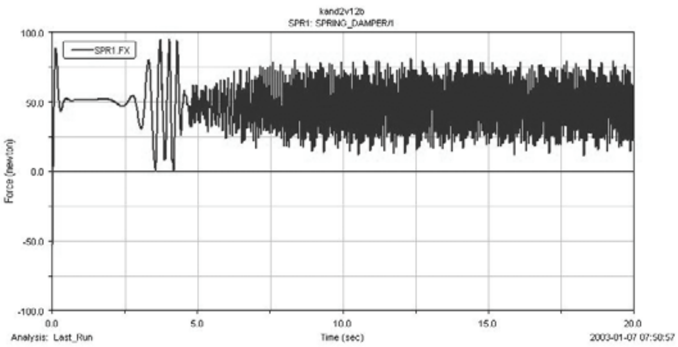

Figure 17. Rod suspension spring force.

The force in the rod suspension spring during the steady state is very useful to predict the vibration transmitted to the cabinet. As bigger the amplitude force as bigger will be the cabinet vibration.

4. Conclusions

In this work it was showed three methods to study the dynamic behavior of a washing machine during the spinning. Unfortunately, we can't get all the results needed to describe the dynamic behavior using only one method.

The analytical method is a good way to get some results very quickly and with some convenience. The cost to use this method is very low and with it we can test some suspension design with no cost. Its problem is the limitation in getting some important results.

The experimental method is very good way to get almost all the results needed to describe the dynamic behavior. Using this method is very useful to get the cabinet vibration of the machine, the main complain of the consumers. The cost to use this method is not so low, it needs to have a good signal analyzer and some accelerometers. The need for physical prototypes is another difficulty in using this method.

The method using simulation through MBS is the most useful method. Once we have the model of the machine, we can get a lot of results for many design configurations with no cost with physical prototypes. The problems using this method are the cost of the simulation software and the knowledge needed to create a good model for numerical simulation.

Unfortunately, due to confidential issue, the comparison of the results gotten through the three methods can not be showed. But, the authors can say that they are in a very good correlation.

5. References

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