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Comparison of Algorithms for Parameter Estimation with Process and Measurement Noise

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Abstract: This work presents the comparison between techniques applied to parameter identification of an aircraft accounting for process and measurement noise based on the following methods: Output error approach and Filtering approaches. The Output error approach, which can account only for measurement noise, uses the iterative Levenberg Marquardt optimization method to estimate the unknown parameters by minimization of a likelihood cost function. In the filtering approaches, accounting for both process and measurement noise, two different methods are considered. In the first, the unknown parameters can be estimated as augmented states using the Extended Kalman Filter. The second concerns Filter Error methods which are similar to the Output Error method and the filter is used only for the natural purpose of obtaining the true state variables from the noisy measurements, i.e., for state estimation. Simulated data is used to evaluate the methods performance.

Keywords: system identification, parameter estimation, Kalman Filter, Output Error, Filter Error

NOMENCLATURE

u = longitudinal component of velocity, m/s
α = angle of attack, rad
q = pitch angular velocity, rad/s
θ = pitch angle, rad
a_z = vertical acceleration, g
δ_e = elevator deflection, rad

G = process noise intensity matrix
F = measurement noise intensity matrix
P = covariance matrix
K = Kalman gain matrix
J = Cost function

Subscripts
ω = relative to process noise
ν = relative measurement noise
k = relative to sample
a = relative to system augmentation
c = relative to change in vector or matrix due to small change in the system parameters

INTRODUCTION

One of the most important phases of the design and evaluation process in the aeronautical design nowadays concerns modeling and simulation. In order to accomplish this task, the system identification and parameter estimation constitutes a fundamental step. System Identification is a general procedure to match the observed input-output response of a dynamic system by a proper choice of an input-output model and its physical parameters. From this point of view, the aircraft system identification or inverse modelling comprises proper choice of aerodynamic models, the development of parameter estimation techniques by optimization of the mismatch error between predicted and real aircraft response and the development of proper tools for integration of the equations of motion within the system simulation and correlated activities. The techniques analyzed in this work can be subdivided in the following way:

- *Output Error Method (OEM)*: method which minimizes the error between the values of output variables predicted by a given model, and the values of these variables measured in the real system. Output error methods do not work properly with process noise;

- *Extended Kalman Filter (EKF)*: the parameters are defined as additional state variables and then a nonlinear filter is used to estimate the augmented state vector;

- *Filter Error Method (FEM)*: similar to the Output Error Method, the parameters are estimated by minimization of a cost function involving the output variables error. In this case, a filter is implemented to estimate the state variables and then, process and measurement noise can be taken into account.

The application of methods that take into account measurement noise only, which is the case of the Output Error analyzed in this work, to extract the aerodynamic coefficients from flight test data have been successfully used in the literature. However, the application of algorithms based on stochastic filtering, that can handle both measurement and process noise, are becoming necessary, because then it is possible to analyze flight test data obtained in turbulent atmosphere or even improve the estimation results compared to the traditional methods.

Thus, three different estimation algorithms that enable parameter estimation are critically evaluated from a viewpoint of computation complexity and time, convergence properties, parameter estimates, and their accuracies. These algorithms are applied to systems with additive process and measurement noise.

This work is structured as follows: first, we present the equations of the dynamic model, which represents the linear longitudinal equations of an aircraft motion with the parameters to be estimated and the covariance matrices of the process and measurement noise. In the sequence, the algorithms for parameter estimation are outlined in the following order: Output Error Method, Extended Kalman Filter and Filter Error Method. Then , the results are presented with the analysis and comparison of the algorithms and the concluding remarks.

DYNAMIC MODEL OF LONGITUDINAL MOTION

The aircraft dynamic system is described by a stochastic linear model. In this section the inverse problem formulation is applied to the longitudinal movement of the aircraft, for which the linear state and output equations can be written as,

State equations:

$$\begin{bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_\alpha & X_q & -g \\ Z_u/U_0 & Z_\alpha/U_0 & 1 & 0 \\ M_u & M_\alpha & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} . \begin{bmatrix} u \\ \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} \\ 0 \end{bmatrix} .\delta_e + G.\omega(t) \tag{1}$$

Observation equations:

$$\begin{bmatrix} u \\ \alpha \\ q \\ \theta \\ a_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{U_0}{g} Z_u & -\frac{U_0}{g} Z_\alpha & 0 & 0 \end{bmatrix} . \begin{bmatrix} u \\ \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{U_0}{g} M_{\delta_e} \end{bmatrix} .\delta_e + F.\nu(t) \tag{2}$$

The intensity matrices of the process and measurement noise are given by,

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad e \quad F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The process noise, $\omega(k)$, and the measurement noise, $\nu(k)$ are defined as Gaussian white noise such that,

$$\begin{aligned} &E[\omega(k)] = 0,\ E[\nu(k)] = 0,\ E[\omega(k)\omega(j)^T] = P_\omega \delta(k-j),\ E[\nu(k)\nu(j)^T] = P_\nu \delta(k-j), \\ &E[x(k)\omega(j)^T] = 0,\ E[\nu(k)\omega(j)^T] = 0,\ E[x(k)\nu(j)^T] = 0. \end{aligned} \tag{4}$$

where the covariance matrices are given by

$$P_\omega = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0 & 1.5e-2 & 0 & 0 \\ 0 & 0 & 1.5e-2 & 0 \\ 0 & 0 & 0 & 1.5e-2 \end{bmatrix} \quad e \quad P_\nu = \begin{bmatrix} 0.25 & 0 & 0 & 0 & 0 \\ 0 & 7.61e-5 & 0 & 0 & 0 \\ 0 & 0 & 1.22e-5 & 0 & 0 \\ 0 & 0 & 0 & 7.61e-5 & 0 \\ 0 & 0 & 0 & 0 & 0.01 \end{bmatrix} \tag{5}$$

The parameter vector Θ to be estimated is defined as

$$\Theta = \left[X_u\ X_\alpha\ X_q\ Z_u\ Z_\alpha\ M_u\ M_\alpha\ M_q\ X_{\delta_e}\ Z_{\delta_e}\ M_{\delta_e}\right]^T \tag{6}$$

OUTPUT ERROR METHOD

In this section, the parametric identification, in particular the parameter estimation applied to a linear causal model of an aircraft, in space state formulation according to eq. (1) and (2) is considered. The Output Error Method is one of the most used estimation methods in aircraft identification and aerodynamic parameter estimation. It has several desirable statistical properties, including its application to nonlinear dynamical systems and the proper accounting of measurements noise.

The structure of the model is considered to be known, and the identification process consists in determining the parameter vector Θ, which gives the best prediction of the output signal $z(t)$, using some sort of optimization criteria. The attainment of an estimate through optimization of a cost function based on the prediction error of the plant requires, usually, the minimization of a nonlinear functional. Thus, the Levenberg-Marquardt method is used here as the optimization algorithm.

The cost function to be minimized involves the prediction error,

$$e(k,\hat{\Theta}) = z(k) - y(k,\hat{\Theta}) \tag{7}$$

where $y(k,\hat{\Theta})$ is the output prediction based on the actual estimate $\hat{\Theta}$ of the parameter vector Θ.

First we define the Likelihood Functional, $p(z|\Theta)$, as the probability Gaussian density function of the variable y for a given parameter vector Θ, with mean $f(\Theta)$ and covariance R^T,

$$p(z|\Theta) = \frac{1}{(2\pi)^{m/2}|R^T|^{n/2}}.exp\left\{-\frac{1}{2}\sum_{k=1}^{n}[z(k)-y(k)]^T[R^T]^{-1}[z(k)-y(k)]\right\} \tag{8}$$

The Maximum Likelihood Estimate (MLE) is the value of Θ that maximizes this functional,

$$\hat{\Theta} = \underset{\Theta\in D_M}{Arg}\ Max\ p(\mathrm{z}|\Theta) \tag{9}$$

Then we define the cost function as:

$$J = \frac{1}{2}\sum_{k=1}^{N}[z(k)-y(k)]^T R^{-1}[z(k)-y(k)] + \frac{N}{2}\ln|R| \tag{10}$$

In the case where R is unknown, equating $\partial J/\partial R$ to zero gives the maximum likelihood estimate of R:

$$\hat{R} = \frac{1}{N}\sum_{k=1}^{N}[z(k)-y(k)]\,[z(k)-y(k)]^T \tag{11}$$

Minimizing $J(\Theta)$ is maximizing $p(y|\Theta)$, because $J(\Theta)$ is equivalent to $-ln\,p(y|\Theta)$ except for a constant term.

The optimization necessary condition is obtained when

$$\nabla_\Theta J(\Theta) = 0 \tag{12}$$

Taylor series expansion about the k-th value provides

$$[\nabla_\Theta J(\Theta)]_{i+1} \cong [\nabla_\Theta J(\Theta)]_i + [\nabla^2_\Theta J(\Theta)]_i.(\Theta_{i+1}-\Theta_i) = 0 \tag{13}$$

which can be used to find the minima of the original cost function through the recursion,

$$\Theta_{i+1} = \Theta_i + [\nabla^2_\Theta J(\Theta)]_i^{-1}[\nabla_\Theta J(\Theta)]_i^T \tag{14}$$

The complexity in the calculation of the Hessian matrix, $\nabla^2_\Theta J(\Theta)$ in (14), is avoided through the Gauss-Newton method, which uses the approximations,

$$\nabla_\Theta J(\Theta) = \sum_{k=1}^{N}\left[\frac{\partial y}{\partial\Theta}(k)\right]^T R^{-1}[z(k)-y(k)] \tag{15}$$

$$\nabla^2_\Theta J(\Theta) = \sum_{k=1}^{N}\left[\frac{\partial y}{\partial\Theta}(k)\right]^T R^{-1}\left[\frac{\partial y}{\partial\Theta}(k)\right] \tag{16}$$

The gradient of the estimated output, $\frac{\partial y}{\partial\Theta}(k)$, is called *Sensibility Function* and can be obtained analytically for a linear system by partial differentiation of its equations. A better approach is to approximate this differentiation by finite differences. In this procedure, the parameters Θ are perturbed one at a time and the corresponding perturbed model response y_c is computed. The sensitivity coefficient is then given by,

$$\frac{\partial y}{\partial\Theta_j} = \frac{(y_c - y)}{\Delta\Theta_j} \tag{17}$$

The Levenberg-Marquardt algorithm is an extension of the Gauss-Newton. The idea is to modify eq. (16) to

$$\nabla^2_\Theta J(\Theta) \approx \sum_{k=1}^{N} \left[\frac{\partial y}{\partial \Theta}(k)\right]^T R^{-1} \left[\frac{\partial y}{\partial \Theta}(k)\right] + \lambda I \tag{18}$$

and the inversion of the Hessian in (14) is not performed in an explicit manner, i.e., typically the original equation

$$\left[\nabla^2_\Theta J(\Theta) + \lambda I\right] \Delta\hat{\Theta} = \nabla^T_\Theta J(\Theta_i) \tag{19}$$

is solved via Singular Value Decomposition (SVD).

The inclusion of λI in (19) solves the problem of an ill conditioned approximated Hessian. The Levenberg-Marquardt algorithm can be interpreted in the following way: for small values of λ it behaves like the Gauss-Newton algorithm, while for high values of λ it behaves like the gradient method.

FILTERING APPROACH (EXTENDED KALMAN FILTER)

The Extended Kalman Filter is a sub-optimal solution for the nonlinear filtering problem. The nonlinear functions f and g are linearized in each new estimated/filtered state. The simultaneous estimation of states and parameters is obtained by augmenting the state vector with unknown parameters (as additional states) and by using the filtering algorithm with the nonlinear augmented model.

By considering the constant vector parameters Θ as the outputs of an auxiliar dynamic system,

$$\dot{\Theta} = 0 \tag{20}$$

and by defining the augmented state vector

$$x_a^T = \left[\begin{array}{cc} x^T & \Theta^T \end{array}\right] \tag{21}$$

the augmented system can be represented by

$$\begin{aligned} x_a^T(t) &= f_a\left[x_a(t), u(t)\right] + G_a \omega_a(t) \\ &= \left[\begin{array}{c} f\left[x(t), u(t), \Theta\right] \\ 0 \end{array}\right] + \left[\begin{array}{c} G \\ 0 \end{array}\right] \omega(t) \\ y(t) &= g_a\left[x_a(t), u(t)\right] \\ z(k) &= y(k) + F.\upsilon(k) \qquad , \quad k = 1, \cdots, N \end{aligned} \tag{22}$$

In this model, F is supposed to be the identity matrix.

The estimation algorithm is obtained linearizing the equations at the current state estimate, at each time step and applying the Kalman Filter algorithm to the linearized model. The linearized matrices of the system are given by

$$A(k) = \left.\frac{\partial f_a}{\partial x_a}\right|_{x_a = \hat{x}_a(k),\, u = u(k)} \tag{23}$$

$$C(k) = \left.\frac{\partial g_a}{\partial x_a}\right|_{x_a = \hat{x}_a(k),\, u = u(k)} \tag{24}$$

and the state transition matrix is calculated as

$$\phi(k) = \exp[-A(k)\Delta t] \tag{25}$$

The filtering algorithm is divided in two parts: (i) time propagation, and (ii) measurement update.

Time propagation:

The current estimate is used to predict the next state, in such a way that the states are propagated from the current state to the next time instant. The predicted state is given by

$$\tilde{x}_a(k+1) = \hat{x}_a(k) + \int_{t_k}^{t_{k+1}} f_a[\hat{x}_a(t), u(t), t]dt \tag{26}$$

In the absence of knowledge about the process noise, eq. (26) gives the state prediction based on the initial/current estimate. The covariance matrix of the state estimation error propagates from instant k to $k+1$ as

$$\tilde{P}(k+1) = \phi(k)\hat{P}(k)\phi^T(k) + G_a(k)QG_a^T(k) \tag{27}$$

Where, $\tilde{P}(k+1)$ is the covariance matriz prediction for instant $k+1$, G_a is the coefficient matrix related to the process noise, and Q is the covariance matrix of the measurement noise.

Measurement update:

The Extended Kalman Filter updates the state estimates incorporating the measurements in the following way:

$$\hat{x}_a(k+1) = \tilde{x}_a(k+1) + K(k+1)\{z_m(k+1) - g_a[\tilde{x}_a(k+1), u(k+1), t]\} \tag{28}$$

where, K is the Kalman matrix gain.

The covariance matrix is updated using the Kalman gain, the linearized measurement matrix and the predicted covariance matrix $\tilde{P}(k+1)$.

The Kalman gain matrix is given by

$$K(k+1) = \tilde{P}(k+1)C^T(k+1)[C(k+1)\tilde{P}(k+1)C^T(k+1) + R]^{-1} \tag{29}$$

The expression for the a posteriori covariance matrix is given by

$$\hat{P}(k+1) = [I - K(k+1)C(k+1)]\tilde{P}(k+1) \tag{30}$$

FILTER ERROR METHOD

The Filter Error Method is the most general approach for parametric estimation taking into account process and measurement noise. In this approach, like the OEM, we also define a cost function to be minimized with respect to the parameter vector using a convenient optimization algorithm. The Levenberg-Marquardt method is used here. The stochastic filtering is used with the only purpose of estimating the state variables, x, through the filtering of the noisy measurements, z.

Among the algorithms presented in this article, the Filter Error with time varying filter is the most complex to implement. In contrast to the stationary filter, he matrices S, K and P are no longer constants and have to be calculated at each sampling time k. Similarly, the matrices A and C obtained through first order linearization of the system equations are calculated at each iteration.

The time varying method, is formulated in the following way. The cost function to be minimized is given by

$$J = \frac{1}{2}\sum_{k=1}^{N}[z(k) - y(k)]^T S^{-1}(k)[z(k) - y(k)] + \sum_{k=1}^{N}\frac{1}{2}\ln|S(k)| \tag{31}$$

where the covariance innovations matrix S is updated at each sampling point k.

The steps for time propagation (prediction) and correction used to obtain the updated values of states, $\hat{x}$, and the state error covariance matrix $\hat{P}$ are given below.

Time propagation:

$$\begin{aligned} \tilde{x}(k) &= \hat{x}(k-1) + \int_{t_{k-1}}^{t_k} f[x(t), u_e(k), \Theta]\, dt \\ \tilde{y}(k) &= g[\tilde{x}(k), u(k), \Theta] \end{aligned} \tag{32}$$

Assuming that Δt is small, the predicted matrix P can be approximated by

$$\tilde{P}(k) \approx \Phi\hat{P}(k-1)\Phi^T + \Delta t GG^T \tag{33}$$

Correction:

$$K(k) = \tilde{P}(k)C^T(k)[C(k)\tilde{P}(k)C^T(k) + R]^{-1} \tag{34}$$

$$\hat{x}(k) = \tilde{x}(k) + K(k)[z(k) - \tilde{y}(k)] \tag{35}$$

$$\begin{aligned} \hat{P}(k) &= [I - K(k)C(k)]\tilde{P}(k) \\ &= [I - K(k)C(k)]\tilde{P}(k)[I - K(k)C(k)]^T + K(k)RK^T(k) \end{aligned} \tag{36}$$

The equation for $\hat{P}$ in eq. (36) with quadratic form is usually preferred, since it is numerically stable converges faster.

Once the filtered states are obtained, the estimates of the parameters are given by

$$\Theta_{i+1} = \Theta_i + [\nabla^2_{\Theta} J(\Theta)]_i^{-1} [\nabla_{\Theta} J(\Theta)]_i^T \tag{37}$$

$$\nabla_{\Theta} J(\Theta) = \sum_{k=1}^{N} \left[\frac{\partial y}{\partial \Theta}(k) \right]^T S^{-1}(k) \left[z(k) - y(k) \right] \tag{38}$$

$$\nabla^2_{\Theta} J(\Theta) = \sum_{k=1}^{N} \left[\frac{\partial y}{\partial \Theta}(k) \right]^T S^{-1}(k) \left[\frac{\partial y}{\partial \Theta}(k) \right] \tag{39}$$

As in the case of OEM, the gradient $\partial y / \partial \Theta$ can be obtained by the introduction of small perturbations in each of the system parameters one at a time. The change in the response of the system due to this small perturbation can be obtained by the following equations:

$$\begin{aligned} \tilde{x}_c(k) &= \hat{x}_c(k+1) + \int_{t_{k-1}}^{t_k} f[x_c(t), u_e(k), \Theta + \Delta\Theta] \, dt \\ y_c(k) &= g[\tilde{x}_c(k), u(k), \Theta + \Delta\Theta] \\ \hat{x}_c(k) &= \tilde{x}_c(k) + K_c[z(k) - y_c(k)] \end{aligned} \tag{40}$$

where the subscripts c represent the change in vector or matrix due to small perturbations in the parameters. Note that the computation of the change in the state variable needs the perturbed Kalman gain matrix K_c, which can be obtained from

$$K_c = P_c C_c^T S^{-1} \tag{41}$$

The state error covariance matrix P_c, necessary for the computation of K_c in eq. (41), can be obtained from eq. (33).

Once the perturbed system response y_c is obtained using the set of equations above, the gradient $\partial y / \partial \Theta$ can be easily obtained. Assuming that y_{c_i} represents the change i the n-th component of the output vector y corresponding to a perturbation in parameter Θ_j, the gradient is given by

$$\begin{aligned} &\left(\frac{\partial y(k)}{\partial \Theta} \right)_{ij} \approx \frac{y_{c_i}(k) - y_i(k)}{\Delta\Theta_j} \\ &for\ i = 1, ..., m\ and\ j = 1, ..., q \end{aligned} \tag{42}$$

where q represents the dimension of the parameter Θ.

In the time varying approach the matrix S is calculated directly from eq. (34):

$$S(k) = C(k)\tilde{P}(k)C^T(k) + R \tag{43}$$

To calculate $S(k)$ from eq. (43) it is necessary the value of noise measurement covariance matrix R, which can be estimated by using Fourier smoothing. In this approach, Fourier series analysis is used to smooth the measured data and separate out the clean signal from noise based on spectral content.

RESULTS

By running a simulation of 30s with sampling time of 0.05s, 600 data samples of input and output signals are obtained, for which the algorithms are applied. The same initial estimate of the parameters are used for all runs, which corresponds to 30 % of the nominal values.

Table 1 presents the results of estimation of the parameters and the estimate of the relative standard deviation, calculated from the Fischer Information matrix. We also compare the final value of the cost function achieved, the number of iterations and the computer time required by each method.

According to the results achieved, it can be observed that some parameters estimates by Output Error Method are biased, as was expected, due to the process noise present in the data.

The Extended Kalman Filter algorithm presents estimates of the parameters that are close to the nominal value. As a recursive parameter estimation algorithm, all data points are processed together at a time, yielding parameters representing average system behavior. The computer memory requirements for this method are small, because storage of past data is not required. In Table 1 it cab be observed the computer time spent by the algorithm, which is only 6.93 s. The disadvantage of the method is the requirement of a fine tuning of the covariance matrices. In this particular case, the estimates are close to nominal because noise characteristics are known.

The best estimates are achieved by the Filter Error Method, which combines the advantages of the non-recursive estimation methods and the stochastic filtering of the states. From the values of parameters estimates, relative standard deviation and number of iterations, its improved convergence can be observed. The disadvantage of the method is the computational complexity, which can be seen from the computer time spent by the algorithm. The figures 1 and 2 present the evolution of the parameter estimate by the number of iterations for the Filter error method. The plots in figure 3 demonstrate the prediction capacity of the method showing the measured variables and their prediction with the estimated parameters.

Table 1 – Comparison of parameter estimation.

Parameters	Nominal	OEM	EKF	FEM
X_u	-0.0091	0.0151 (16.20)	-0.088 (17.15)	-0.080 (6.26)
X_α	9.43	18.8525 (8.78)	7.0608 (94.04)	8.1864 (2.20)
X_q	0	12.3708 (14.70)	-0.3603 (623)	-0.6321 (5.62)
Z_u	-0.088	-0.084 (2.79)	-0.088 (3.78)	-0.088 (0.41)
Z_α	-34.68	-35.392 (0.89)	-34.216 (1.05)	-34.364 (0.29)
M_u	0	0.0003 (78.78)	-0.0001 (93.08)	-0.0003 (4.94)
M_α	-3.49	-3.6061 (0.99)	-3.2727 (1.11)	-3.3323 (0.28)
M_q	-2.04	-1.8644 (2.53)	-1.8446 (2.91)	-1.7208 (0.60)
X_{δ_e}	0	25.5723 (11.71)	-2.9457 (539.90)	-1.8619 (6.51)
Z_{δ_e}	-0.11	-0.1298 (6.06)	-0.0977 (8.88)	-0.1039 (3.08)
M_{δ_e}	-5.09	-5.0205 (1.49)	-4.8050 (1.78)	-4.7196 (0.34)
Cost function	-	5.8450e-14	5.68e-15	2.47e-15
Iterations	-	7	1	5
Time [s]	-	18.18	6.93	69.49

CONCLUDING REMARKS

Three algorithms for parameter estimation were applied to simulated flight data with process and measurement noise. Despite its successful use along the years, the Output Error Method does not show good convergence properties due the process noise in the data. Some estimates are inconsistent and biased compared to the expected values.

In the case of the Extended Kalman Filter, to be able to use this approach for systems with unknown noise statistics, it is necessary to extend the filtering algorithm to include simultaneous estimation of noise covariances, often referred to in the literature as “adaptive filtering”.

The performance of the Filter Error algorithm was the best among the methods considered here. Despite of its complexity, the algorithm showed good convergence properties and is the most adequate for parameter estimation for systems with process and measurement noise.

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REFERENCES

Góes, L.C.S., Hemerly, E.M., Maciel, B.C.O., Rios Neto, W. and Mendonca, C.B., 2006, “Aircraft Parameter Estimation using Output Error Methods” ,Inverse Problems in Science and Engineering Journal, Vol. 14, No. 6, pp. 651-664.

Jategaonkar, R.V. and Plaetschke, E., 1989, “Algorithms for Aircraft Parameter Estimation Accounting for Process and Measurement Noise”, Journal of Aircraft, Vol. 26, No. 4, pp. 360-372.

Jategaonkar, R.V., 2006, “Flight Vehicle System Identification in Time Domain”, AIAA Professional Development Tutorial, Keystone, CO, United States.

Maine, R.E. and Iliff, K.W., 1981, “Formulation and Implementation of a Practical Algorithm for Parameter Estimation with Process and Measurement Noise”, Journal of Applied Mathematics, Vol. 41, No. 3, pp. 558-579.

Raol, J.R., Girija, G. and Singh, J., 2004, “Modelling and Parameter Estimation of Dynamic Systems”, The Institution of Electrical Engineers, London, United Kingdom, 388 p.

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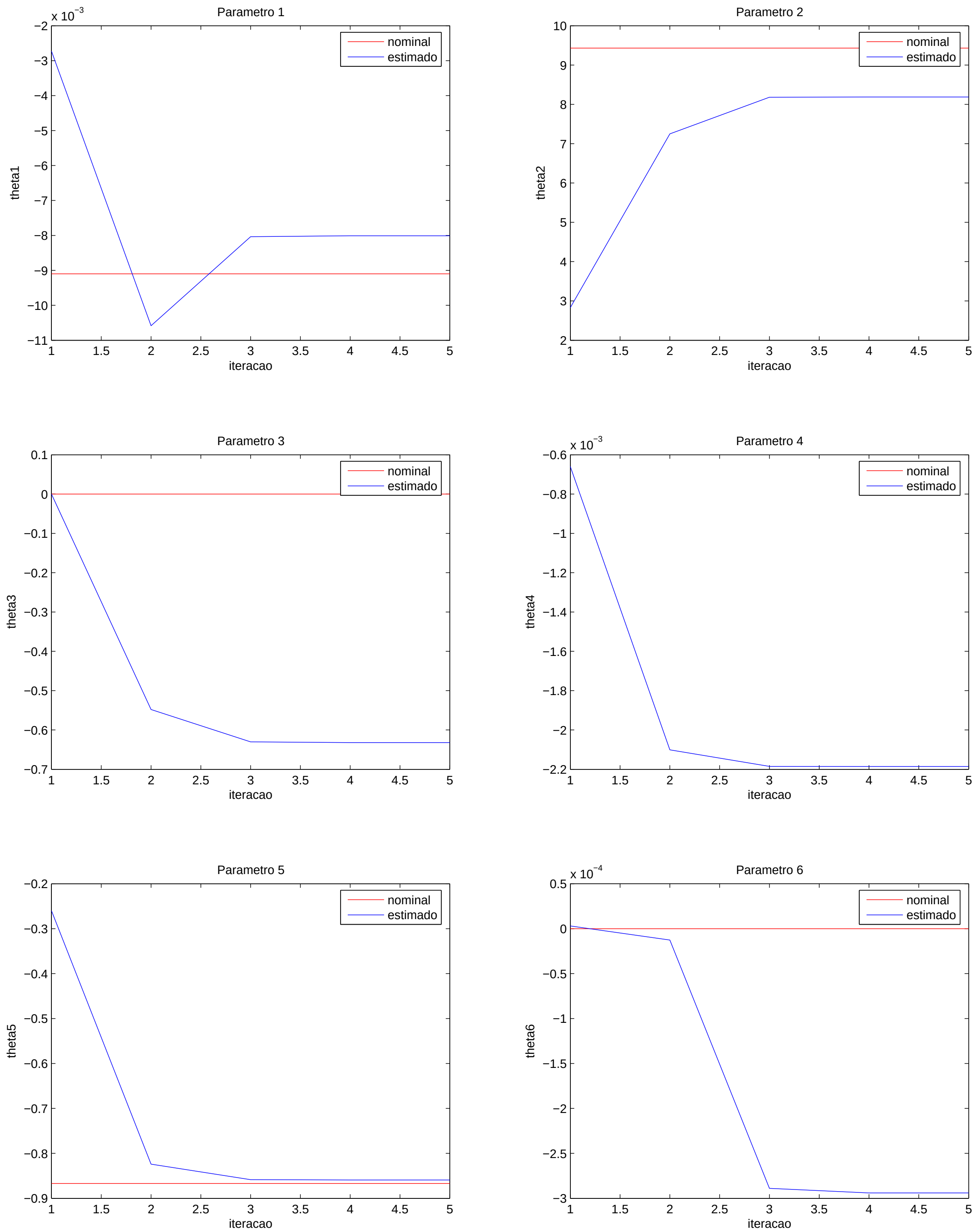

Figure 1 – Evoluo da estimativa dos parmetros.

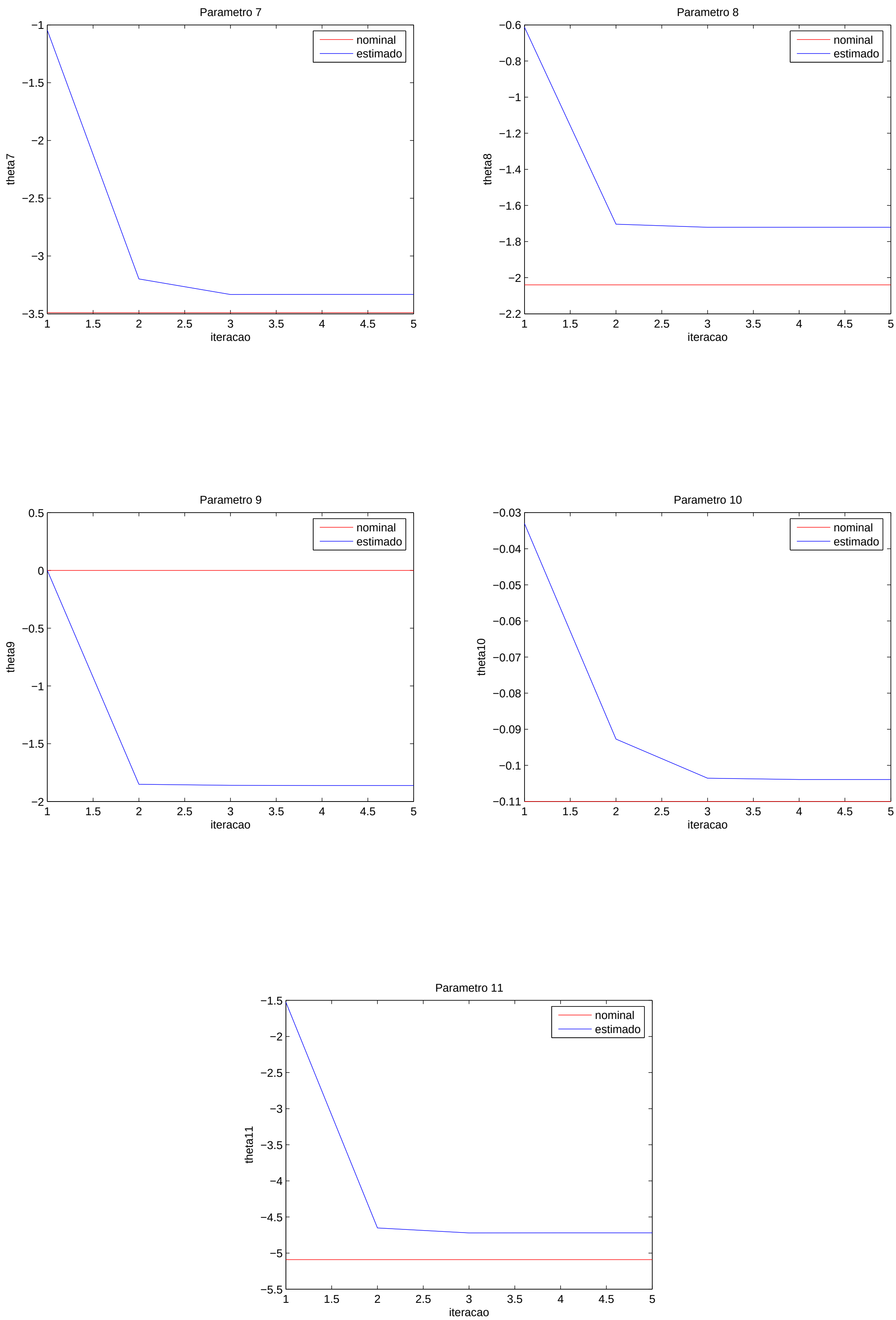

Figure 2 – Evoluo da estimativa dos parmetros.

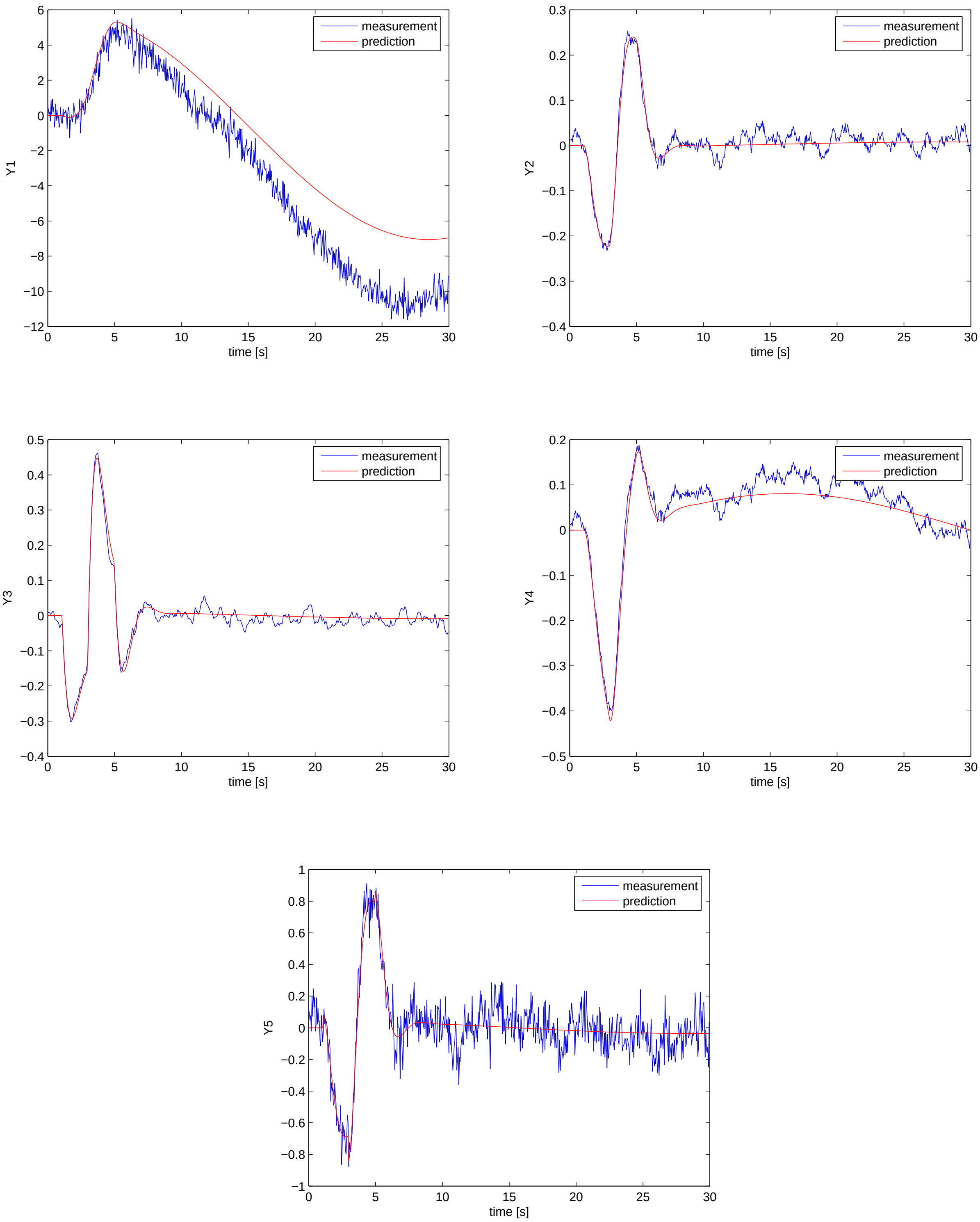

Figure 3 – Fitting of the output values measured and predicted by the Filter error method.