

# INTERFACIAL HEAT TRANSFER COEFFICIENT IN A PACKED BED MODELED AS A PERIODIC CELL SUBJECTED TO PRESCRIBED HEAT FLUX

Marcelo B. Saito<sup>1</sup>

Marcelo J.S. De-Lemos<sup>2\*</sup>

Departamento de Energia - IEME

Instituto Tecnológico de Aeronáutica - ITA

12228-900 - São José dos Campos - SP - Brasil

\* Corresponding author, <sup>1</sup>mbsaito@mec.ita.br, <sup>2</sup>delemos@mec.ita.br.

**Abstract.** *The interfacial heat transfer coefficient in a packed bed is analyzed using a periodic cell with prescribed heat flux. This work intends to obtain functional relationships for the interfacial convective heat transfer coefficient closely packed beds. A periodic fully developed condition is used to represent the entire domain. The numerical methodology employed is based on the control-volume approach with a boundary-fitted non-orthogonal coordinate system. The dimensionless temperature profile and the Nusselt number for the steady periodic state are presented.*

**Keywords.** *Porous Media, Heat Transfer Coefficient, Thermal Non-Equilibrium.*

## 1. Introduction

In recent years, the problem of local thermal non-equilibrium has received considerable attention due to its relevance in a wide variety of engineering applications such as electronic cooling, heat pipes, nuclear reactors, drying technology, multiphase catalytic reactors and others. The use of two-equation model is required for these types of problems. Kuwahara et. al (2001) proposed a numerical procedure to determine macroscopic transport coefficients from a theoretical basis without any empiricism. They used a single unit cell and determined the interfacial heat transfer coefficient for the asymptotic case of infinite conductivity of the solid phase. Nakayama et. al (2001) extended the conduction model of Hsu (1999) for treating also convection in porous media. Having established the macroscopic energy equations for both phases, useful exact solutions were obtained for two fundamental heat transfer processes associated with porous media, namely, steady conduction in a porous slab with internal heat generation within the solid, and also, thermally developing flow through a semi-infinite porous medium.

In all of the above, only laminar flow has been considered. When treating turbulent flow in porous media, however, difficulties arise because the flow fluctuates with time and a volumetric average is applied Gray & Lee (1977). For handling such situations, a new concept called *double decomposition* has been proposed for developing a macroscopic model for turbulent transport in porous media Pedras & de Lemos (2000) Pedras & de Lemos (2001a) Pedras & de Lemos (2001c) Pedras & de Lemos (2001b) Pedras & de Lemos (2003). This methodology has been extended to non-buoyant heat transfer Rocamora & de Lemos (2000), buoyant flows by de Lemos & Braga (2003) and mass transfer by de Lemos & Mesquita (2003). Based on this same concept, de Lemos & Rocamora (2002) have developed a macroscopic turbulent energy equation for a homogeneous, rigid and saturated porous medium, considering local thermal equilibrium between the fluid and the solid matrix. A general classification of all methodologies for treating turbulent flow and heat transfer in porous media has been recently published de Lemos & Pedras (2001).

This work proposes a macroscopic heat transfer analysis using a two-energy equation model for conduction and convection mechanisms in porous media. Saito & de Lemos (2004) proposed an extension of the transport model of de Lemos & Rocamora (2002) considers local thermal non-equilibrium. The contribution herein consists in documenting and testing a detailed numerical model for obtaining the interfacial heat transfer coefficient in a packed bed modeled as a periodic cell subjected to prescribed heat flux.

The next sections details the basic mathematical model, including the mean and turbulent fields for turbulent flows. Although the discussion of turbulent motion in porous media isn't present in this work the definition and concept to calculating the interfacial heat transfer coefficient for macroscopic flows are presented.

## 2. Microscopic Transport Equations

Microscopic transport equations for incompressible fluid flow in a rigid homogeneous porous medium have been already presented in the literature and for that they are here just presented (e.g. reference de Lemos & Rocamora (2002)). They read,

$$\text{Continuity: } \nabla \cdot \mathbf{u} = 0 . \tag{1}$$

$$\text{Momentum: } \mathbf{r} \left[ \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) \right] = -\nabla p + \mathbf{m}\nabla^2 \mathbf{u} . \quad (2)$$

$$\text{Energy - Fluid Phase: } (\mathbf{r} c_p)_f \left\{ \frac{\partial T_f}{\partial t} + \nabla \cdot (\mathbf{u} T_f) \right\} = \nabla \cdot (k_f \nabla T_f) + S_f . \quad (3)$$

$$\text{Energy - Solid Phase (Porous Matrix): } (\mathbf{r} c_p)_s \frac{\partial T_s}{\partial t} = \nabla \cdot (k_s \nabla T_s) + S_s . \quad (4)$$

where the subscripts  $f$  and  $s$  refer to fluid and solid phases, respectively. Here,  $\mathbf{r}$  is the fluid density,  $\mathbf{u}$  is the fluid instantaneous velocity,  $p$  is the pressure,  $\mathbf{m}$  represents the fluid viscosity,  $T$  is the temperature  $k_f$  is the fluid thermal conductivity,  $k_s$  is the solid thermal conductivity,  $c_p$  is the specific heat and  $S$  is the heat generation term. If there is no heat generation either in the solid or in the fluid, one has further  $S_f = S_s = 0$ .

### 3. Decomposition of Flow Variables in Space and Time

Macroscopic transport equations for turbulent flow in a porous medium are obtained through the simultaneous application of time and volume average operators over a generic fluid property  $\mathbf{j}$  Gray & Lee (1977). Such concepts are mathematically defined as,

$$\bar{\mathbf{j}} = \frac{1}{\Delta t} \int_{t_i}^{t_i + \Delta t} \mathbf{j} dt, \text{ with } \mathbf{j} = \bar{\mathbf{j}} + \mathbf{j}' \quad (5)$$

$$\langle \mathbf{j} \rangle^i = \frac{1}{\Delta V_f} \int_{\Delta V_f} \mathbf{j} dV; \langle \mathbf{j} \rangle^v = \mathbf{f} \langle \mathbf{j} \rangle^i; \mathbf{f} = \frac{\Delta V_f}{\Delta V}, \text{ with } \mathbf{j} = \langle \mathbf{j} \rangle^i + \mathbf{j}'' \quad (6)$$

where  $\Delta V_f$  is the volume of the fluid contained in a Representative Elementary Volume (REV)  $\Delta V$ .

The *double decomposition* idea first introduced and fully described in Pedras & de Lemos (2000) Pedras & de Lemos (2001a) Pedras & de Lemos (2001c) Pedras & de Lemos (2001b) Pedras & de Lemos (2003), combines Eqs. (5)-(6) and can be summarized as:

$$\overline{\langle \mathbf{j} \rangle^i} = \langle \bar{\mathbf{j}} \rangle^i; \bar{\mathbf{j}} = \overline{\mathbf{j}}; \langle \mathbf{j} \rangle^i = \langle \mathbf{j} \rangle^i \quad (7)$$

and,

$$\left. \begin{array}{l} \mathbf{j}' = \langle \mathbf{j} \rangle^i + \mathbf{j}'' \\ \overline{\mathbf{j}} = \overline{\mathbf{j}} + \mathbf{j}'' \end{array} \right\} \text{ where } \mathbf{j}'' = \mathbf{j}' - \langle \mathbf{j} \rangle^i = \mathbf{j} - \overline{\mathbf{j}} . \quad (8)$$

Therefore, the quantity  $\mathbf{j}$  can be expressed by either,

$$\mathbf{j} = \overline{\langle \mathbf{j} \rangle^i} + \langle \mathbf{j} \rangle^i + \overline{\mathbf{j}} + \mathbf{j}'' , \quad (9)$$

or

$$\mathbf{j} = \langle \bar{\mathbf{j}} \rangle^i + \bar{\mathbf{j}} + \langle \mathbf{j} \rangle^i + \mathbf{j}'' . \quad (10)$$

The term  $\mathbf{j}''$  can be viewed as either the temporal fluctuation of the spatial deviation or the spatial deviation of the temporal fluctuation of the quantity  $\mathbf{j}$ .

### 4. Macroscopic Flow and Energy Equations

When the average operators (5)-(6) are applied over Eqs. (1)-(2), macroscopic equations for turbulent flow are obtained. Volume integration is performed over a Representative Elementary Volume (REV), Gray & Lee (1977) and Slattery (1967) resulting in,

$$\text{Continuity: } \nabla \cdot \bar{\mathbf{u}}_D = 0 . \quad (11)$$

where,  $\bar{\mathbf{u}}_D = \mathbf{f} \langle \bar{\mathbf{u}} \rangle^i$  and  $\langle \bar{\mathbf{u}} \rangle^i$  identifies the intrinsic (liquid) average of the time-averaged velocity vector  $\bar{\mathbf{u}}$ .

Momentum:

$$\mathbf{r} \left[ \frac{\partial \bar{\mathbf{u}}_D}{\partial t} + \nabla \cdot \left( \frac{\bar{\mathbf{u}}_D \bar{\mathbf{u}}_D}{\mathbf{f}} \right) \right] = -\nabla (\mathbf{f} \langle \bar{p} \rangle^i) + \mathbf{m}\nabla^2 \bar{\mathbf{u}}_D - \nabla \cdot (\mathbf{r} \mathbf{f} \langle \bar{\mathbf{u}} \bar{\mathbf{u}} \rangle^i) - \left[ \frac{\mathbf{m}\mathbf{f}}{K} \bar{\mathbf{u}}_D + \frac{c_F \mathbf{f} \mathbf{r} \bar{\mathbf{u}}_D / \bar{\mathbf{u}}_D}{\sqrt{K}} \right], \quad (12)$$

where the last two terms in Eq. (12), represent the Darcy and Forchheimer contributions by Forchheimer (1901). The symbol  $K$  is the porous medium permeability,  $c_F$  is the form drag or Forchheimer coefficient,  $\langle \bar{p} \rangle^i$  is the intrinsic average pressure of the fluid, and  $\mathbf{f}$  is the porosity of the porous medium.

The macroscopic Reynolds stress  $-\mathbf{r}\mathbf{f}\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i$  appearing in Eq. (12) is given as,

$$-\mathbf{r}\mathbf{f}\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i = \mathbf{m}_f 2\langle\overline{\mathbf{D}}\rangle^v - \frac{2}{3}\mathbf{f}\mathbf{r}\langle k\rangle^i \mathbf{I}, \quad (13)$$

where,

$$\langle\overline{\mathbf{D}}\rangle^v = \frac{1}{2}[\nabla(\mathbf{f}\langle\overline{\mathbf{u}}\rangle^i) + \nabla(\mathbf{f}\langle\overline{\mathbf{u}}\rangle^i)^T], \quad (14)$$

is the macroscopic deformation tensor,  $\langle k\rangle^i = \langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i/2$  is the intrinsic turbulent kinetic energy, and  $\mathbf{m}_f$ , is the turbulent viscosity, which is modeled in de Lemos & Pedras (2001) similarly to the case of clear flow, in the form,

$$\mathbf{m}_f = \mathbf{r}c_m \frac{\langle k\rangle^i}{\langle \mathbf{e}\rangle^i}, \quad (15)$$

The intrinsic turbulent kinetic energy per unit mass and its dissipation rate are governed by the following equations,

$$\mathbf{r}\left[\frac{\partial}{\partial t}(\mathbf{f}\langle k\rangle^i) + \nabla \cdot (\overline{\mathbf{u}}_D \langle k\rangle^i)\right] = \nabla \cdot \left[\left(\mathbf{m} + \frac{\mathbf{m}_f}{\mathbf{s}_k}\right) \nabla(\mathbf{f}\langle k\rangle^i)\right] - \mathbf{r}\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i : \nabla \overline{\mathbf{u}}_D + c_k \mathbf{r} \frac{\mathbf{f}\langle k\rangle^i \overline{\mathbf{u}}_D}{\sqrt{K}} - \mathbf{r}\mathbf{f}\langle \mathbf{e}\rangle^i. \quad (16)$$

$$\mathbf{r}\left[\frac{\partial}{\partial t}(\mathbf{f}\langle \mathbf{e}\rangle^i) + \nabla \cdot (\overline{\mathbf{u}}_D \langle \mathbf{e}\rangle^i)\right] = \nabla \cdot \left[\left(\mathbf{m} + \frac{\mathbf{m}_f}{\mathbf{s}_e}\right) \nabla(\mathbf{f}\langle \mathbf{e}\rangle^i)\right] + c_1 (-\mathbf{r}\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i : \nabla \overline{\mathbf{u}}_D) \frac{\langle \mathbf{e}\rangle^i}{\langle k\rangle^i} + c_2 c_k \mathbf{r} \frac{\mathbf{f}\langle \mathbf{e}\rangle^i \overline{\mathbf{u}}_D}{\sqrt{K}} - c_2 \mathbf{r}\mathbf{f} \frac{\langle \mathbf{e}\rangle^{i^2}}{\langle k\rangle^i}. \quad (17)$$

where,  $c_k$ ,  $c_1$ ,  $c_2$  and  $c_m$  are nondimensional constants.

Similarly, macroscopic energy equations are obtained for both fluid and solid phases by applying time and volume average operators to Eqs. (3)- (4). As in the flow case, volume integration is performed over a Representative Elementary Volume (REV) resulting in,

$$\begin{aligned} (\mathbf{r}c_p)_f \left[ \frac{\partial \mathbf{f}\langle \overline{T}_f \rangle^i}{\partial t} + \nabla \cdot \left\{ \mathbf{f} \left( \langle \overline{\mathbf{u}} \rangle^i \langle \overline{T}_f \rangle^i + \langle \overline{\mathbf{u}'} \rangle^i \langle \overline{T}_f' \rangle^i + \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i \right) \right\} \right] &= \nabla \cdot [k_f \nabla(\mathbf{f}\langle \overline{T}_f \rangle^i)] + \\ \nabla \cdot \left[ \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i k_f \overline{T}_f dA \right] + \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i \cdot k_f \nabla \overline{T}_f dA, & \end{aligned} \quad (18)$$

$$(\mathbf{r}c_p)_s \left[ \frac{\partial (1-\mathbf{f})\langle \overline{T}_s \rangle^i}{\partial t} \right] = \nabla \cdot [k_s \nabla[(1-\mathbf{f})\langle \overline{T}_s \rangle^i]] - \nabla \cdot \left[ \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i k_s \overline{T}_s dA \right] - \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i \cdot k_s \nabla \overline{T}_s dA, \quad (19)$$

where  $\langle \overline{T}_s \rangle^i$  and  $\langle \overline{T}_f \rangle^i$  denote the intrinsically averaged temperature of solid and fluid phases, respectively,  $A_i$  is the interfacial area within the REV and  $\mathbf{n}_i$  is the unit vector normal to the fluid-solid interface, pointing from the fluid towards the solid phase. Eqs. (18) and (19) are the macroscopic energy equations for the fluid and the porous matrix (solid), respectively.

Further, using the *double decomposition* concept, Rocamora & de Lemos (2000) have shown that the fourth term on the left hand side of Eq. (18) can be expressed as:

$$\langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i = \langle (\langle \overline{\mathbf{u}} \rangle^i + \langle \overline{\mathbf{u}'} \rangle^i) (\langle \overline{T}_f' \rangle^i + \langle \overline{T}_f \rangle^i) \rangle^i = \langle \overline{\mathbf{u}} \rangle^i \langle \overline{T}_f' \rangle^i + \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i. \quad (20)$$

Therefore, in view of Eq. (20), Eq. (18) can be rewritten as:

$$\begin{aligned} (\mathbf{r}c_p)_f \left[ \frac{\partial \mathbf{f}\langle \overline{T}_f \rangle^i}{\partial t} + \nabla \cdot \left\{ \mathbf{f} \left( \langle \overline{\mathbf{u}} \rangle^i \langle \overline{T}_f \rangle^i + \langle \overline{\mathbf{u}'} \rangle^i \langle \overline{T}_f \rangle^i + \langle \overline{\mathbf{u}} \rangle^i \langle \overline{T}_f' \rangle^i + \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i \right) \right\} \right] &= \\ \nabla \cdot [k_f \nabla(\mathbf{f}\langle \overline{T}_f \rangle^i)] + \nabla \cdot \left[ \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i k_f \overline{T}_f dA \right] + \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i \cdot k_f \nabla \overline{T}_f dA. & \end{aligned} \quad (21)$$

## 5. Interfacial Heat Transfer Coefficient

In Eqs. (19) and (21) the heat transferred between the two phases can be modeled by means of a film coefficient  $\overline{h}_i$  such that,

$$\overline{h_i a_i} (\langle \overline{T_s} \rangle^i - \langle \overline{T_f} \rangle^i) = \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i \cdot k_f \nabla \overline{T_f} dA = \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i \cdot k_s \nabla \overline{T_s} dA. \quad (22)$$

where,  $h_i$  is known as the interfacial convective heat transfer coefficient and  $a_i = A_i/\Delta V$  is the surface area per unit volume and  $A_i$  is the interfacial heat transfer area.

For determining  $\overline{h_i}$ , Kuwahara et. al (2001) modeled a porous medium by considering an infinite number of solid square rods of size  $D$ , arranged in a regular triangular pattern (see Fig. (1)). They numerically solved the governing equations in the void region, exploiting to advantage the fact that for an infinite and geometrically ordered medium a repetitive cell can be identified. Periodic boundary conditions were then applied for obtaining the temperature distribution under fully developed flow conditions. A numerical correlation for the interfacial convective heat transfer coefficient was proposed by Kuwahara et. al (2001) as,

$$\frac{\overline{h_i} D}{k_f} = \left( 1 + \frac{4(1-f)}{f} \right) + \frac{1}{2} (1-f)^{1/2} Re_D Pr^{1/3}, \text{ valid for } 0.2 < f < 0.9, \quad (23)$$

Eq. (23) is based on porosity dependency and is valid for packed beds of particle diameter  $D$ .

This same physical model will be used here for obtaining the interfacial heat transfer coefficient  $\overline{h_i}$  for macroscopic flows.

### 6. Periodic Cell and Boundary Conditions

The macroscopic hydrodynamic and thermodynamic behavior of practical interest can be obtained from the direct application of the first principles to viscous flow and heat transfer at a pore scale. In reality, however, it is impossible to resolve the details of the flow and heat transfer fields within a real porous medium Nakayama et. al (2001) and Kuwahara et. al (2001) modeled a porous medium in terms of obstacles arranged in regular pattern, and solved the set of the microscopic governing equations, exploiting periodic boundary conditions. Moreover, the main objective of this research is, in fact, to enhance the reliability of the numerical results with respect to those obtained from experiments and one step forward to this direction is the formulation of the two-equation model for turbulent flows.

In order to evaluate the numerical tool to be used in the determination of the film coefficient given by Eq. (22), a test case was run for obtaining the flow field in a periodic cell, which is here assumed to represent the porous medium. Consider a macroscopically uniform flow through an infinite number of square rods of lateral size  $D$ , placed in a staggered fashion and maintained at constant wall heat flux. The periodic cell or representative elementary volume,  $\Delta V$ , is schematically showed in Fig. (1) and has dimensions  $2H \times H$ . Computations within this cell were carried out using a non-uniform grid of size  $90 \times 70$  nodes, as shown in Fig. (2), to ensure that the results were grid independent. The Reynolds number  $Re_D = \mathbf{r} \mathbf{u}_D D / \mathbf{m}$  was varied from 40 to 400 and the porosity,  $f = 1 - (D/H)^2$ , was equal to 0.65, which was used in Kuwahara et. al (2001) to show the velocity and temperature fields.

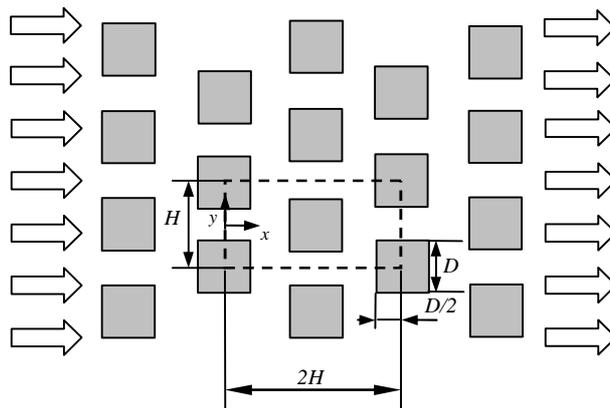


Figure 1. Physical model and coordinate system.

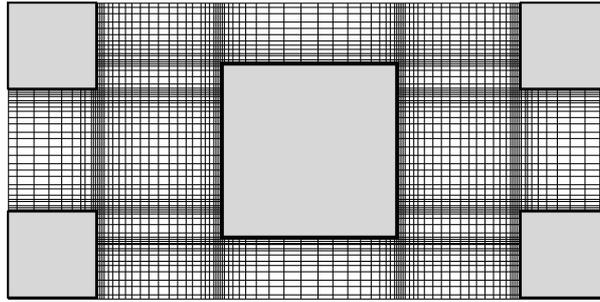


Figure 2. Non uniform computational grid.

The numerical method utilized to discretize the microscopic flow and energy equations in the unit cell is the finite control volume. The SIMPLE method of Patankar (1980) was used for the velocity-pressure coupling. Convergence was monitored in terms of the normalized residue for each variable. The maximum residue allowed for convergence check was set to  $10^{-9}$ , being the variables normalized by appropriate reference values.

For fully developed flow in the cell of Fig. (1), the velocity at exit ( $x/H = 2$ ) must be identical to that at the inlet ( $x/H = 0$ ). Temperature profiles, however, are only identical at both cell exit and inlet if presented in terms of an appropriate non-dimensional variable. The situation is analogous to the case of forced convection in a channel with isothermal walls. Thus, boundary conditions and periodic constraints are given by:

On the solid walls:

$$\mathbf{u} = 0, \quad q_w'' = \text{constant} . \quad (24)$$

On the periodic boundaries:

$$\mathbf{u}|_{inlet} = \mathbf{u}|_{outlet} , \quad (25)$$

$$\int_0^H u \, dy \Big|_{inlet} = \int_0^H u \, dy \Big|_{outlet} = H|\mathbf{u}_D| , \quad (26)$$

$$\mathbf{q}|_{inlet} = \mathbf{q}|_{outlet} \Leftrightarrow \frac{T - T_w}{T_B(x) - T_w} \Big|_{inlet} = \frac{T - T_w}{T_B(x) - T_w} \Big|_{outlet} , \quad (27)$$

The bulk mean temperature of the fluid is given by:

$$T_B(x) = \frac{\int u T \, dy}{\int u \, dy} \quad (28)$$

Computations are based on the Darcy velocity, the length of structural unit  $H$  and the temperature difference ( $T_B(x) - T_w$ ), as references scales.

## 7. Preliminary Laminar Results and Discussion

### 7.1 Periodic Flow

Preliminary results for velocity and temperature fields were obtained for different Reynolds numbers. In order to assure that the flow is hydrodynamically and thermally developed in the periodic cell of Fig. (1), the governing equations were solved repetitively in the cell, taking the outlet profiles for  $\mathbf{u}$  and  $\mathbf{q}$  at exit and plugging them back at inlet. In the first run, uniform velocity and temperature profiles were set at the cell entrance for  $Pr = 1$  and  $Re_D = 100$ , giving  $\mathbf{q} = 1$  at  $x/H=0$ . Then, after convergence of the flow and temperature fields,  $\mathbf{u}$  and  $\mathbf{q}$  at  $x/H=2$  were used as inlet profiles for a second run, corresponding to solving again the flow for a similar cell beginning in  $x/H=2$ . Similarly, a third run was carried out and again outlet results, this time corresponding to an axial position  $x/H=4$ , were recorded. This procedure was repeated several times until  $\mathbf{u}$  and  $\mathbf{q}$  did not differ substantially at both inlet and outlet positions. Resulting non-dimensional temperature profiles are shown in Fig. (3) showing that the periodicity constraints imposed by Eqs. (25)-(27) was satisfied for  $x/H > 2$ . For the entrance region ( $0 < x/H < 2$ ),  $\mathbf{q}$  profiles change with length  $x/H$  being essentially invariable after this distance. Under this condition of constant  $\mathbf{q}$  profile, the flow was considered to be macroscopically developed for  $Re_D$  up to 400.

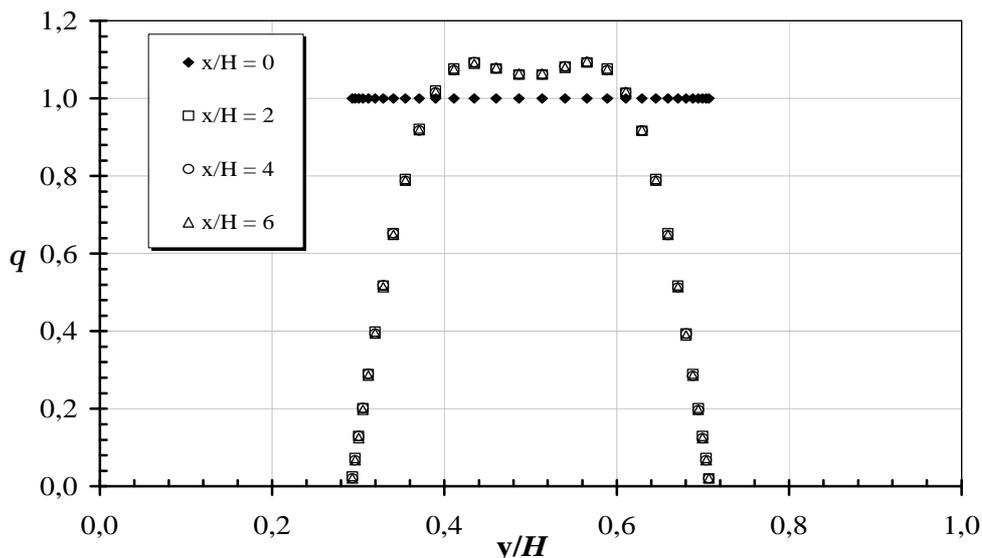


Figure 3. Dimensionless temperature profile for  $Pr=1$  and  $Re_D = 100$ .

## 7.2 Developed Flow and Temperature Fields

Macroscopically developed flow field for  $Pr=1$  and  $Re_D = 10$  is presented in Fig. (4), corresponding to  $x/D=6$  at the cell inlet (see Fig. (3)); for  $Pr=1$  and  $Re_D = 100$  is presented in Fig. (5). The expression “macroscopically developed” is used herein to account for the fact that periodic flow has been achieved at that axial position.

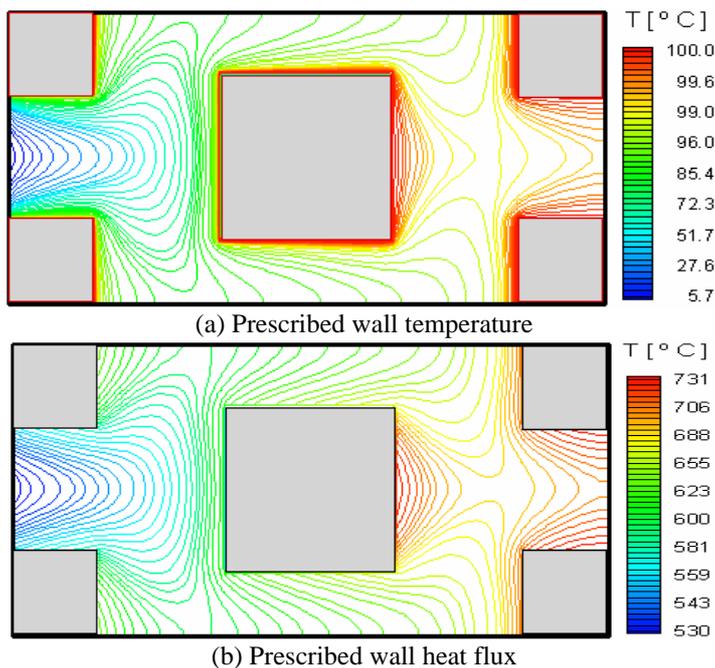


Figure 4. Isotherms for  $Pr=1$  and  $Re_D = 10$ .

Temperature distribution pattern is shown in Fig. (4) and Fig. (5), colder fluid impinges on the left surface yielding strong temperature gradients on that face. Downstream the obstacle, fluid recirculation smooths temperature gradients and deforms isotherms within the mixing region. When the Reynolds number is sufficiently high, the thermal boundary layers covering the rod surfaces indicate that convective heat transfer overwhelms thermal diffusion for prescribed wall temperature and prescribed wall heat flux.

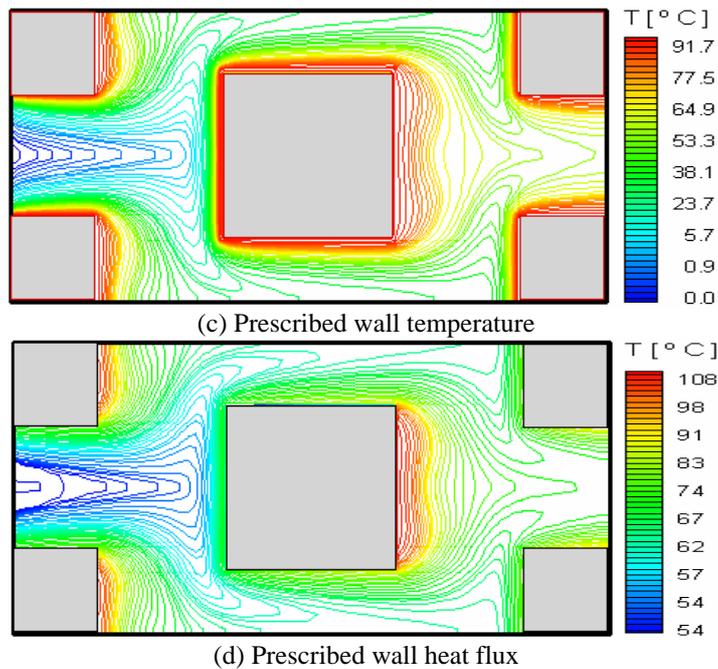


Figure 5. Isotherms for  $Pr=1$  and  $Re_D = 100$ .

### 7.3 Film Coefficient $\bar{h}_i$

Determination of  $\bar{h}_i$  is here obtained by calculating, for the unit cell of Fig. (1), an expression given as,

$$\bar{h}_i = \frac{1}{A_i} \int h_i dA_i \quad (29)$$

where,

$$h_i = \frac{q''}{T_w - \bar{T}_B} \quad (30)$$

and,  $q''$  is local heat flux (constant),  $h_i$  is known as the interfacial convective heat transfer coefficient and  $\bar{T}_B$  is given by:

$$\bar{T}_B = \frac{\int_{cell} uTdy}{\int_{cell} udy}, \quad (31)$$

Once fully developed flow and temperature are fields are achieved, for the fully developed condition ( $x > 6H$ ), bulk temperatures were calculated according to Eq. (28), at both inlet and outlet positions. They were then used to calculate  $\bar{h}_i$  using Eqs. (29)-(31). Results for  $\bar{h}_i$  are plotted in Fig. (6) for  $Re_D$  up to 400. Also plotted in this figure are results computed with correlation (23) using  $f = 0.65$ . The figure seems to indicate that both computations show a reasonable agreement with prescribed wall temperature. However, it was observed that overall values of prescribed wall heat flux are lower than those obtained from correlation of Kuwahara et. al (2001). It is thus seen that the convection conductance depends to some degree on the type of the surface temperature variation.

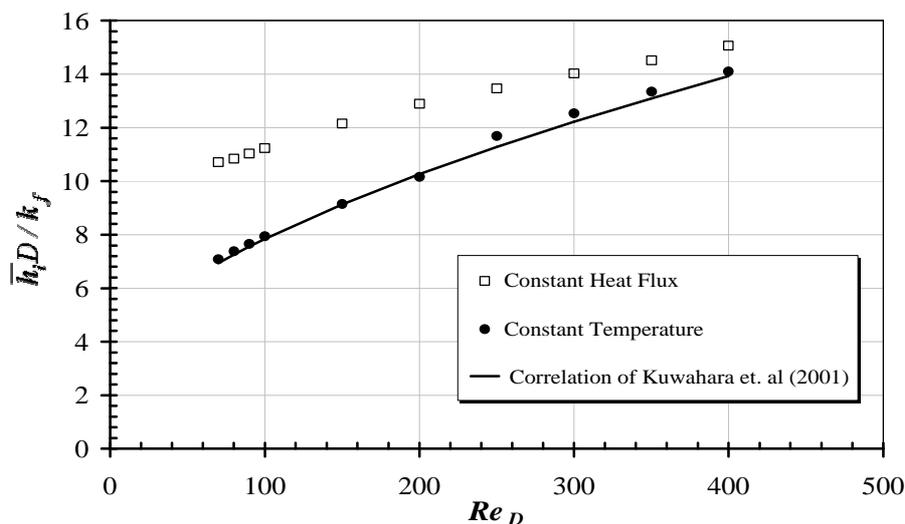


Figure 6. Effect of  $Re_D$  on  $h_i$  for  $Pr = 1$ .

## 8. Concluding remarks

Kuwahara et. al (2001) proposed a numerical procedure to determine the transport coefficients considering constant wall temperature, however a comprehensive investigation of convective coefficient of heat exchanger between the porous substrate considering constant wall heat flux has been presented in this work. As a preliminary result, a macroscopically uniform laminar flow through a periodic model of square rods was computed, considering periodic velocity and temperature fields. Quantitative agreement was obtained when comparing the preliminary results for prescribed wall temperature with simulations by Kuwahara et. al (2001). However, it was observed that overall values of prescribed wall heat flux are lower. Further work will be carried out in order to simulate fully turbulent flow and heat transfer in porous media by means of the proposed two-energy equation. Ultimately, it is expected that a correlation for the heat transfer coefficient be obtained so that the exchange energy between the solid and the fluid can be accounted for.

## 9. Acknowledgements

The authors are thankful to FAPESP and CNPq Brazil, for their financial support during the course of this research.

## 10. References

- de Lemos, M.J.S., Pedras, M.H.J., "Recent mathematical models for turbulent flow in saturated rigid porous media", *Journal of Fluids Engineering*, 2001, 123 (4), 935 – 940.
- de Lemos, M.J.S., Braga, E.J., "Modeling of turbulent natural convection in porous media", *International Communications in Heat and Mass Transfer*, 2003, 30 (5), 615 – 624.
- de Lemos, M.J.S., Mesquita, M.S., "Turbulent mass transport in saturated rigid porous media", *International Communications in Heat and Mass Transfer*, 2003, 30 (1), 105 – 115.
- de Lemos, M.J.S., Rocamora Jr, F.D., "Turbulent Transport Modeling for Heated Flow in Rigid Porous Media", *Proc. of IHTC12 - 12th International Heat Transfer Conference, Grenoble, França, 18-23 August 2002*.
- Forchheimer, P., 1901, "Wasserbewegung durch Boden", *Z. Ver. Deutsch. Ing.*, vol. 45, pp. 1782-1788.
- Gray, W. G. & Lee, P. C. Y., "On the theorems for local volume averaging of multiphase system", *Int. J. Multiphase Flow*, vol. 3, pp. 333-340, 1977.
- Hsu, C.T., 1999, "A Closure Model for Transient Heat Conduction in Porous Media", *J. Heat Transfer* 121. pp. 733-739.
- Kuwahara, F., Shirota, M., Nakayama, A., 2001, "A Numerical Study of Interfacial Convective Heat Transfer Coefficient in Two-Energy Equation Model for Convection in Porous Media", *Int. J. Heat Mass Transfer* 44. pp. 1153-1159.
- Nakayama, A., Kuwahara, F., Sugiyama, M., Xu, G., 2001, "A Two-Energy Equation Model for Conduction and Convection in Porous Media", *Int. J. Heat Mass Transfer* 44. pp. 4375-4379.
- Patankar, S.V., 1980, "Numerical Heat Transfer and Fluid Flow", Mc-Graw Hill.

- Pedras, M.H.J., de Lemos, M.J.S., “On the definition of turbulent kinetic energy for flow in porous media”, *International Communications in Heat and Mass Transfer*, 2000, 27 (2), 211 – 220.
- Pedras, M.H.J., de Lemos, M.J.S., “Macroscopic turbulence modeling for incompressible flow through undeformable porous media”, *International Journal of Heat and Mass Transfer*, 2001a, 44 (6), 1081 – 1093.
- Pedras, M.H.J., de Lemos, M.J.S., “Simulation of turbulent flow in porous media using a spatially periodic array and a low Reynolds two-equation closure”, *Numerical Heat Transfer Part A – Applications*, 2001b, 39 (1).
- Pedras, M.H.J., de Lemos, M.J.S., “On the mathematical description and simulation of turbulent flow in a porous medium formed by an array of elliptic rods”, *Journal of Fluids Engineering*, 2001c, 123 (4), 941 – 947.
- Pedras, M.H.J., de Lemos, M.J.S., “Computation of turbulent flow in porous media using a Low Reynolds  $k - \epsilon$  model and an infinite array of transversally-displaced elliptic rods”, *Numerical Heat Transfer Part A – Applications*, 2003, 43 (6), 585 – 602.
- Rocamora Jr, F.D., de Lemos, M.J.S., “Analysis of convective heat transfer for turbulent flow in saturated porous media”, *International Communications in Heat and Mass Transfer*, 2000, 27 (6), 825-834.
- Saito, M.B., de Lemos, M.J.S., “Interfacial Heat Transfer Coefficient for Non-Equilibrium Convective Transport In Porous Media”, *International Communications in Heat and Mass Transfer*, 2004 (submitted).
- Slattery, J. C., “Flow of viscoelastic fluids through porous media”, *A.I.Ch.E. J.*, vol. 13, pp. 1066-1071, 1967.