# NEUTRON & THERMO - HYDRAULIC MODEL OF A REACTIVITY TRANSIENT IN A NUCLEAR POWER PLANT FUEL ELEMENT

#### José de Jesús Rivero Oliva, rivero@poli.ufrj.br

Departamento de Engenharia Nuclear, Universidade Federal do Rio de Janeiro, Av. Horácio Macedo, 2030, Bloco G, Sala 201, Cidade Universitária, Ilha do Fundão, Rio do Janeiro, RJ, Brasil

Abstract. A reactivity transient without reactor scram was modeled and calculated using analytical expressions for the space distributions of the temperature fields, combined with discrete numerical calculations for the time dependences of thermal power and temperatures. The transient analysis covered the time dependencies of reactivity, global thermal power, fuel heat flux and temperatures in fuel, cladding and cooling water. The model was implemented in Microsoft Office Excel, dividing the Excel file in several separated worksheets for input data, initial steady-state calculations, calculation of parameters non-depending on eigenvalues, eigenvalues determination, calculation of parameters depending on eigenvalues, transient calculation and graphical representation of intermediate and final results. The results show how the thermal power reaches a new equilibrium state due to the negative reactivity feedback derived from the fuel temperature increment. Nevertheless, the reactor mean power increases 40% during the first second and, in the hottest channel, the maximum fuel temperature goes to a significantly high value, slightly above 2100 °C, after 8 seconds of transient. Consequently, the results confirm that certain degree of fuel damage could be expected in case of a reactor scram failure. Once the basic model has being established the scope of accidents for future analyses can be extended, modifying the nuclear power behavior (reactivity) during transient and the boundary conditions for coolant temperature. A more complex model is underway for an annular fuel element.

Keywords: Non-stationary heat transfer, analytical method, nuclear fuel rod, temperature field, reactivity transient

### **1. INTRODUCTION**

Non-stationary heat transfer problems emerge in several engineering areas (Behera and Kumar, 2009). They are part of the education process in Universities, where solutions are implemented with teaching purposes (Janáčová et al., 2011). Specifically, they are of major interest for nuclear power plants where thermal regime of nuclear fuel is crucial for safety (Blinkov et al., 2010).

Methods from Mathematical Physics constitute the basic theoretical framework for the analyses of non-stationary heat transfer problems (Tijonov and Samarsky 1972). Unfortunately, analytical solutions can be obtained only for a limited amount of problems involving relatively simple geometries and mathematical functions as initial and boundary conditions (Blomberg, 1996).

The present paper provides an application of analytical methods from Mathematical Physics to the solution of a transient heat transfer problem in a cylindrical nuclear fuel rod due to a positive reactivity jump without reactor scram. Analytical expressions were obtained for the space distributions of the temperature fields. They were combined with discrete numerical calculations for the time dependences of thermal power and temperatures. The transient analysis covered the time dependencies of reactivity, global thermal power, fuel heat flux and temperatures in fuel, cladding and cooling water.

# 2. MODEL DESCRIPTION

Figure 1 shows the two regions for which the time dependant one-dimensional heat transfer equation was solved: the fuel region ( $\mathbf{r} \leq \mathbf{r}_{f_0}$ ) and the cladding region ( $\mathbf{r}_{ci} \leq \mathbf{r} \leq \mathbf{r}_{co}$ ). It is assumed the existence of angular symmetry and that the domain of interest is sufficiently far from the rods extremes, so that the z-dependence can be neglected. The materials properties are assumed uniform and not temperature dependant.



Figure 1. Diagram of the modeled fuel element.

The boundary value problems for the fuel temperature  $T_f(r, t)$  and the cladding temperature  $T_c(r, t)$  are similar to those solved in (Jian and Cotta, 2001) using the improved lumped parameter formulation.

For fuel:

$$\rho_f c_f \frac{\partial T_f}{\partial t} = k_f \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_f}{\partial r} \right) + g(t); \qquad r \le r_{fo}$$
(1)

$$T_f(r,0) = T_{f0}(r)$$
 (2)

$$\left|T_f(0,t)\right| < \infty \tag{3}$$

$$\left[-k_f r_{fo} \frac{\partial T_f}{\partial r}\right]_{r=r_{fo}} = h_g r_{ci} \left[T_f \left(r_{fo}, t\right) - T_c \left(r_{ci}, t\right)\right]$$
<sup>(4)</sup>

For cladding:

$$\rho_c c_c \frac{\partial T_c}{\partial t} = k_c \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_c}{\partial r} \right); \qquad \mathbf{r_{ci}} \le \mathbf{r} \le \mathbf{r_{co}}$$
(5)

$$T_c(r,0) = T_{c0}(r)$$
 (6)

$$\left[-k_{c}\frac{\partial T_{c}}{\partial r}\right]_{r=r_{ci}} = h_{g}\left[T_{f}(r_{fo},t) - T_{c}(r_{ci},t)\right]$$
<sup>(7)</sup>

$$\left[-k_c \frac{\partial T_c}{\partial r}\right]_{r=r_{co}} = h \left[T_c(r_{co}, t) - T_m(t)\right]$$
<sup>(8)</sup>

In the preceding expressions, " $\rho$ " is the material density, "c" is the material specific heat, "k" is the thermal conductivity, "h<sub>g</sub>" is the heat transfer coefficient through the gap between fuel and cladding, "h" is the heat transfer coefficient between cladding and cooling water, "g(t)" is the internal heat source density in the fuel, " $T_{f0}(r)$ " is the steady-state fuel temperature field, " $T_{c0}(r)$ " is the steady-state cladding temperature field and " $T_m(t)$ " is the water coolant mean temperature inside the reactor core. The parameters with subscript "f" correspond to fuel and with subscript "c" to cladding. Note that the boundary conditions (4), (7) and (8) are non-homogeneous of third kind, corresponding to a thermal heat transfer following the Newton law. The model is solved using the following steps:

# 2.1 Determination of the steady-state initial temperature distributions $T_{f0}(r)$ and $T_{c0}(r)$

Starting from the global initial thermal power, the number of fuel elements, the reactor core height, the mass coolant flow ant the initial coolant temperature, ordinary differential equations are solved for fuel and cladding, with the proper boundary conditions, leading to expressions (9) and (10), where A, B and C are known constants determined for fuel (subscript "f") and cladding (subscript "c"):

$$T_{f0}(r) = B_f + C_f \left( r_{fo}^2 - r^2 \right)$$
(9)

$$T_{c0}(r) = B_c + A_c \ln \frac{r_{c0}}{r}$$
(10)

#### 2.2 Analytical Solution for fuel

The solution for  $T_f(r, t)$  is proposed as  $T_f(r, t) = U(r, t) + v(r, t)$ , where U(r, t) is constructed in such a way that the resulting auxiliary problem for v(r, t) had homogenous boundary conditions (Tijonov and Samarsky, 1972). For this geometry, the function:

$$U(r,t) = \frac{h_g T_c(r_{ci},t) r}{h_g r_{fo} + k_f \frac{r_{fo}}{r_{ci}}}$$
(11)

leads to the following boundary value problem for v(r, t):

$$\rho_f c_f \frac{\partial v}{\partial t} = k_f \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v}{\partial r} \right) + S(r, t); \quad r \leq r_{fo}$$
<sup>(12)</sup>

$$v(r,0) = T_{f0}(r) - W_f r$$
<sup>(13)</sup>

$$|v(0,t)| < \infty \tag{14}$$

$$\left[-k_{f}r_{fo} \frac{\partial v}{\partial r}\right]_{r=r_{fo}} = h_{g}r_{ci} v\left(r_{fo}, t\right)$$
<sup>(15)</sup>

where  $W_f$  is a known constant for fuel and S(r, t) is given by the expression:

$$S(r,t) = \frac{g(t)}{\rho_{f}c_{f}} - \frac{\frac{dT_{c}(r_{ci},t)}{dt}}{r_{fo} + \frac{k_{f}r_{fo}}{h_{g}r_{ci}}} r + \frac{k_{f}}{\rho_{f}c_{f}} \frac{T_{c}(r_{ci},t)}{\left[r_{fo} + \frac{k_{f}r_{fo}}{h_{g}r_{ci}}\right]} \frac{1}{r}$$
(16)

The auxiliary problem for v(r, t) is a problem with homogenous boundary conditions, leading to eigenfunctions for the space variable "r", in terms of the Bessel function of zero order and first kind  $J_0(\sqrt{\lambda}r)$ . The eigenvalues  $\lambda$  are the solutions of the transcendent equation:

$$J_0\left(\sqrt{\lambda}r_{f0}\right) = \sqrt{\lambda} \ \frac{k_f r_{f0}}{h_g r_{ci}} J_1\left(\sqrt{\lambda}r_{f0}\right) \tag{17}$$

There is no analytical expression for the eigenvalues, which are first located by a search algorithm and afterwards determined with the desired precision using a bisection method algorithm.

Actually, v(r, t) is the sum of the solutions for two auxiliary problems  $v_1(r, t)$  and  $v_2(r, t)$ . The results are:

$$v_1(r,t) = \sum_{n=1}^{\infty} D_n e^{-k_n^2 t} J_0\left(\sqrt{\lambda_n} r\right)$$
<sup>(18)</sup>

$$v_2(r,t) = \sum_{n=1}^{\infty} \left[ \int_0^\tau S_n(\tau) \, e^{-k_n^2(t-\tau)} \, d\tau \right] J_0\left(\sqrt{\lambda_n} r\right)$$
<sup>(19)</sup>

$$k_n = \sqrt{\lambda_n} \frac{k_f}{c_f \rho_f};$$
 n = 1, 2, 3, 4... (20)

In the expression (19),  $S_n(t)$  are the coefficients of the series development of S(r, t), given by (16), in terms of the eigenfunctions  $J_0(\sqrt{\lambda_n} r)$ .  $v_1(r, t)$  represents the contribution of the initial steady-state temperature field, vanishing after just a few seconds.  $v_2(r, t)$  considers the cumulative temperature effect at the time "t" coming from the heat produced by the sources at all the previous times " $\tau$ ".

Several integrals had to be solved for the determination of the coefficients  $D_n$  and  $S_n(t)$ ; integrals of the type  $\int_0^{r_f \circ} J_0(\sqrt{\lambda_n} r) dr$ ,  $\int_0^{r_f \circ} J_0(\sqrt{\lambda_n} r) r dr$ ,  $\int_0^{r_f \circ} J_0(\sqrt{\lambda_n} r) r^2 dr$ ,  $\int_0^{r_f \circ} J_0(\sqrt{\lambda_n} r) r^3 dr$ . Unfortunately, the integrals with even exponents of "r" cannot be solved using simple integration formulas. They are solved in terms of the Struve functions (Abramowitz and Stegun, 1970). Some details on Struve functions can be found in (Newman, 1984). For the numerical solution of the problem there were used algorithms for the evaluation of the Struve functions. The norms of the Bessel eigenfunctions were evaluated by the usual formula (Abramowitz and Stegun, 1970).

#### 2.3 Analytical Solution for cladding

The solution for cladding can be obtained, following a similar process, as explained for the fuel region in section 2.2. The main difference is that the eigenfunctions  $Z_0(\sqrt{\lambda}r)$  of the problem for cladding are linear combinations of the zero order Bessel functions of first and second kind, known as Bessel and Neumann functions, respectively. In the case of cladding, integrals are of the same type of those for fuel and are solved in a similar way. The formulas for  $J_0(\sqrt{\lambda}r)$  are also valid for  $Z_0(\sqrt{\lambda}r)$ . An additional integral was solved analytically:  $\int_{r_{ci}}^{r_{co}} Z_0(\sqrt{\lambda_n}r) r \ln r dr$ . The corresponding expressions of the analytical solution for cladding are the followings:

$$U(r,t) = T_f(r_{fo},t) + \frac{r - r_{ci} + \frac{k_c}{h_g}}{r_{co} - r_{ci} + \frac{k_c}{h_g} + \frac{k_c}{h}} \left[ T_m(t) - T_f(r_{fo},t) \right]$$
(21)

$$v_1(r,t) = \sum_{n=1}^{\infty} D_n e^{-k_n^2 t} Z_0\left(\sqrt{\lambda_n} r\right)$$
<sup>(22)</sup>

$$Z_0\left(\sqrt{\lambda_n}r\right) = J_0\left(\sqrt{\lambda_n}r\right) + E_n N_0\left(\sqrt{\lambda_n}r\right); \ n = 1, 2, 3, 4, \dots$$
<sup>(23)</sup>

 $\sqrt{\lambda_n}$  are the roots of the transcendent expression:

$$\frac{J_0\left(\sqrt{\lambda_n}r_{ci}\right) + \frac{k_c}{h_g}\sqrt{\lambda_n}J_1\left(\sqrt{\lambda_n}r_{ci}\right)}{J_0\left(\sqrt{\lambda_n}r_{co}\right) - \frac{k_c}{h}\sqrt{\lambda_n}J_1\left(\sqrt{\lambda_n}r_{co}\right)} = \frac{N_0\left(\sqrt{\lambda_n}r_{ci}\right) + \frac{k_c}{h_g}\sqrt{\lambda_n}N_1\left(\sqrt{\lambda_n}r_{ci}\right)}{N_0\left(\sqrt{\lambda_n}r_{co}\right) - \frac{k_c}{h}\sqrt{\lambda_n}N_1\left(\sqrt{\lambda_n}r_{co}\right)}$$
(24)

 $E_n$  is given by:

$$E_n = -\frac{J_0 \left(\sqrt{\lambda_n} r_{co}\right) - \frac{k_c}{h} \sqrt{\lambda_n} J_1 \left(\sqrt{\lambda_n} r_{co}\right)}{N_0 \left(\sqrt{\lambda_n} r_{co}\right) - \frac{k_c}{h} \sqrt{\lambda_n} N_1 \left(\sqrt{\lambda_n} r_{co}\right)}$$
(25)

$$v_2(r,t) = \sum_{n=1}^{\infty} \left[ \int_0^t S_n(\tau) \ e^{-k_n^2(t-\tau)} \ d\tau \right] Z_0\left(\sqrt{\lambda_n} r\right)$$
<sup>(26)</sup>

$$k_n = \sqrt{\lambda_n} \frac{k_c}{c_c \rho_c};$$
 n = 1, 2, 3, 4... (27)

In (26)  $S_n(t)$  are the coefficients of the series development of S(r,t) in terms of the eigenfunctions  $Z_0(\sqrt{\lambda_n}r)$ . S(r,t) is determined by the expression:

$$S(r,t) = -\frac{dT_{f}(r_{fo},t)}{dt} - \left[\frac{dT_{m}(t)}{dt} - \frac{dT_{f}(r_{fo},t)}{dt}\right] \frac{r - r_{ci} + \frac{k_{c}}{h_{g}}}{r_{co} - r_{ci} + \frac{k_{c}}{h_{g}} + \frac{k_{c}}{h}} + \frac{k_{c}}{\rho_{c}c_{c}} \frac{[T_{m}(t) - T_{f}(r_{fo},t)]}{[r_{co} - r_{ci} + \frac{k_{c}}{h_{g}} + \frac{k_{c}}{h}]} \frac{1}{r}$$
(28)

#### 2.4 Coolant temperature model

Based on the thermal balance of heat transferred from cladding and the one evacuated by the circulating coolant, assuming a constant coolant inlet temperature at the reactor core, the following ordinary differential equation problem was stated for coolant temperature:

$$\frac{dT_m(t)}{dt} + (A_m + B_m) T_m(t) = A_m T_c(r_{co}, t) + B_m T_{mi}$$
(29)

$$A_m = \frac{hS_c}{M_m c_m}; \ B_m = \frac{2G_m}{M_m} \tag{30}$$

$$T_m(0) = T_{m0} \tag{31}$$

where " $S_c$ " is the cladding outer surface, " $M_m$ " is the total coolant mass in the primary circuit, " $c_m$ " is the coolant specific heat, " $G_m$ " is the coolant mass flow, " $T_{mi}$ " is the fix coolant inlet temperature and " $T_{m0}$ " is the initial steady-state coolant mean temperature.

The solution of (29) with (31) gives an expression of the type:

$$T_{m}(t) = T_{m0} e^{-(A_{m}+B_{m})t} + A_{m} \int_{0}^{t} T_{c}(r_{co},\tau) e^{-(A_{m}+B_{m})(t-\tau)} d\tau + \frac{B_{m}T_{mi}}{A_{m}+B_{m}} \left[1 - e^{-(A_{m}+B_{m})t}\right]$$
(32)

#### 2.5 Reactivity model

It was implemented the point kinetics model with 6 groups of delayed neutrons, leading to the following problem:

$$\frac{dP(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \sum_{j=1}^{6} \lambda_j C_j(t); \quad \beta = \sum_{j=1}^{6} \beta_j$$
(33)

$$\frac{dC_j(t)}{dt} = \frac{\beta_j}{\Lambda} P(t) - \lambda_j C_j(t) \quad \text{with } C_{j0} = \frac{\beta_j}{\Lambda} P_0; \ j = 1, 2, \dots, 6$$
(34)

After the integration of (34), the following expressions were obtained:

$$C_j(t) = \frac{\beta_j}{\Lambda} \left[ \frac{P_0 e^{-\lambda_j t}}{\lambda_j} + \int_0^t P(\tau) e^{-\lambda_j (t-\tau)} d\tau \right]; \qquad j = 1, 2, \dots, 6$$
(35)

$$\frac{dP(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \sum_{j=1}^{6} \frac{\beta_j}{\Lambda} \left[ P_0 e^{-\lambda_j t} + \lambda_j \int_0^t P(\tau) e^{-\lambda_j (t-\tau)} d\tau \right]; \quad \beta = \sum_{j=1}^{6} \beta_j$$
(36)

where "P(t)" is the instant neutron power, " $\rho(t)$ " is the instant reactivity, " $\beta$ " is the total fraction of delayed neutrons, " $C_j(t)$ ,  $\beta_j$  and  $\lambda_j$ " are the concentration, the fraction of delayed neutrons and the radioactivity decay constant corresponding to the precursor nuclide of the group "j" of delayed neutrons, respectively; " $\Lambda$ " is the mean lifetime of a neutron generation, " $P_0$ " is the initial steady-state neutron power and " $C_{j0}$ " is the initial steady-state concentration of the precursor nuclide of the group "j" of delayed neutrons.

#### 2.6 Transient model

The model is constituted by the interrelated expressions for reactivity and temperatures fields in fuel, cladding and coolant that were previously described. For each time "t" the power from the neutron model determines the fuel temperature field via "g(t)" in equation (1). Then follows that the fuel temperature determines the cladding temperature via " $T_f(\eta_o, t)$ " in expressions (21) and (28) and, lastly, the cladding temperature determines the coolant temperature via " $T_c(r_{co}, t)$ " in expression (32). At the same time the cladding temperature " $T_c(r_{ci}, t)$ " is required in expressions (11) and (16) of the fuel model and, on the other hand, the coolant temperature " $T_m(t)$ " is required in expressions (21) and (28) of the cladding model. Finally, the fuel temperature introduces a feedback into the neutron model due to the fuel temperature reactivity coefficient via " $\rho(t)$ " in expression (36).

Under the conditions previously explained, it is impossible to find analytical explicit expressions for the time dependencies of thermal power and temperature fields. Consequently, the transient was calculated evaluating numerically the analytical spatial solutions for discrete time values, using a time step of 0.01 second. New heat fluxes and temperatures are determined for the successive times, using the variables previous values for the precedent time, together with the thermal power for the current calculation time. Thermal power is estimated by the expression  $P(t) = P(t - \Delta t) + \Delta t \frac{dP(t)}{dt}$ , where  $\frac{dP(t)}{dt}$  is derived from (36), with  $\rho(t)$  corrected by the fuel temperature reactivity coefficient.

The integrals of the type " $I = \int_0^{t_s} f(\tau) e^{-k(t_s - \tau)} d\tau$ ", appearing in several terms of the model (see (19), (26), (32) and (36)) where solved numerically, using the trapezium method, leading to the following expression:

$$I = \frac{1}{k} \left[ f(0) e^{-kt_s} \left( e^{\frac{k\Delta t}{2}} - 1 \right) + \sum_{l=1}^{s-1} f(t_l) e^{k[2(l-s)-1]\frac{\Delta t}{2}} (e^{k\Delta t} - 1) + f(t_s) \left( 1 - e^{-\frac{k\Delta t}{2}} \right) \right]$$
(37)

where " $\Delta t$ " is the interval used for the time dependant calculations, and  $l = 1, 2, 3, \dots s - 1$ , are the index corresponding to the calculated times  $t_l$ , previous to the current time  $t_s$ .

The integrals of the type " $I_1 = \int_0^{t_s} \frac{df(\tau)}{d\tau} e^{-k(t_s - \tau)} d\tau$ " were transformed into integrals of the type "I", using:

$$I_{1} = \int_{0}^{t_{s}} \frac{df(\tau)}{d\tau} e^{-k(t_{s}-\tau)} d\tau = f(t) - f(0) e^{-kt_{s}} - k \int_{0}^{t_{s}} f(\tau) e^{-k(t_{s}-\tau)} d\tau$$
(38)

## **3. MAIN RESULTS**

The model was implemented in Microsoft Office Excel, dividing the Excel file in several separated worksheets for input data, initial steady-state calculations, calculation of parameters non-depending on eigenvalues, eigenvalues determination, calculation of parameters depending on eigenvalues, transient calculation and graphical representation of intermediate and final results.

A case study was stated, based on the cells parameters used in (Hang and Chang, 2003), but considering a solid fuel rod instead of an annular one. Once the input data are introduced the program automatically calculates the steady-state variables and the parameters non-depending on the eigenvalues. Using a command button the user gives the order for the eigenvalues calculation and the program automatically calculates the eigenvalues and also the parameters depending on them. Figure 2 illustrates the eigenvalues determination in fuel and cladding. They are the zero-values of the represented functions. Finally, the user presses the command button for the transient calculation to run the complete model. All the intermediate and final results can be easily inspected and checked for consistence.



Figure 2. Graphics illustrating the eigenvalues determination in fuel and cladding.

The eigenvalues are determined in a few seconds. Ten eigenvalues for cladding and one hundred eigenvalues for fuel are sufficient for temperature calculations with numerical precision up to the second decimal place. For these amount of eigenvalues the whole transient of 400 seconds was calculated in about 10 minutes, using a time step of 0,01 second (40000 points evaluation). Table 1 is a partial copy of the numerical results for the hottest channel, available in the transient calculation worksheet.

| t [s] | <i>P(t)</i><br>( <i>MW</i> ) | q<br>(kW/m) | $Tf(r_{fo},t)$ $(^{\bullet}C)$ | $Tf_{max}(t)$ (•C) | $\frac{Tc(r_{c\dot{\nu}}t)}{(^{\bullet}C)}$ | $Tc(r_{co},t)$ (•C) | <i>Tm(t)</i> (• <i>C</i> ) | ρ(t)     | $Tf_{med}(t)$<br>(°C) |
|-------|------------------------------|-------------|--------------------------------|--------------------|---|---------------------|----------------------------|----------|-----------------------|
| 0,00  | 7037,50                      | 51,16       | 636,21                         | 1993,14            | 415,21                                      | 379,63              | 343,47                     | 0,002    | 1314,677              |
| 0,10  | 8697,17                      | 63,22       | 637,28                         | 1993,31            | 415,39                                      | 379,70              | 343,47                     | 0,001959 | 1316,049              |
| 0,20  | 9266,31                      | 67,36       | 639,21                         | 1996,16            | 415,83                                      | 379,92              | 343,47                     | 0,001875 | 1318,828              |
| 0,30  | 9433,25                      | 68,57       | 641,21                         | 1999,49            | 416,32                                      | 380,17              | 343,48                     | 0,00178  | 1322,018              |
| 0,40  | 9450,19                      | 68,69       | 643,07                         | 2002,94            | 416,79                                      | 380,41              | 343,49                     | 0,001683 | 1325,257              |
| 0,50  | 9410,70                      | 68,41       | 644,75                         | 2006,36            | 417,22                                      | 380,64              | 343,50                     | 0,001588 | 1328,412              |
| 0,60  | 9349,82                      | 67,96       | 646,25                         | 2009,71            | 417,61                                      | 380,84              | 343,52                     | 0,001497 | 1331,44               |
| 0,70  | 9281,02                      | 67,46       | 647,60                         | 2012,97            | 417,97                                      | 381,03              | 343,54                     | 0,001411 | 1334,327              |
| 0,80  | 9209,74                      | 66,95       | 648,82                         | 2016,12            | 418,29                                      | 381,21              | 343,56                     | 0,001328 | 1337,069              |
| 0,90  | 9138,37                      | 66,43       | 649,91                         | 2019,18            | 418,58                                      | 381,37              | 343,58                     | 0,00125  | 1339,672              |
| 1,00  | 9068,09                      | 65,92       | 650,90                         | 2022,13            | 418,84                                      | 381,51              | 343,61                     | 0,001176 | 1342,138              |

Table 1. Results for the first second of transient analysis.

Figure 3 shows the power and fuel temperatures behavior in the hottest channel during the first 50 seconds of the transient.



Figure 3. Behavior of Thermal Power and Temperature fields during the first 50 seconds of the transient.

The results show how the thermal power reaches a new equilibrium state due to the negative reactivity feedback derived from the fuel temperature increment. Nevertheless, the reactor mean power increases 40% during the first second and, in the hottest channel, the maximum fuel temperature goes to a significantly high value, slightly above 2100 <sup>o</sup>C, after 8 seconds of transient. Consequently, the results confirm that some fuel damage could be expected in case of a reactor scram failure.

#### 4. CONCLUSIONS

A reactivity transient without reactor scram was modeled and calculated using analytical expressions for the space distributions of the temperature fields, combined with discrete numerical calculations for the time dependences of thermal power and temperatures. The results show the self-regulation of nuclear reactor power. The importance of the reactor scram to avoid fuel damages was confirmed.

Once the basic model has being established the scope of accidents for future analyses can be extended, modifying the nuclear power behavior (reactivity) during transient and the boundary conditions for coolant temperature. A more complex model is underway for an annular fuel element.

## 5. REFERENCES

Abramowitz, M., Stegun, A., 1970. Handbook of Mathematical Functions. Dover Publications, New York.

- Behera, P. and Kumar Sahu, S., 2009. Analysis of transient heat conduction in different geometries. Master thesis, National Institute of Technology, Rourkela, Orissa, India.
- Blinkov, V. N., Boltenko, E. A., Elkin I. V., Melikhov O. I. and S. L. Solov'ev, 2010. "Prospects for Using Annular Fuel Elementsin Nuclear Power Engineering". Thermal Engineering, Vol. 57, No. 3, pp. 213–218.
- Blomberg, T., 1996. Heat conduction in two and three dimensions. Computer modelling of Building Physics applications. Ph. D. Thesis, Lund University, Lund, Sweden.
- Han, H.K., Chang, S. H., 2003. "Development of a thermal-hydraulic analysis code for annular fuel assemblies". Nuclear Engineering and Design, Vol. 226, Issue 3, pp. 267–275.
- Janáčová, D., Charvátová, H., Kolomazník, K., Vašek V and Mokrejš, P., 2011. "Solving of non-stationary heat transfer in a plane plate". Recent Researches in Automatic Control. 13th WSEAS International Conference on Automatic Control, Modelling & Simulation (ACMOS '11), Lanzarote, Canary Islands, Spain, pp. 287–291.
- Jian, S., Cotta. R. M., 2001. "Improved lumped parameter formulation for simplified LWR Thermohydraulic analysis". Annals of Nuclear Energy, Vol. 28, Issue 10, pp. 1019–1031.
- Newman, N. J., 1984. "Approximations for the Bessel and Struve Functions". Mathematics of Computation, Vol. 43, No. 168, pp. 551-556.

Tijonov, A.N., Samarsky, A.A, 1972. Ecuaciones de la Física Matemática. Editorial MIR, Moscow.

#### 6. RESPONSIBILITY NOTICE

The author is the only responsible for the printed material included in this paper.