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Topology Optimization of Multiple Load Case Structures

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ABSTRACT

This paper investigates a topology optimization strategy for structures under multiple load cases. Typically compliance is calculated for each of the load cases in analysis and minimization is computed for a weighted average of the compliances, resulting in time consuming algorithms. A way to increase convergence speed is updating density on the element level. Instead of having an averaged compliance for the whole structure, it is proposed to compute it individually for every element. All load cases are analyzed, however only the ones of the maximum compliances are considered for sensitivity analysis. Thus compliance gradient is function of a few load cases at each element, reducing the processing time without weight penalty. The efficiency of the proposed technique is exemplified and compared to the one of a classical approach of multiple load case problem, solved using optimality criteria.

Keywords: topology optimization, multiple load cases, density update on element level

1 INTRODUCTION

Optimization techniques are very important in many fields of engineering, such as aerospace, mechanical and naval. Problems in these fields are often function of several variables and optimization should be carried out for each of them in order to achieve global minima. As loading is typically the input in structural optimization, an efficient strategy for multiple load case problems is quite useful in engineering activities.

The objective function of topology optimization problems is typically minimize compliance, which may be defined by displacement and stiffness of the structure. When the problem accounts for multiple load cases, the objective function is usually set as a linear combination of the compliance of each load case, calculated separately. Hence the more load cases there are in the problem, the longest is the optimization process.

Reducing the amount of load cases is the straight answer to save processing time. However, evaluating which load cases of the problem are relevant is not always an easy task, since it depends on the geometry of the structure that changes at each optimization step. Instead of determining which load cases shall be considered, it is proposed to define the region of the structure under the influence of each load case. This strategy also contributes to prevent interference of non relevant load cases that could drive the solution out of the optimum.

The approach developed in this work is an adaptation to topology optimization of the solution strategy proposed by A. Faria [1], which a is robust optimization proposal based in a directional search of the critical load case.

The 99 line code of topology optimization written in Matlab [2] has been a starting point for the development of the new technique. Optimality Criteria is the solving approach employed, however the new technique may be easily adapted to any optimization solver.

2 THEORETICAL DEVELOPMENT

O. Sigmund [2] defines a single load case topology optimization problem of an isotropic material by the power-law approach that can be mathematically written as:

$$\min_x:\ c(x) = U^T K U = \sum_{e=1}^{N} p(x_e)^p u_e^T k_0 u_e \quad (1a)$$

$$subject\ to\ a\ prescribed\ volume\ fraction:\ \frac{V(x)}{V_0} = f \quad (1b)$$

$$subject\ to\ the\ loading\ vector{:}\ KU = F \quad (1c)$$

Where, *U* is the vector of displacement, *K* the global stiffness matrix, *F* the applied loading, *N* is the number of elements, *x* is the density and *p* is the penalization power.

The sensitivity analysis of the objective function (1a) is formulated by:

$$\frac{\partial c}{\partial x_e} = -p(x_e)^{p-1} u_e^T k_0 u_e \quad (2)$$

The typical compliance formulation for a problem of "*L*" load cases is the summation of the compliance of each load case *i*:

$$C(x) = \sum_{i=1}^{L} c_i(x) = \sum_{i=1}^{L} U_i^T K U_i \quad (3)$$

Hence the compliance gradient is also calculated for every load case (equation 4), no matter whether they are relevant or not.

$$\frac{\partial C}{\partial x_e} = \sum_{i=1}^{L} -p(x_e)^{p-1} (u_e^T k_0 u_e)_i \quad (4)$$

However, calculating compliance for every load cases provides a ranking of the most critical ones. The higher is the compliance, the less worthy is the element contribution for that load case. In addition, compliance of each load case derives from the compliance of all the elements. Instead of summing all of them, we could compare the compliance of each element through all load cases and identify which load case is the most critical for every element. Sensitivity analysis would then be calculated only for the critical load cases.

Equation (5) defines the compliance of each element for a load case *i*, where $1 \leq i \leq L$

$$c_i(x_e) = p(x_e)^p (u_e^T k_0 u_e)_i \quad (5)$$

The worst compliance for each element reads:

$$c_w(x_e) = max(c_1, c_2, \ldots c_L) \quad (6)$$

The objective function becomes to minimize the compliance c_w and the sensitivity analysis is performed for gradient of the worst compliance of each element, that is:

$$\frac{\partial c_w}{\partial x_w} = -p(x_w)^{p-1} u_w^T k_0 u_w \tag{7}$$

Where x_w and u_w are the elements density and the displacement matrix associated to the load case of the maximum compliance.

Minimizing the worst compliance of each element drives the solution to minimize the maximum compliance of the structure. However, the instability of the load case of maximum compliance may increase the number of iterations. Hence instead of considering only one load case for each element as presented in equation (6), it may be considered a few ones. For example, equation (8) defines the group of the critical compliances of an element as sum of first and second worse load cases.

$$c_{2c}(x_e) = max_{1st}(c_i) + max_{2nd}(c_i) \tag{8}$$

It is also considered in this work a group of "*n*" worse compliances in such that *n* is the load case whose compliance is greater or equal to 90% of the worst compliance (equation 9).

$$c_{mc}(x_e) = max_{1st}(c_i) + max_{2nd}(c_i) + \cdots + max_{nth}(c_i) \tag{9}$$

Where: $max_{nth}(c_i) \geq c_w = max_{1st}(c_i)$

3 ALGORITHM IMPLEMENTATION

Few modifications have been implemented into the original code of reference [2]. The same volume fraction is prescribed (40%), the same filtering (mesh-independency) and solver (optimality criteria) are employed to focus the comparison in the objective function. The three approaches for the element compliance (one, two and multiple critical load cases) described in the previous section are implemented and compared to the multiple load case version of reference [2], named in this work as "all load cases".

Stability of the answer is an important issue for "multiple critical load cases" approach. Since the number of load cases accounted at each step may vary, the gradient of the compliance function is normalized by the ratio of the load cases accounted and the total load cases of the problem.

Eventually the function of maximum embedded in Matlab Software is for simplicity the one employed to calculate compliance in the proposed approaches (equations 6, 8 and 9).

4 NUMERICAL EXAMPLES

Let us consider a cantilever beam fixed in the left extremity and discretized in a mesh of 84 by 20 squared elements. The structure is subjected up to nine load conditions, illustrated in Figures 1-3.

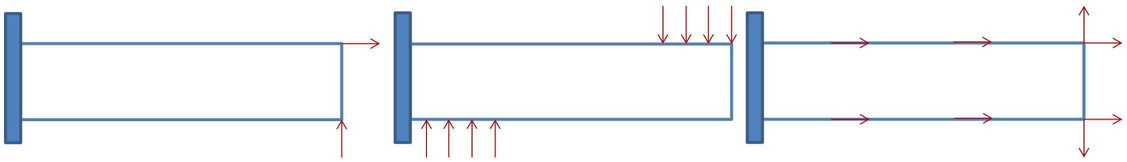

Figure 1: Load cases 1 (L) to 3 (R) applied to example 01.

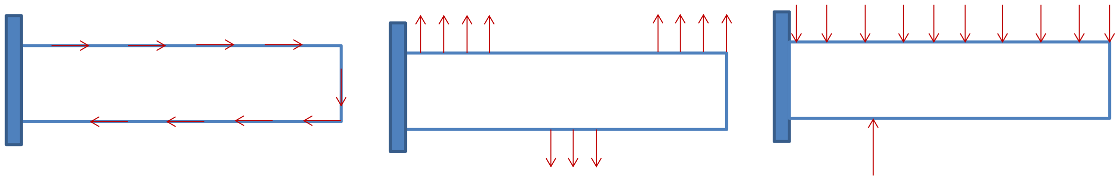

Figure 2: Load Cases 4 (L) to 6 (R) applied together with LC1 to LC3 on example 02.

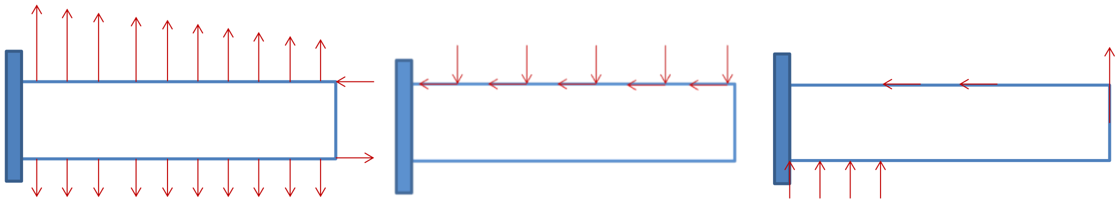

Figure 3: Load Cases 7 (L) to 9 (R) applied together with LC1 to LC6 on example 03.

Analysis will be performed for loading sets of 3, 6 and 9 load cases (examples 01, 02 and 03 respectively). Table 1 presents the flexibility (compliance) of the optimum structure for each load case calculated individually on the original code of reference [2]. Since the compliance of LC 1, LC 5 and LC 8 are higher; it is expected that solution shall be driven by them.

Table 1: Compliance of the optimized structure for each load case individually

LC 1	LC 2	LC 3	LC 4	LC 5	LC 6	LC 7	LC 8	LC 9
5.81E+04	3.45E+04	3.46E+03	3.05E+03	5.13E+04	1.91E+04	2.38E+04	5.88E+04	2.37E+04

Figures 4-6 present the results of the topology optimization for the 3 proposed examples. Results of each approach are presented side by side, respectively by one maximum load case, two maximum load cases, multiple maximum load cases and all load cases.

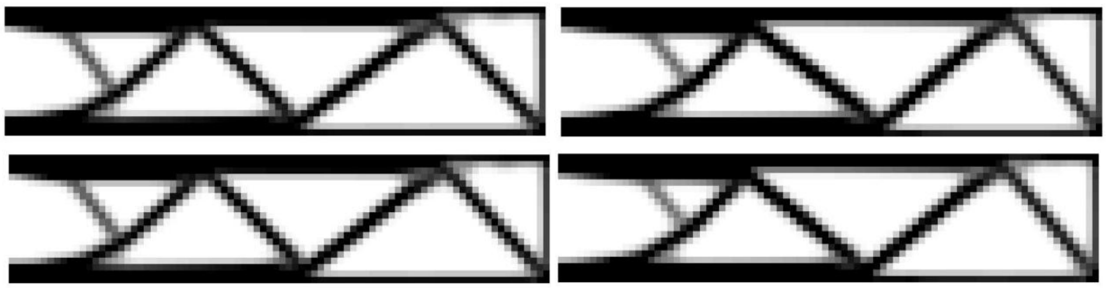

Figure 4: solution to 3 load cases problem provided by the strategies of 1LC (top-left), 2LC (top-right), Mult-LC (bottom-left) and All-LC (bottom-right)

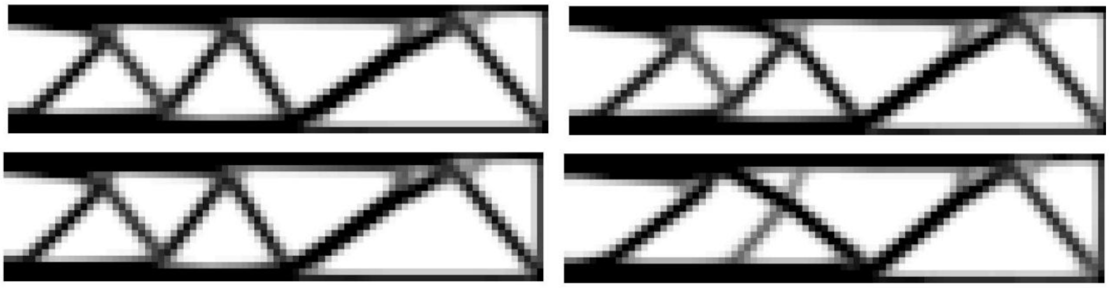

Figure 5: solution to 3 load cases problem provided by the strategies of 1LC (top-left), 2LC (top-right), Mult-LC (bottom-left) and All-LC (bottom-right)

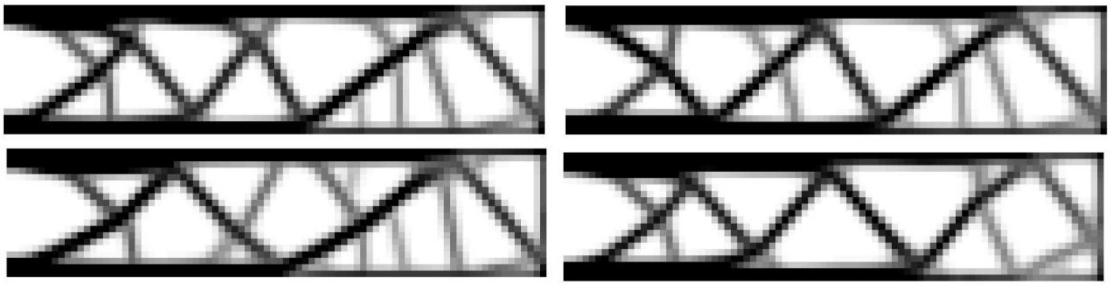

Figure 6: solution to 9 load cases problem provided by the strategies of 1LC (top-left), 2LC (top-right), Mult-LC (bottom-left) and All-LC (bottom-right)

The proposed strategies and reference approach delivered similar solutions as expected. Performance of each strategy is presented on Tables 2 to 4. Compliance of each load case is calculated according to equation (1a). Eventually, the elapsed time for running each solution is presented as reference only, since none of the algorithms have been developed focused on computational efficiency. Examples of enhancements that could be applied to the algorithms are presented on Ref. [3].

Table 2: Results for 3 load cases - original stop criteria

	One	Two	Multiple	All
C1:	59945	61270	60039	61341
C2:	38721	37205	38761	37410
C3:	5714	5884	5717	5788
Iterations:	337	101	277	114
Elapsed time:	223	69	184	76

Table 3: Results for 6 load cases - original stop criteria

	One	Two	Multiple	All
C1:	64125	64824	64127	65131
C2:	40053	39477	40075	38410
C3:	6349	6409	6345	6249
C4:	5096	5301	5090	5002
C5:	56045	56476	56036	55458
C6:	23242	23170	23251	23053
Iterations:	132	82	136	77
Elapsed time:	112	71	116	65

Table 4: Results for 9 load cases - original stop criteria:

	One	Two	Multiple	All
C1:	69345	68071	70213	69482
C2:	43391	42258	44159	40444
C3:	7486	7022	7792	7698
C4:	6106	5496	6386	5694
C5:	60671	59940	61786	60705
C6:	24734	23611	25830	22668
C7:	35292	36581	36766	37057
C8:	69427	67450	70016	66828
C9:	31796	30285	32088	30791
Iterations:	80	140	887	78
Elapsed time:	81	136	857	75

Comparing the compliance of each load case after the optimization, it is found that compliance of load case 1 ended up being the highest for all examples. It means that application of load case 1 demands more of the optimized structures than all the other load cases. Hence compliance variation of the other load cases is not as critical, though the ideal optimization point would be a structure with the same compliance for all the load cases.

Results point that a small reduction of the flexibility has been achieved on the critical load, except for example 3 (9 load cases) where strategy for multiple load cases reads about 1.1% higher in compliance. However a first reading on the amount of iterations may mislead to a reduction in convergence rate. Examination of Figures 7 to 9, which plot the first 45 iterations of the compliance evolution of the critical load case, allows understanding the development of compliance convergence.

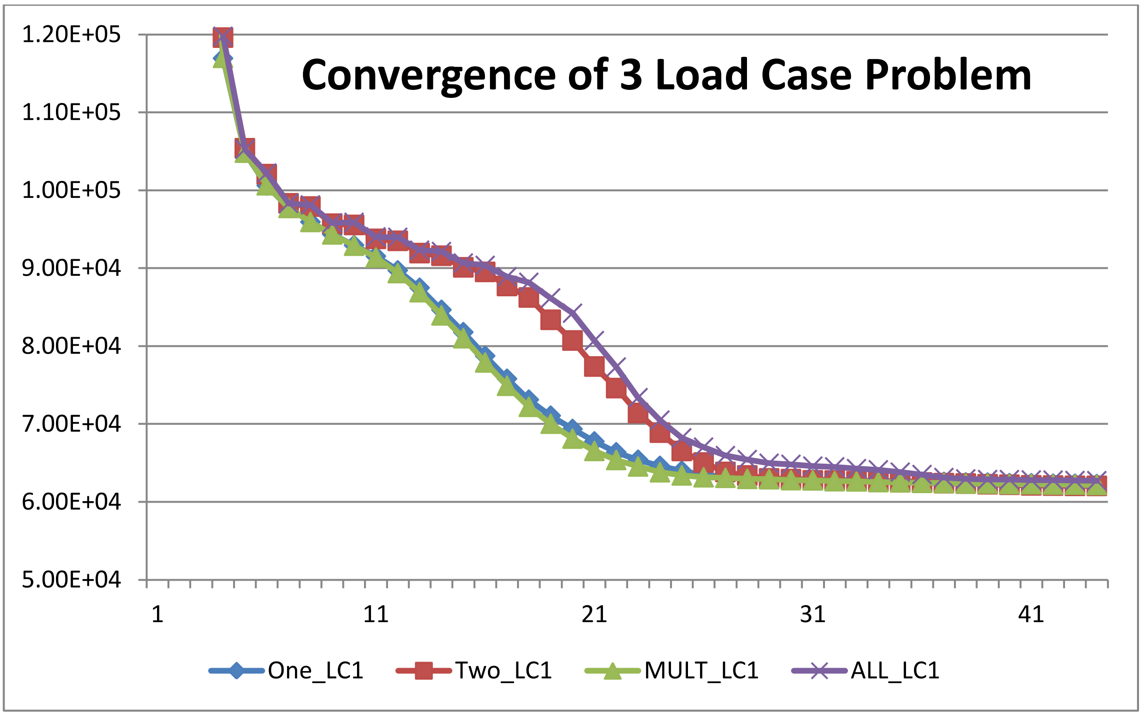

Figure 7: initial 45 iterations of the compliance variation of the critical load case (LC1) - convergence of the proposed techniques is faster than typical approach

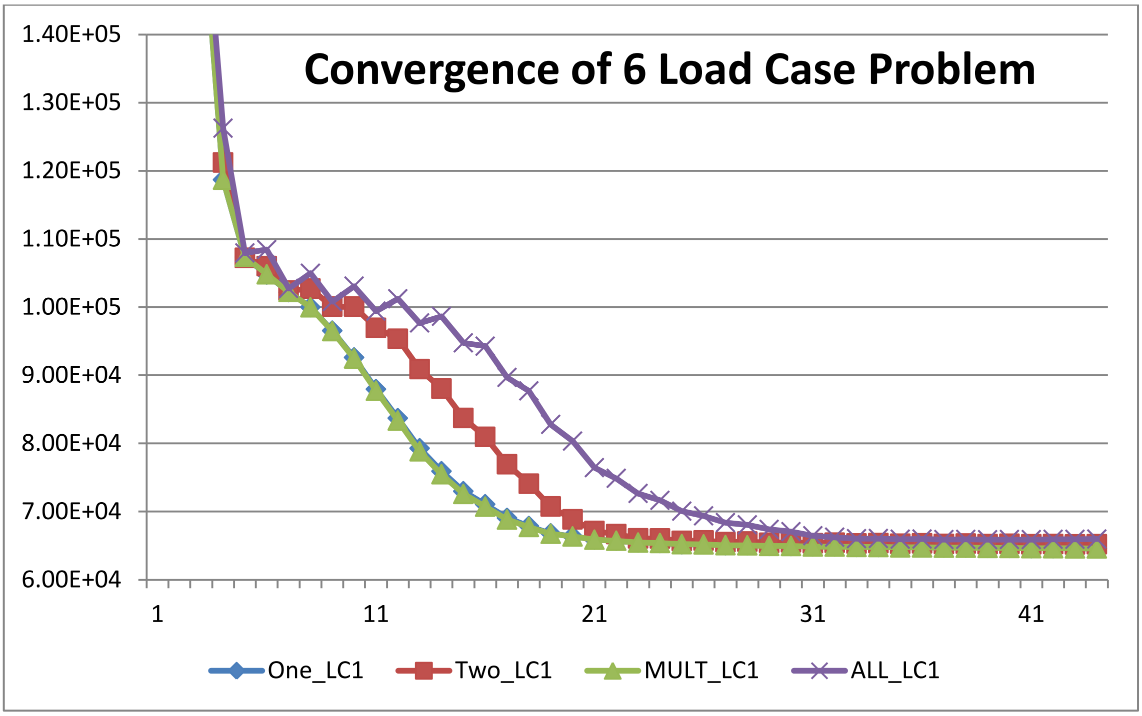

Figure 8: initial 45 iterations of the compliance variation of the critical load case (LC1) - convergence of the proposed techniques is faster than typical approach

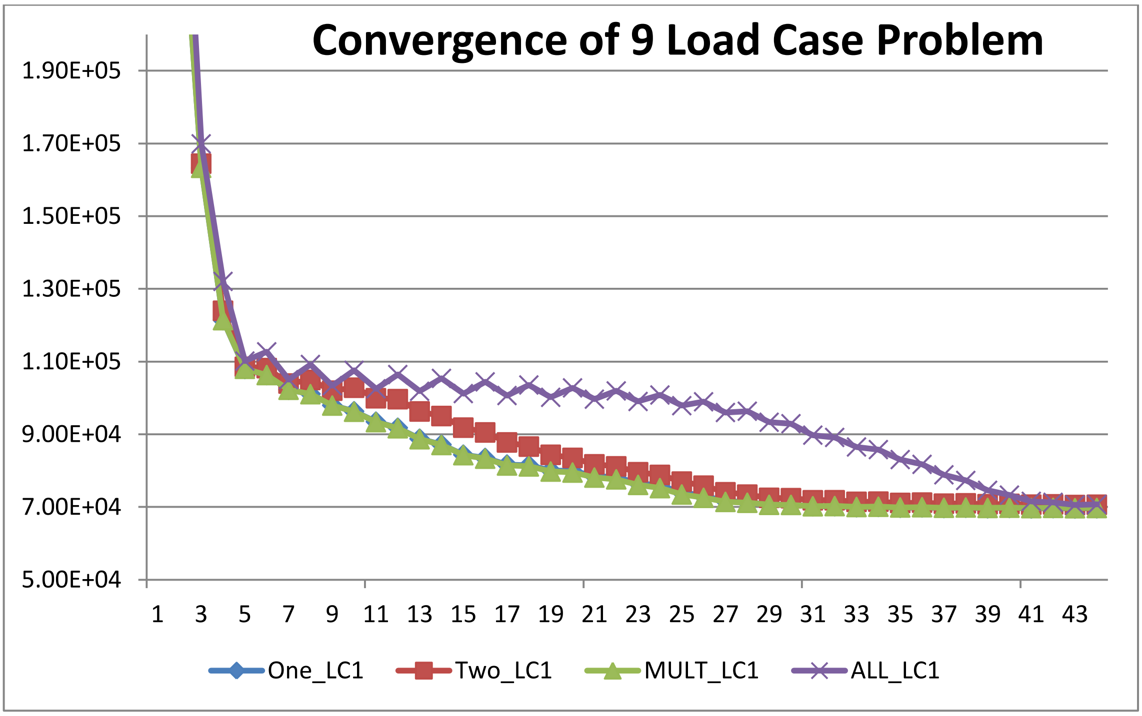

Figure 9: initial 45 iterations of the compliance variation of the critical load case (LC1) - convergence of the proposed techniques is faster than typical approach

The convergence rates for the proposed strategies are significantly better for the first 25 iterations. The reason why the proposed approaches demand more iterations to converge is the stop criteria. The original code of reference [2] proposes to end the optimization when maximum density variation of the elements is smaller than 1%. As critical load case of each element may alternate in the proposed approaches, the element density variation is not as smooth as in the all load cases approach. Consequently the optimization process of the proposed strategies demands more iterations to converge.

Instead of developing an improved filtering that could fix this issue, it is proposed a stop criteria based in the convergence of compliance. One possibility is to run the optimization until the maximum compliance variation is smaller than a percentage of the compliance of the previous iteration. The compliance employed in each strategy is the compliance of the structure, defined as the sum of the compliance of each element for the computed load cases. The following results have been run for a compliance variation smaller than 0.001%.

Table 5: Results for 3 load cases - modified stop criteria

	One	Two	Multiple	All
C1:	61772	62112	60229	61076
C2:	39767	37492	38801	37425
C3:	5871	6136	5842	5699
Iterations:	82	41	90	195
Time elapsed:	56	28	61	132

Table 6: Results for 6 load cases - modified stop criteria

	One	Two	Multiple	All
C1:	66801	65419	64627	64745
C2:	41894	39752	40176	38474
C3:	6579	6516	6421	6144
C4:	5733	5631	5296	4819
C5:	57988	56623	56250	55463
C6:	24668	23267	23307	22966
Iterations:	19	29	42	170
Time elapsed:	17	25	36	141

Table 7: Results for 9 load cases - modified stop criteria

	One	Two	Multiple	All
C1:	69381	70729	69537	68054
C2:	43429	43788	43639	40658
C3:	7460	7735	7332	7474
C4:	6133	6477	6034	5505
C5:	60677	61180	60964	59437
C6:	24766	24511	24913	22664
C7:	35446	37419	35349	36920
C8:	69487	69649	69789	65639
C9:	31852	31636	32094	31225
Iterations:	67	42	99	181
Elapsed time:	67	42	99	180

The compliance stop criteria have not been as effective for the reference approach as it has been to the proposed strategies. The amount of iterations reduced for them and increased for the "all load cases" approach. A reasonable comparison would consider the results of the proposed approaches for the compliance stop criteria and the results of the reference approach for the density stop criteria. This comparison, presented on Tables 8-10, still point advantages for the proposed approaches: reduction in the number of iterations without weight penalty.

Table 8: Variation in the number of iterations and compliance for critical load case - Ex. 1

Example 1 - three load cases applied					
Objective function strategy:	Compliance of LC 1:		Iterations:		Stop Criteria:
One critical load case	61772	0.7%	82	-28.1%	Compliance convergence
Two critical load cases	62112	1.3%	41	-64.0%	Compliance convergence
Multiple critical load cases	60229	-1.8%	90	-21.1%	Compliance convergence
All load cases (reference)	61341	0.0%	114	0.0%	Density convergence

Table 9: Variation in the number of iterations and compliance for critical load case – Ex. 2

Example 2 -six load cases applied					
Objective function strategy:	Compliance of LC 1:		Iterations:		Stop Criteria:
One critical load case	66801	2.6%	19	-75.3%	Compliance convergence
Two critical load cases	65419	0.4%	29	-62.3%	Compliance convergence
Multiple critical load cases	64627	-0.8%	42	-45.5%	Compliance convergence
All load cases (reference)	65131	0.0%	77	0.0%	Density convergence

Table 10: Variation in the number of iterations and compliance for critical load case – Ex. 3

Example 3 - nine load cases applied					
Objective function strategy:	Compliance of LC 1:		Iterations:		Stop Criteria:
One critical load case	69381	-0.1%	67	-14.1%	Compliance convergence
Two critical load cases	70729	1.8%	42	-46.2%	Compliance convergence
Multiple critical load cases	69537	0.1%	99	26.9%	Compliance convergence
All load cases (reference)	69482	0.0%	78	0.0%	Density convergence

The geometry for the optimized structure with compliance convergence criterion is presented on Figures 10-12. The strategy for only one critical load case delivered inferior solutions with compliance convergence criterion. In example 2, the gray cloud on the top of the structure (Figure 11) is consequence of the premature stop in iteration 19. The multiple critical load cases approach seems to be the most promising: cleaner structural layout and faster convergence (Figures 7-9).

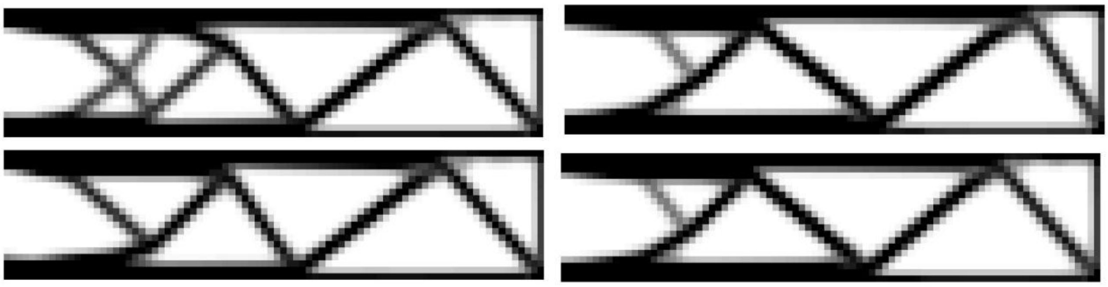

Figure 10: solution to 3 load cases problem provided by the strategies of 1LC (top-left), 2LC (top-right), Mult-LC (bottom-left) and All-LC (bottom-right) – Modified Stop criteria

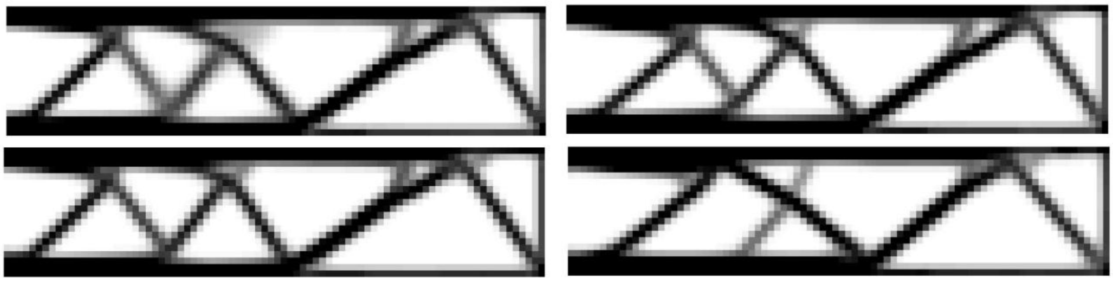

Figure 11: solution to 6 load cases problem provided by the strategies of 1LC (top-left), 2LC (top-right), Mult-LC (bottom-left) and All-LC (bottom-right) – Modified Stop criteria

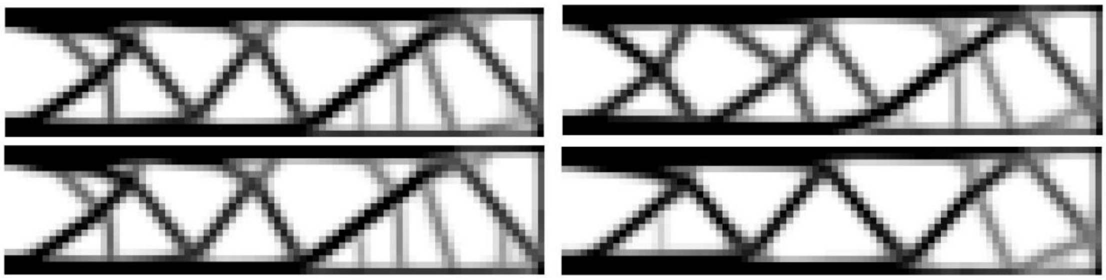

Figure 12: solution to 9 load cases problem provided by the strategies of 1LC (top-left), 2LC (top-right), Mult-LC (bottom-left) and All-LC (bottom-right) – Modified Stop criteria

5 CONCLUSION

This paper investigated a new methodology to deal with multiple load case problems in topology optimization. Instead of either running all the load cases or selecting a few ones, it is proposed to identify the critical load cases for each region of the problem. Compliance is thus accounted individually at each element for the critical load cases only, reducing the computational cost. Comparing to the typical approach [2], it presents a faster convergence without weight penalty.

The instability of the results has not taken into significant changes in the structure geometric shape and has been addressed by adequate stop criteria. Implementing other filtering techniques, such as density filtering [6, 3, 7], shall reduce the regions of gray density.

Reference [3] may be a starting point for computational improvements in algorithms. This associated with the implementation of functions of maximum and ordering more effective than the one embedded in Matlab will boost the reduction in computational cost.

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