Topological Optimization and Control for Reducing Vibration in a Beam Using Piezoelectric Material

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ABSTRACT

The growing need for lighter and adaptable structures especially in applications such as aerospace, automotive and robotics show the importance of advanced methods for structural optimization and active control. This paper shows the development of an optimal design of smart structures, with the aim of controlling the vibrations induced by external forces. In this study, the structural topology and the location of actuators are conducted simultaneously, for non-piezoelectric and piezoelectric material. A short cantilever beam is modeled by solid finite element. The formulation seeks for structural optimization by minimizing the compliance; control optimization is accomplished through the maximization of a controllability measure and the parameters of LQR controller. Numerical examples show that the optimal topologies found can be capable of improving the active damping of the structure.

Keywords: simultaneous design; topology optimization; vibration control; piezoelectricity.

1. INTRODUCTION

With the evolution has increased the need for lighter structures and adaptable, these structures require the development of methods designed to add value, knowledge, technology, among others, or also decrease its manufacturing costs.

Smart Structures employ three basic elements, sensors that record information internal and external, actuators that perform work or apply forces and control systems that make decisions. These structures have numerous applications, for example in spacecraft, aircraft, automobiles, ships, robots, equipped with sensors and actuators that allow reaction to changes proactively.

The topology optimization method contributes positively in the design of structures, reducing material usage and thereby reducing costs which is important even in relation to environmental sustainability. Therefore, it is important to study management techniques appropriate for lighter structures are subject to present problems of excessive vibration.

A material used as sensor/actuator is a piezoelectric material that can act both as a sensor and as actuator using the piezoelectric effect. The piezoelectric effect is a form of electromechanical coupling in which some crystals generate electrical charge when deformed, or deform when subjected to the action of an electric field.

Some papers study the modeling and optimal location of actuators and sensors in smart structures and vibration control these structures. [1], presented a study of the positioning of piezoelectric actuators in smart structures using measures of modal controllability and space, obtained by finite element method and singular values. These values are used to obtain an index that quantifies the controllability of the system so as to position the actuator while minimizing the effort of the driver. [2], present analytical and experimental results on the optimal placement of piezoelectric actuators for the control of beam structures in order to determine the voltage (Volt) that minimizes the error between the desired shape and form obtained using the model Euler-Bernoulli to predict the deformation of the beam.

[3], considered the location of pairs (actuator-sensor) piezoelectric, placed in a flexible beam using a model LQR with the objective of finding the optimal location of the actuator-sensor pair, in this paper were used genetic algorithms to minimize the index LQR performance. [4] and [5], present a simultaneous design for structural topology and the location of actuators. The topology optimization problem is formulated for three material phases (two solid and one empty), the isotropic elastic material no piezoelectric form the structural part, while a piezoelectric material form the active part. It was proposed a approach nested solution, where the locations of actuators are considered as main subprocess optimization, where main loop optimization decides where must be solid material and where must be empty spaces, and then, the subprocess defines where the material must have piezoelectric properties by optimizing a control law.

[6], presented a numerical study on the active vibration control beams with piezoelectric material. Where it was realized a comparison between classical control strategies (constant gain and velocity feedback) and optimal control strategy (LQR and LQG) to investigate its effectiveness by acting as sensors and actuators.

Since what was exposed this work aims to develop a simultaneous design of structural topology optimization and vibration control using piezoelectric actuators, therefore, it's proposed a methodology for determining the matrices Q and R used in optimal control LQR.

2. F INITE ELEMENT FORMULATION

The finite element method (FEM) consists of various numeric methods that divide a geometry into in smaller elements which the approximation of the exact solution can be obtained by interpolating an approximate solution. Furthermore, one advantage that's can used in structures with more complex geometry.

The behavior of sensors and actuators made of piezoelectric materials can be modeled by the following constitutive equations:

$$T = [c^E]S - [e]^T E \tag{1}$$

$$D = [e]S - [\epsilon^S]E \tag{2}$$

where T and S are the vectors of mechanical stress and strain, E and D are the vectors of field and electric displacement, $[c^E]$, $[\epsilon^S]$ e $[e]$ are matrices of elastic, dielectric and piezoelectric coupling coefficients, respectively. The piezoelectric matrix $[e]$ considers the piezoelectric effect, i.e., the coupling between mechanical and electrical fields [7].

The Hamilton variational principle can be used to establish the equations of finite elements for piezoelectric structures. Therefore, the overall balance equation that governs a structural system can be written as follows:

$$\begin{bmatrix} M_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{\emptyset} \end{bmatrix} + \begin{bmatrix} K_{uu} & K_{u\emptyset} \\ K_{u\emptyset}^T & K_{\emptyset\emptyset} \end{bmatrix} \begin{bmatrix} u \\ \emptyset \end{bmatrix} = \begin{bmatrix} f \\ q \end{bmatrix} \tag{3}$$

where u is the structural displacement, $\emptyset$ is the electric potential, and two points above the variable represents the time derivative, M_{uu} is the structural mass matrix, K_{uu} is the structural stiffness matrix, $K_{u\emptyset}$ is the piezoelectric coupling matrix, $K_{\emptyset\emptyset}$ is the dielectric matrix, f is the structural force vector e q is the electric of load vector.

According to [8], the degrees of freedom can be divided into degrees of freedom in the electrode potential at the grounded electrode and internal electrical degrees of freedom, $\emptyset_p$, $\emptyset_g$ e $\emptyset_i$, respectively. Thus, the potential of grounded electrodes are canceled, and the internal electrical degrees of freedom are condensed, so we get:

$$\begin{bmatrix} M_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{\emptyset}_e \end{bmatrix} + \begin{bmatrix} H_{uu} & H_{up} \\ H_{up}^T & H_{pp} \end{bmatrix} \begin{bmatrix} u \\ \emptyset_e \end{bmatrix} - \begin{bmatrix} f \\ q_e \end{bmatrix} \tag{4}$$

where H_{uu} is condensed stiffness matrix, H_{up} is the condensed and mapped piezoelectric coupling matrix and H_{pp} is condensed and mapped of electrical capacitance matrix, the 'e' subscript refers to the potential values to the electrodes.

2.1 Free vibration and modal analysis

There are two possible configurations for the electrodes on the faces of the structure with piezoelectric material in modal analysis, known as short circuit and open circuit. Both atarting the same principle, however, the initial equation differs by structural movement. Considering a harmonic excitation, where $u = u_o e^{jwt}$ and $\emptyset_e = \emptyset_o e^{jwt}$, the system of equations 4 becomes:

$$\begin{bmatrix} H_{uu} - \Omega^2 M_{uu} & H_{up} \\ H_{up}^T & H_{pp} \end{bmatrix} \begin{bmatrix} u_o \\ \emptyset_o \end{bmatrix} e^{jwt} = \begin{bmatrix} f_o \\ q_o \end{bmatrix} e^{jwt} \tag{5}$$

where the subscript 'o' refers to the amplitude of the harmonic motion. Considering $q_o = \mathbf{0}$ and taking only the first line we have:

$$(H_{uu} - \Omega^2 M_{uu})u_o = f_o - H_{up}\phi_o \tag{6}$$

Isolating ϕ_o the second line can be condensed by replacing ϕ_o the first line gives the generalized eigenvalue problem.

$$(H_{uu} - H_{up}H_{pp}^{-1}H_{up}^T)\Psi = \Omega^2 M_{uu}\Psi \tag{7}$$

where Ψ are the vibration modes and Ω are the natural frequencies modal corresponding to configuration open circuit. In the configuration with grounded electrodes $(\emptyset_o = 0)$, known as short circuit configuration, the generalized problem of eigenvalues is given by equation 8.

$$H_{uu}\Psi = \Omega^2 M_{uu} \tag{8}$$

2.2 Model in state space

As presented in the previous section, the degrees of freedom in the electrode had grounded electrical potential zero and weren't considered further degrees of freedom internal electrical condensates were statically and will be used as inputs to the actuator control system [5], thus, isolating the first line of Equation 4 has equation 9, while the second equation of equation 4 can be used to determine the electrical load on the electrodes.

$$M_{uu}\ddot{u} + H_{uu}u = -H_{up}\emptyset_e + f \tag{9}$$

When a structure in this vibration presents the vibration modes dependent excitation spectrum, therefore, it's assumed that the lowest order modes have a lowest energy, i.e., are more easily excited. A truncated modal matrix can be used in the processing of generalized coordinates u to modal coordinates η, because, when a structure is in vibrating shows the vibration modes dependent excitation spectrum, with that, it's assumed that the lower order modes has the lowest energy, ie are more easily excited. Thus, the displacement vector can be approximated by the superposition of the m first modes, by:

$$u = \Psi\eta \cong \sum_{i=1}^{m} \Psi_i \eta_i \tag{10}$$

where Ψ is the truncated modal matrix and η is the modal of coordinates vector. Considering a simple model of viscous damping and the m first vibration modes is represented by equation 9:

$$\ddot{\eta} + 2Z\Omega\dot{\eta} + \Omega^2\eta = B_m u^c + f_m \tag{11}$$

where $\boldsymbol{Z}$ and $\boldsymbol{\Omega}$ are diagonal modal damping and natural frequency matrices, respectively. For the model truncated in state variables the modal displacements and velocities are given by:

$$x = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} \eta \\ \dot{\eta} \end{Bmatrix} \tag{12}$$

Therefore, the dynamical system in state space is described by a first order differential equation, ignoring the observation, we have:

$$\dot{x} = Ax + B_{\phi}u_{\phi}^{c} + B_{u}u_{u}^{c} \tag{13}$$

where $\boldsymbol{A}$ is the system matrix, $\boldsymbol{B}_{\phi}$ and $\boldsymbol{B}_{u}$ are electrical and mechanical of inputs matrices and u_{ϕ}^{c} and u_{u}^{c} are the electrical and mechanical of inputs vectors, given by equation 14.

$$A = \begin{bmatrix} \mathbf{0} & I \\ -\Omega^2 & -2Z\Omega \end{bmatrix}, \quad B_{\phi} = \begin{bmatrix} \mathbf{0} \\ \Psi^T H_{up} \end{bmatrix}, \quad B_u = \begin{bmatrix} \mathbf{0} \\ \Psi^T \end{bmatrix}, \tag{14}$$

$$u_{\phi}^{c} = \phi \quad \text{e} \quad u_{u}^{c} = f$$

3. OPTIMAL CONTROL LQR DESIGN

The use of a controller is required for a device that has an ideal behavior, i.e., the controller must manage and to ensure the dynamic behavior. According [6], the feedback gains are chosen to change the dynamics of the system, aiming to reduce the motion of the mechanical system, acting as a regulator.

The LQR method is based on the minimization of a quadratic performance index that is associated with the energy of the state variables and control signals. The goal of LQR controller design is to establish a compromise between the energy state and control by minimizing a cost function defined by the equation 15.

$$J = \int_0^{t_f} \left(\boldsymbol{x}^T Q x + \boldsymbol{u}_{\phi}^{c\,T} \boldsymbol{R} \boldsymbol{u}_{\phi}^{c} \right) dt \tag{15}$$

where $\boldsymbol{Q}$ is a real symmetric or Hermitian matrix positive definite or positive semidefinite, expresses the weight of the state variables and $\boldsymbol{R}$ is a Hermitian or real symmetric positive definite matrix and express energy expenditure derived control signal [9]. It is assumed in this problem, the control vector $u(t)$ is unrestricted. A well-constructed project must take into account a consistent choice for these matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$, where a widely used choice is the identity matrix, or a multiple thereof.

As [10], the linear control law given by equation 16 is the optimal control law. Consequently, if the matrix of elements $\boldsymbol{G}$ are determined to minimize the performance index, then $u(t)$ is great for whatever the initial state $x(0)$.

$$u = -Gx \tag{16}$$

The optimal gain matrix is given by $G = -R^{-1}B_{\phi}^{T}P$, where P is the solution of the Riccati equation given by:

$$A^{T}P + PA - PB_{\phi}R^{-1}B_{\phi}^{T}P + Q = 0 \tag{17}$$

Considering the state feedback and the feedback gains matrix the equation of state in open-loop is given for equation 18.

$$\dot{x} = \left(A - B_{\phi}G\right)x + B_{u}u_{u}^{c} \tag{18}$$

As a hypothesis, it's assumed that all states are completely observable and are related with the outputs [11].

4. SIMULTANEOUS DESIGN OF STRUCTURAL TOPOLOGY OPTIMIZATION AND CONTROL FOR VIBRATION REDUCTION

Optimization is defined as a set of procedures that aims to minimize or maximize a particular function, known as objective function, subject or not to restrictions, thus obtaining a better use of available resources. A common practice in projects indicate sequence, however some authors [5], [12] argue that a combined project obtains better results than the traditional sequential design.

Therefore the formulation for the simultaneous optimization project according to [5], can be formulated as follows:

$$\begin{aligned} &\min_{p_c} \; \alpha f_e(p_e, p_c) + \beta f_c(p_e, p_c), \\ &\quad sujeito\ a \begin{cases} g(p_e, p_c) \leq 0, \\ h(p_e, p_c) = 0, \end{cases} \end{aligned} \tag{19}$$

where α and β are respectively the weights weighted for the structural and control function, f_e is the structural objective function, f_c is the control objective function, p_e is the structural variable design, p_c is the system design control design variable, g is the restriction of inequality and h is the equality restriction. According [13], the advantage of this formulation is that minimization of the sum of two objectives separate is always less or equal than the sum of two objective individually, which makes the simultaneous optimization most advantageous.

This work follows the one proposed by [5], was presented a simultaneous structural design optimization and control, this procedure uses the method of topological optimization to design the structure by minimizing flexibility with volume restrictions through topology optimization, secondly, it seeks to maximize the trace of gramiano controllability of the control LQR system for optimal location of actuators.

Even the two stages being resolved separately and in sequence, there is interaction between the two objective functions characterizing a simultaneous design. Thus, the project is set as a linear programming problem where a cost function is minimized with respect to structural parameters p_e and control p_c and all minimizations are able to find their optimal global [5], thus, minimizing nested is given by:

$$\min_{p_e}\min_{p_c} f(p_e, p_c) \tag{20}$$

What according [14], makes optimization of control a subprocess of the nested optimization. Can be written as follows:

$$\min_{p_e} \square f_1(p_e). \tag{21}$$

where

$$f_1(p_e) \triangleq \min_{p_c} f_2(p_e, p_c). \tag{22}$$

while the equation 21 refers to structure optimization, the equation 22, which is a sub-optimization, relates to the optimization of control, i.e., always that the structural variables are changed the subprocess is modified.

In this article, structural optimization seeks to minimize the mean flexibility, i.e., where the structure should be put solid, elastic or piezoelectric, and where shouldn't have material. For this model material for the case optimization with optimum location of piezoelectric material is given by equation 23.

$$\begin{gathered}
[c^E] = p_e^{p1}\left(p_c^{p2}\left[c_{pzt}^E\right] + (1 - p_c^{p2})\left[c_{elas}^E\right]\right). \\
[\epsilon^S] = p_e^{p1} p_c^{p2}\left[\epsilon_{pzt}^S\right]. \\
[e] = p_e^{p1} p_c^{p2}[[e]_{pzt}]. \\
\gamma = p_e\left(p_c\gamma_{pzt} + (1 - p_c)\gamma_{elas}\right).
\end{gathered} \tag{23}$$

where the effective properties of the material interpolated are: $[c^E]$ which defines the elastic properties, $[\epsilon^S]$ which defines the piezoelectric of coupling properties, [e] that defines the dielectric properties and γ is the specific weight. $[c_{elas}^E]$, $[c_{pzt}^E]$ are the elastic properties of the material non-piezoelectric and piezoelectric, respectively. $[\epsilon_{pzt}^S]$, $[[e]_{pzt}]$ are the dielectric properties and electromechanical coupling of piezoelectric material properties, respectively.

The minimization of flexibility that is proportional to the work of external forces is the structural objective function given by equation 24.

$$W(p_e, p_c) = f_e = \mathbf{f}^T\mathbf{u} \tag{24}$$

where $\mathbf{f}$ and $\mathbf{u}$ are the vectors of static forces and the displacements global of the structure, respectively. Even the function of flexibility depending on the design variable p_c the project includes only the minimization of flexibility, therefore it can be written as:

$$\min_{p_e} f_e(p_e, p_c)$$
$$sujeito\ a \begin{cases} 0 < p_{ei} \leq 1, & (i = 1,2,\dots,N_e), \\ V_e = \dfrac{\int_\Omega p_{ei}\, d\Omega}{\int_\Omega d\Omega} \leq V_e^{max}, \end{cases} \tag{25}$$

where p_{ei} is the component for the ith element and N_e is the number of structural design variable, which is equal to the number of finite elements. The second constraint limits the total volume of material at a volume fraction V_e^{max} predetermined.

In the second phase, the control system is designed with an objective function that maximizes the trace of Gramiano controllability LQR control system, the index great is:

$$f_s(p_e, p_c) = f_c = tr(W_c) \tag{26}$$

where W_c is the controllability of Gramiano, before that, the control optimization is written as follows:

$$\max_{p_c} f_c(p_e, p_c)$$
$$sujeito\ a \begin{cases} 0 < p_{ci} \leq 1, & (i = 1,2,\dots,N_c), \\ V_c = \dfrac{\int_\Omega p_{ci}\, d\Omega}{\int_\Omega d\Omega} \leq V_c^{max}, \end{cases} \tag{27}$$

where p_{ci} is the i-th component of the vector of design variables control, N_c is the number of design variables that control equals the number of finite elements, and the second constraint limits the total volume of piezoelectric material at a fraction of the volume V_c^{max} preset.

The sequential mathematical programming method (SLP) is used to solve optimization problems, which requires the sensitivities of the objective function and constraints regarding the design variables. The sensitivities of the material, flexibility, controllability and Gramiano the eigenvalues and eigenvectors model are calculated analytically as [5].

5. METHODOLOGY FOR CHOICE OF MATRICE Q AND R

One issue that contributes to the effectiveness of the LQR controller is the correct determination of the weighting of state matrices Q and weighting of control matrices R seeking to satisfy certain conditions of controller design and to implementation limitations. The determination of these matrices has a direct influence on the calculation of the gain. A reduction arbitrarily fast state can be achieved at the expense of an increase in control signal employed, implying in some cases the practical impossibility to implement such a solution. On the other hand, an arbitrarily large reduction in the control can cause a increase large of the state, situation often undesirable in certain control processes. Soon, we want to determine values that best meet certain criteria, such as: percentage of response, maximum stabilization control, stabilization of time, that when achieved reflect a better system performance, aiming at a value of commitment to reality practical problem.

In this work was developed a methodology based on a scan between Q and R values predetermined, generating a map of compromise that takes into account the settling time of the system versus the amount of energy used in controlling which includes the following steps:

1. Choose values for scanning Q and R.
2. Reading the values of Q and R defined in stage one (1), each possible combination is used in the dynamic system and used to control LQR.
3. Identification of the control signal used for each of the combinations between the values of Q and R.
4. Presentation of the map with all possible combinations of the values of Q and R, presentation control signal and the settling time of the system in each case.

Based on these stage it is possible to have performance factors generate compromise between the value of the control signal used for fixing the system and the time required for such establishment. As the map generated in stage 4 is possible to see the result of the methodology and choose which is the best set of values of Q and R to the specific needs of the problem as: settling time, actuator limits for employee for control, among others. To the scan the values, we used the range of $R = 0.05{:}0.05{:}1$, (ranging from 0.05 to 1 at an interval of 0.05), while for the values of $Q = 10^{(n-1)}$ (with $n = 1{,}2\ \dots\ 20$).

6. RESULTS

To verify the performance of the proposed project was implemented in MATLAB in a beam balance with measures 210 mm x 70 mm x 20 mm (Figure 1). The structures are discretized in 294 (21 x 7 x 2) solid isoparametric finite elements of 8 nodes with three degrees of freedom mechanical and electrical degree of freedom per node. The value of static force is 1000 N and was distributed in the three end nodes of the beam. The values considered for the modal damping are 1.71%, 0.72%, 0.42% and 0.41% [6]. For optimization design the volume constraints are equal the 50% and 5% for the restricting total volume and volume restriction piezoelectric, respectively. The pseudo densities are uniform for all elements and equal to $p_e = 0.40$ and $p_c = 0.04$, taking as stopping criterion the minimum number of 40 iterations and the optimization process should stop when the change of design variables from one iteration to another is less to 4%. To the sensitivity filter was considered a radius of filtering of $r_1\text{min}\uparrow e = 14.2\ mm$ for structural analysis and for control analysis a radius of filtering of $r_1\text{min}\uparrow c = 5\ mm$.

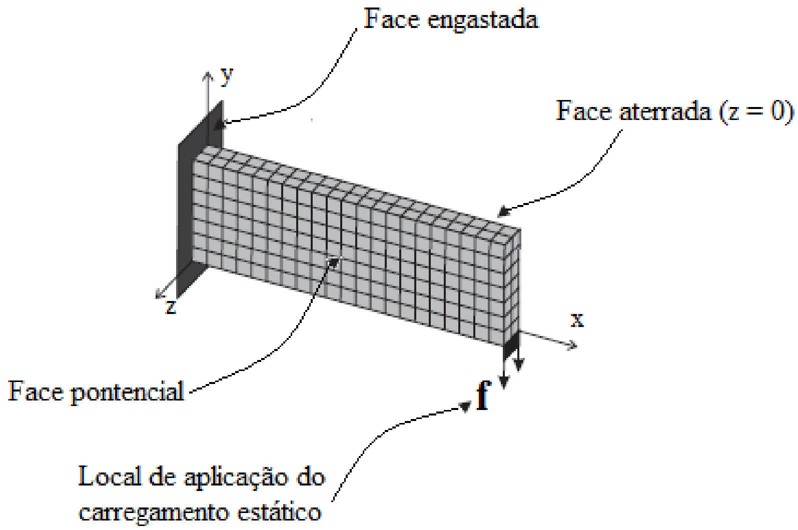


Figure 1 - Cantilever beam discretized in 294 solid isoparametric finite elements.

The values of the elastic constants, piezoelectric and dielectric are obtained from the work [15], [16], and Morgan Electro Ceramics Web site and can be seen in Table 1.

Table 1 - Properties of materials.

PZT5A	
Constantes elásticas	(10^{10} N/m^2)
c_{11}^E	12.1
c_{12}^E	7.54
c_{13}^E	7.52
c_{33}^E	11.1
c_{44}^E	2.11
c_{66}^E	2.26
Constantes piezelétricas	(C/m^2)
d_{31}	-171
d_{33}	374
d_{51}	584
Constantes dielétricas	(F/m)
ϵ_0	8.85 x 10^{-12}
$\frac{\epsilon_{31}}{\epsilon_0}$	916
$\frac{\epsilon_{33}}{\epsilon_0}$	830

Densidade	7750 kg/m^3
ALUMÍNIO	
Módulo de elasticidade	71 x 10^9 **N**/m^2
Densidade	2700 kg/m^3
Coeficiente de Poisson	0.33

The optimal topologies for distribution of solid material (isotropic elastic or piezoelectric) and the distribution of piezoelectric material analyzing the first, second and fourth vibration modes are shown in Figure 2.

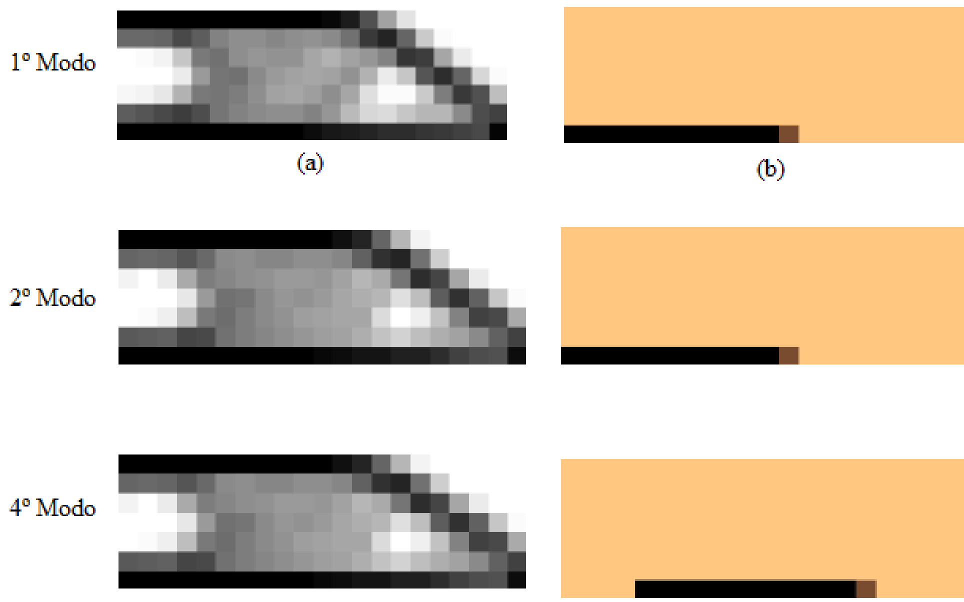


Figure 2: Optimal topologies for distribution of material solid (a) and distribution of piezoelectric material (b) for 1st, 2nd and 4th vibration modes.

Where it can be seen that the distribution of piezoelectric material for the first two vibration modes occurs closest to end (left side of figure) and in the middle of the beam when the fourth mode is analyzed.

For the responses of the optimized structures the a impulsive load on the tip of the beam was applied the methodology described in-section (5), which aims to determine the best combination of Q and R that satisfies the criteria of maximum tension or settling time. The Figure 3 shows the map without considers the voltage limits, but rather, all possible combinations of Q and R.

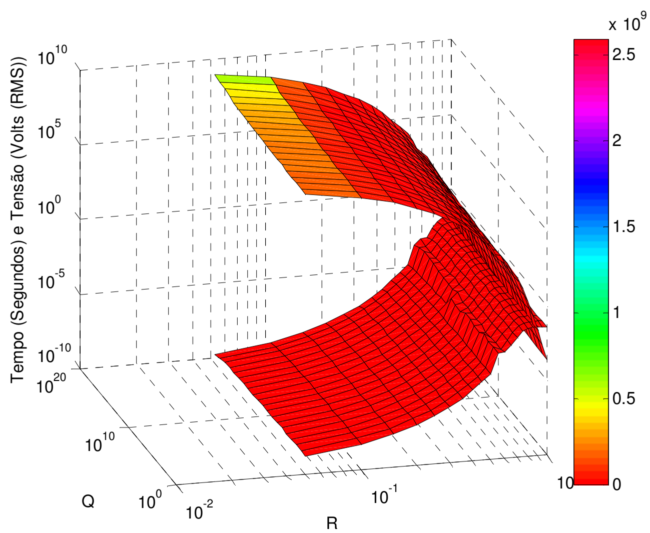


Figure 3: Map of possibilities for the parameters Q and R.

The Figure 4 shows the map of the combinations (Q and R) and corresponding control signals and settling time for a maximum voltage of 1500 Volts. Where the values can be determined matrices Q and R which most closely match the maximum voltage.

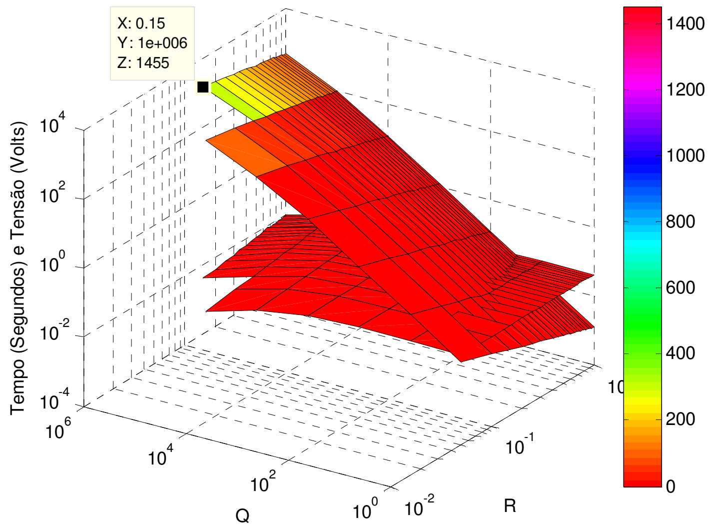


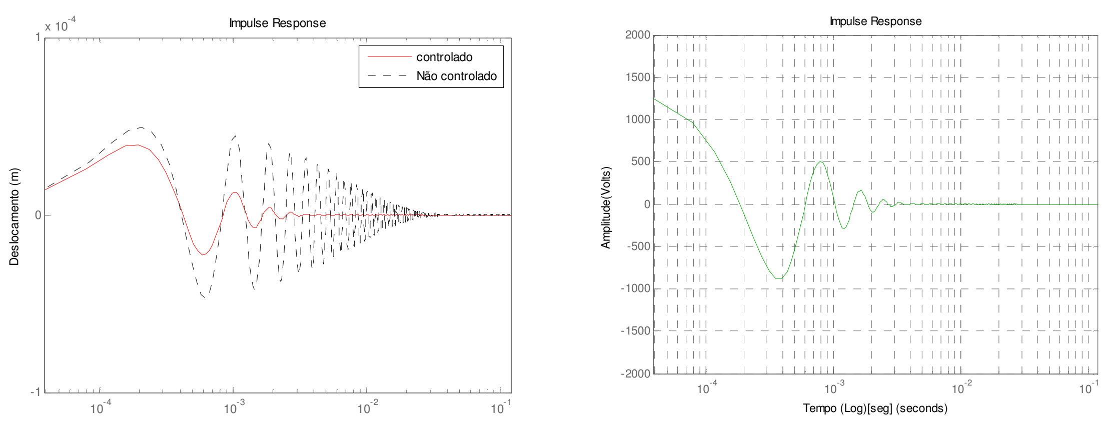
Figure 4: Map of the possibilities of the parameters Q and R with maximum voltage of 1500 Volts.

As can be seen in Figures 3 and 4 the higher the value of the control signal is low the time required for stabilizing the movement. As for the slightest sign of control, time to stabilization of the system increases. Looking at Figure 4 the values for the matrices of weights that came over the control signal usage limit actuator are given by:

$$R - 0.15$$

$$Q = 10^6.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The Figure 5 shows the results for the displacement and voltage to the control model.



(a) (b)

Figure 5: Response to a impulse load: (a) displacement, (b) control signal.

As can be seen in Figure 5 (a), the LQR control system closed loop (red line) significantly contributes to attenuate the displacement at the tip of the beam in Figure 5 (b) can observe the respective control signal is within the limits established in the methodology.

7. CONCLUSION

This paper presented a simultaneous design of structural topology optimization and vibration control using piezoelectric material. Furthermore, presented a methodology for determining the best matrices Q and R which influence the LQR controller gain, i.e., the energy spent in the process control and the settling time of the system.

It was presented results that show the optimal topologies found, as well, results that show the importance of the correct determination of the matrices Q and R, which influence the gain LQR, which alters the damping capacity of structures by using piezoelectric actuators.

Thus, using as base which was developed can be suggested for further work in the inclusion of sensors and study design concepts of observability, also the analyze other control model, for example, the LQG (Linear Controller quadratic Gaussian).

8. ACKNOWLEDGMENT

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