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KINEMATIC CONTROL OF MOBILE ROBOTS TO PRODUCE CHAOTIC TRAJECTORIES

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Abstract. *This article introduces a path-planning strategy concerning mobile robots' missions for terrain exploration, with specific purposes of search or surveillance. The proposed method to achieve fast and complete scanning of the entire robot workspace consists of imparting chaotic motion behavior to the mobile robot using a path-planner based on the Standard map. This strategy can ensure high unpredictability of robot trajectories, resembling a non-planned motion for external observers. The kinematic modeling of a mobile robot, and two closed-loop locomotion control schemes (continuous and discontinuous) are described, as well the proposed strategy of Standard map-based path-planning. Results and analysis of numerical simulations, testing the robot kinematic control and the path-planning procedure, close the article.*

Keywords*: mobile robots, kinematic control, nonlinear systems, chaos*

1. Introduction

Mobile robotics, after decades of important developments, stays as an interesting research issue in consequence of its ever-increasing applications on different domains, and the relevance of its economic and technological impacts. Motion control of robot locomotion, using diverse strategies, is no longer a restrictive difficulty. Some recent studies focus on dealing with complex missions and tasks that these machines are capable to accomplish.

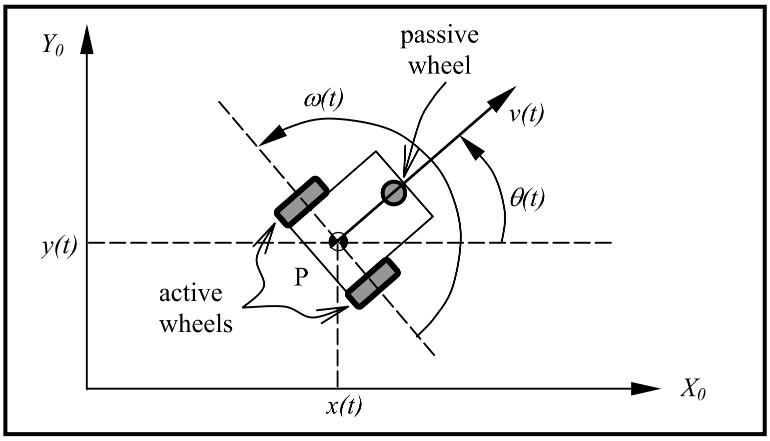
This work is interested on the specific problem of terrain exploration with search or vigilance goals. In this type of missions, in addition to regular navigation competencies (planning and reacting), complementary features like high unpredictability of motion trajectories and fast scanning of the entire workspace are strongly suitable. Chaotic behavior, typical of a class of nonlinear dynamical systems, can guarantee an unpredictable robot motion that scans the whole connected workspace, without terrain map requirements.

In Nakamura & Sekiguchi (2001), integration between the robot motion system and a chaotic system, the Arnold dynamical system, is used to impart a chaotic behavior to the robot. In Martins-Filho et al. (2004), an open-loop control approach is proposed to produce unpredictable trajectories, using state variables of the Lorenz chaotic system to command the robot wheels velocities. Conversely, this article proposes a path-planning strategy on a closed-loop locomotion control scheme to produce trajectories by point-to-point path following, i.e. the design and the execution of trajectories that will cause the robot to reach a sequence of partial targets' locations. The partial targets' are defined in real-time by an auxiliary system based on chaos theory. Other articles connecting chaos theory and mobile robotics discuss the identification of chaos dynamics on robot behavior as an effect of interactions with environment and of the control structure (Islam & Murase, 2005; Nehmzow, 2003; Nehmzow, 2004).

2. Kinematic control

The mobile robot considered in this work is a typical differential motion robot with two degrees-of-freedom, composed by two active, parallel and independent wheels, a third passive wheel with exclusively equilibrium functions (a sort of free steered standard wheel), and proximity sensors capable of obstacles detection. The active wheels are independently controlled on velocity and rotation sense. The sensors provide short-range distances to obstacles. For instance, these sensors can be infrared devices commonly used in mobile robots, with adequate accuracy. Additionally, the robot is supposed to be equipped with specific sensors for detection and recognition of searched objects.

The robot chassis is considered as a rigid body operating on a horizontal plane, and its motion is obtained by driving the active wheels. The resultant motion is described in terms of linear velocity $v(t)$ and direction $\theta(t)$, describing an instantaneous linear motion of the medium point of the wheel axis and a rotational motion (rotational velocity $\omega(t)$) of the robot body over this same point. The geometry of this motion scheme is shown in Fig. (1).



Figure(1): Geometry of the robot motion on Cartesian plane.

The robot motion control can be done providing the wheels velocities, $\omega_l(t)$ and $\omega_r(t)$ or, equivalently, $v(t)$ and $\omega(t)$, called input or control variables. The mathematical model of this kinematic problem considers these two control variables and three state variables: the robot position and orientation ($x(t)$, $y(t)$, $\theta(t)$):

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{1}$$

This class of systems, nonholonomic one, is not obvious to control, however it has been studied deeply by various research groups. For instance, interesting and adequate solutions are available in Astolfi (1995), Campion et al. (1996), Canudas de Wit & Sordalen (1993), Sordalen & Canudas de Wit (1993), and Lee et al. (2000).

2.1. Feedback control laws

The motion control adopted in this work involves a real-state feedback controller, an appropriate approach to produce a desired trajectory described by a sequence of coordinates (x,y). It means that the path-planning task is given by a specialized robot module, independent of the motion control module, that sets intermediate positions lying on the requested path.

The control law, proposed in Lee et al. (2000), considers the geometric situation shown in Fig. (2). The robot is placed at an arbitrary configuration (position and orientation), and a desired position is defined by the robot path-planner. In the robot reference frame (X_RY_R), the configuration error vector is $e=[\rho \;\; \varphi]^T$, where ρ and φ define the target coordinates.

The control design problem can be stated as to find a control gain K_c, constant or not, to provide the control action in terms of the error vector, and that is capable to stabilize the closed-loop motion system, i.e. this feedback makes error go asymptotically to zero.

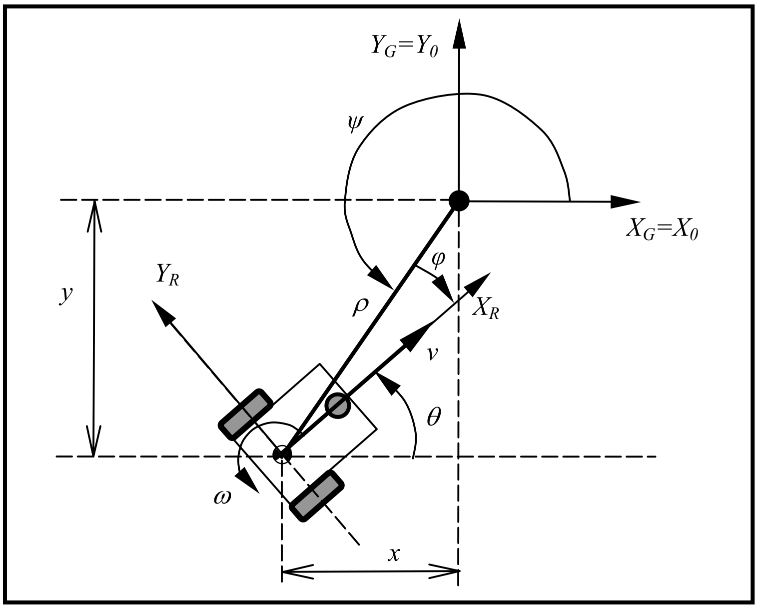


Figure (2): Configuration of the kinematic control.

The robot kinematic model is described by Eq. (1), where *dx(t)/dt* and *dy(t)/dt*, the linear velocity components on absolute reference frame (fixed on the workspace). We define the angle φ between the X_R axis of body reference frame and the vector connecting the robot center and the desired position. The other configuration variables, ρ and ψ, describe respectively the distance between present and desired positions, and the angle between the direction to the target and the axis X_0. Considering a coordinates change defined by

$$\begin{aligned} \rho &= \sqrt{\Delta x^2 + \Delta y^2} \\ \varphi &= 180 + \theta - \psi \end{aligned} \tag{2}$$

the description of the motion in the new coordinates become

$$\begin{bmatrix} \dot{\rho} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} -\cos\varphi & 0 \\ \frac{1}{\rho} sin\varphi & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{3}$$

Concerning these polar coordinates system descriptions, it's necessary to remark that the coordinate transformation is not defined at $x = y = 0$. The proposed control law defines a feedback to determine the system inputs, v and ω:

$$v = k_1 \rho \cos\varphi \quad ; \quad \omega = -k_1 sin\varphi \cos\varphi - k_2 \varphi \tag{4}$$

It can be shown that this control law stabilizes the systems, i.e. leads its state variables to the origin. The details of this prove, based on Lyapunov functions, can be found in Lee et al. (2000).

A second kinematic control is based on a very simple and discontinuous control scheme. In this scheme, the robot executes two phases control action. The first one consists of an exclusive rotation motion, with constant angular velocity about its own center, to point the robot straightforward to the next target location:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \tag{5}$$

Completed the robot-pointing phase, the robot can execute a straight trajectory with constant velocity toward the desired position:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta.v \\ sin\theta.v \\ 0 \end{bmatrix} \tag{6}$$

Evidently, if an obstacle is found on this trajectory, a specific navigation competency (obstacle avoidance) is necessary to accomplish this locomotion task. In this work, the obstacle avoidance problem is not treated, nevertheless a simple solution can be easily implemented, like the BUG2 algorithm (Lumelsky & Skews, 1990): the robot follows the obstacle contour, but departs immediately when it is able to move directly toward the target.

Using these two different control laws, we intend to validate the proposed path planning, testing the terrain covering by the robot motion, and examining the geometric characteristics of trajectories.

3. Chaotic trajectories

Deterministic chaos is a dynamical behavior of a nonlinear systems class with special features: (i) unpredictability, i.e. the knowledge of the system state during an arbitrary time does not allow to predict its posterior trajectory, it's a consequence of the initial conditions dependence or sensitivity; (ii) continuous frequencies spectrum, that characterizes a non periodic behavior; (iii) nevertheless non periodic, the trajectories pattern tends to repetition.

The dimension of phase space condition for deterministic chaos occurrence is dimension ≥ 3, and a robot motion trajectory executed on a flat terrain is defined on a 2D plane. Consequently, a chaotic system cannot directly provide a robot path planning. We propose a method based on Poincaré sections. A Poincaré section defines a phase space with arbitrary dimension, and composes a map with chaotic behavior. Differential equations generate a discrete map through its flow intercepting a Poincaré section (or return map) at each unity of time.

3.1. The Standard map

The Standard or Taylor-Chirikov map is a family of area-preserving maps, $z_{n+1} = f(z_n, z_{n+1})$ with $z = (x, y)$, given by

$$\begin{aligned} x_{n+1} &= x_n + K.sin(y_n) \\ y_{n+1} &= y_n + x_{n+1} \end{aligned} \tag{7}$$

where x is a periodic configuration variable, and y is the momentum variable, usually computed $mod(2\pi)$. The map has a single parameter K that represents the strength of the nonlinear kick. This map was proposed by Bryan Taylor and then independently obtained by Boris Chirikov to describe the dynamics of magnetic field lines on a kicked rotor (Lichtenberg & Lieberman, 1983).

4. Results of numerical simulations

In the first step of the proposed strategy validations, we simulated the Standard map to verify if its covering properties satisfy the mission request of fast and complete scan of the considered terrain. We define a square terrain with dimensions 200m × 200m. The result of this scan simulation begins with an arbitrary initial position. The result of partial goals' locations planned after 100 and 1200 iterations, considering a Standard map with gain value $K=6$, are shown in Fig. (3).

In the following step of validation tests, we simulated the robot motion applying the two closed-loop control laws, continuous and discontinuous laws discussed in Section 2, to track the point-to-point path planned using the Standard map. The robot was commanded to follow the same point-to-point planned path. The results of the application of the continuous control law (with different values of control gains) are shown in Fig. (4) and (5), and the results of discontinuous control law are shown in Fig. (6).

Before analyzing the simulation results, a necessary comment should be placed: the robot can perceive the target (a searched object on a search mission, or an intruder on a patrol mission) inside the sensor range region, the dimensions of this perception field depends on the properties of the device used to perceive external objects. Therefore, the perception field trajectory has a width centered on the robot body trajectory.

Comparing the two control laws, continuous and discontinuous, we can conclude that the first approach presents better results and the effective robot trajectory seems more unpredictable (discontinuous law uses only straight trajectories between the planned points). In addition, a continuous law is easier to implement in terms of experimental realization.

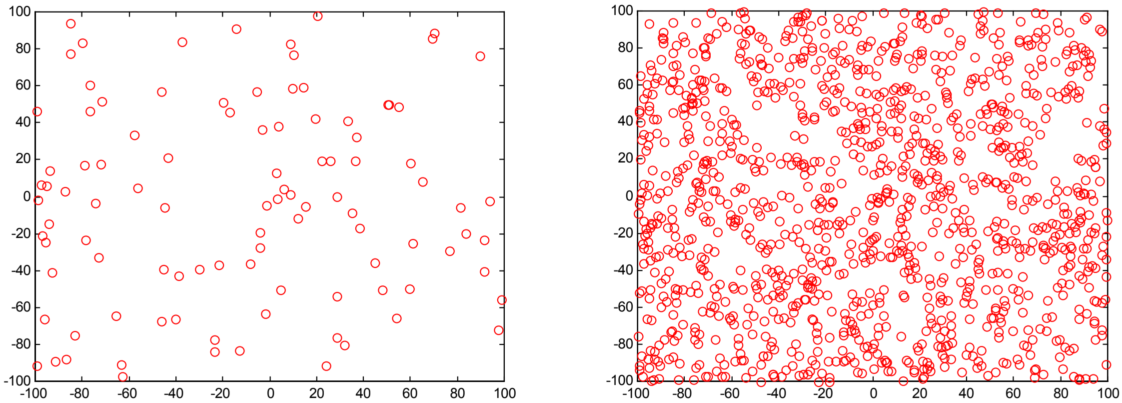


Figure (3): Terrain covering using Standard maps respectively with 100 points and 1200 points.

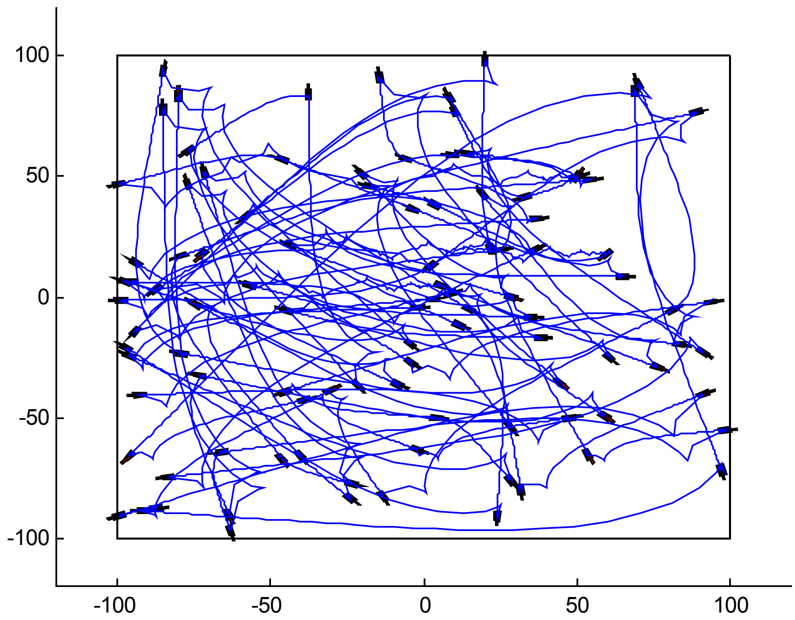


Figure (4): The mobile robot trajectory to achieve 100 planned points using continuous control law (control gains $k_1=1.5$ and $k_2=5.5$).

The proposed strategy can be considered questionable with respect to the terrain covering and unpredictability since the results are very similar to a path generated using uniformly distributed random numbers on the terrain area. Considering the discontinuous control laws, the results can be really similar, however, using the continuous control law, a first advantage can be remarked. As already analyzed in Nakamura & Sekiguchi (2001), the comparison between chaotic and random walk motion showed that the density of resultant trajectory of random walk is lower than that of the chaotic robot because the robot motion is more discontinuous and consequently it must spend more time on repetitive turning maneuvers.

Moreover, we consider that the fundamental difference between random and chaotic strategies is the planned nature of the point-to-point path. Considering the chaotic approach, the robot navigation system maintains complete control and knowledge of the planning process for the reason that it is a deterministic system and the dynamical behavior is precisely defined. In terms of navigation competencies, including aspects of the robot localization function, this path-planning determinism represents an important advantage comparing with navigation based on a sort of random walk point-to-point trajectory.

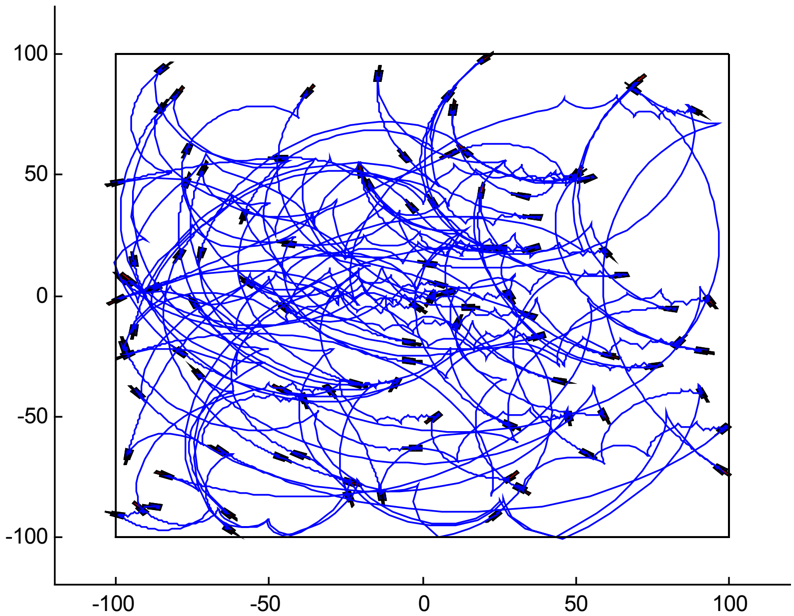


Figure (5): The mobile robot trajectory to achieve 100 planned points using continuous control law (control gains $k_1=1.5$ and $k_2=1.5$).

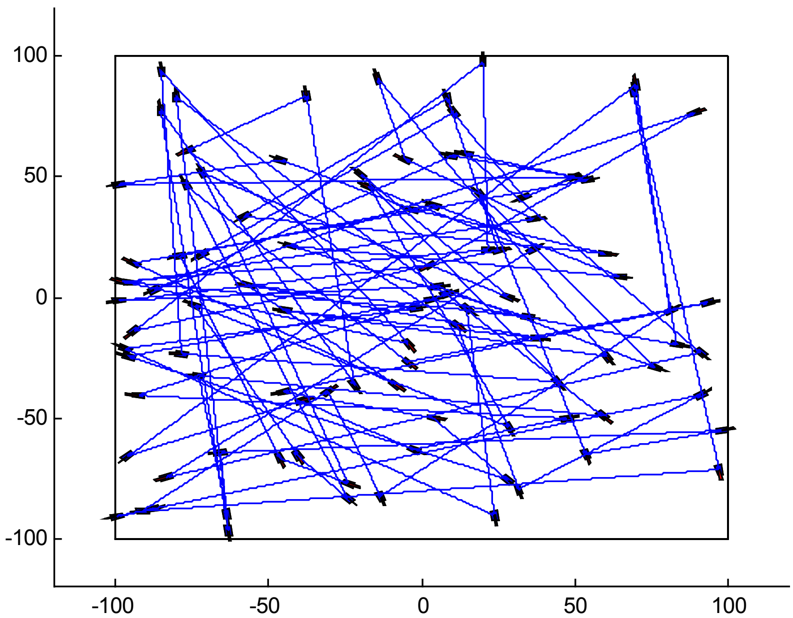


Figure (6): The mobile robot trajectory to achieve 100 planned points using discontinuous control law.

5. Conclusion

This article introduced a strategy to deal with a special type of mission for mobile robots, the terrain exploration with particular purpose of search or surveillance. The proposed method to achieve the main mission requirements consists of a scheme to impart a chaotic motion behavior to the mobile robot using a path-planner based on the Standard map. This map defines a discrete sequence of partial targets' locations or, in other words, a point-to-point robot trajectory. This strategy ensures high unpredictability of robot trajectories, resembling a non-planned motion from external observers point of view.

The kinematic modeling of a mobile robot, and two closed-loop control schemes (continuous and discontinuous) adopted in the work were described in details. A brief discussion about the Standard map was presented to explain the path-planning strategy.

Results of numerical simulations confirm that the chaotic planning procedure, with appropriate gain values, can result in a fast and complete scan of the entire robot workspace. This work shows that the application of dynamical behaviors of nonlinear systems on solutions for mobile robots control problems can represent an interesting interdisciplinary interface for researchers of both scientific domains, with positive perspectives of future works including experimental realizations. This project will continue through studies about geometric properties characteristics of diverse robot chaotic trajectories, and tests concerning the mission efficiency to find arbitrary targets using statistical analysis.

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